Market Timing and Predictability in FX Markets

Thomas A. Maurer* Thuy-Duong Tô† Ngoc-Khanh Tran‡

December 29, 2021

Abstract

We study the economic value of market timing in FX markets, i.e., using information about the conditional Sharpe ratio to adjust the notional value of a conditionally mean-variance efficient currency portfolio. Our strategy trades more (less) aggressively when the conditional risk-return trade-off is more (less) favorable. This leads to a significant improvement in the out-of-sample unconditional Sharpe ratio, skewness and maximum drawdown per 1% expected excess return. The strategy's market timing predicts returns, volatility and skewness in FX markets. Popular currency pricing factors do not explain the strategy's high average excess returns. Our findings suggest that it is costly to impose leverage or risk (i.e., conditional volatility) limits or other inferior market timing policies when constructing currency trading strategies.

JEL-Classification: F31, F37, G11, G12, G15, G17.

Keywords: Market Timing, Leverage Limits, Risk Limits, FX markets, Mean-Variance Optimization, Exchange Rates, Estimation Errors.

*Faculty of Business and Economics, The University of Hong Kong, +852 5236 2121, maurer@hku.hk.

†School of Banking and Finance, UNSW Business School, The University of New South Wales, td.to@unsw.edu.au.

‡Pamplin College of Business, Virginia Tech, nktran7@vt.edu.

1 Introduction

Mean-variance optimized portfolios earn high out-of-sample (OOS) returns in foreign exchange (FX) markets (Baz et al., 2001; Della Corte et al., 2009; Ackermann et al., 2016; Daniel et al., 2017). We investigate the time-series performance of these portfolios and show that the conditional Sharpe ratio and crash risks are time-varying and predictable. A mean-variance optimized currency strategy that exploits this information, referred to as MV hereafter, trades more (less) aggressively when prices of risks in FX markets are higher (lower). This market timing is valuable and significantly improves the performance compared to other mean-variance optimized strategies that use inferior market timing policies. MV has a desirable unconditional return distribution with high average excess returns, a high unconditional Sharpe ratio, positive skewness, and low maximum drawdown per 1% expected excess return.

The performance of MV has important implications for asset allocations, pricing risks, and parameter estimations in FX markets. Regarding asset allocations, our results suggest that leverage or risk limits, (i.e., targeting a constant notional value or volatility) are costly when market conditions are time-varying and market timing is highly profitable. In practice, such limits are often imposed on portfolio managers (in addition to less tight regulatory constraints). Thus, it is important to understand the implicit costs associated with such leverage or risk (i.e., conditional volatility) limits. Regarding the pricing of risks, our paper shows that the high excess returns generated by the market timing of MV cannot be explained by known currency risk factors, indicating the existence of important FX market risk sources that are not yet well understood. Regarding the parameter estimation, our paper shows that a combination of forward discounts (which serve as useful proxies for conditional expected excess returns of currencies) and principal component analysis (PCA, which is useful to obtain a robust estimate of the exchange rate covariance matrix) are key elements for MV's superior OOS performance.

All conditionally mean-variance efficient strategies have proportional portfolio weights, i.e., they invest in the same risky asset portfolio. Accordingly, their conditional Sharpe ratios are identical. However, these strategies generally differ with respect to the asset allocation, i.e., the time-variation in the notional value or leverage. We denote this time-variation as market timing. We analyze the differences in the market timing of popular mean-variance optimized strategies. In FX markets, Baz et al. (2001) minimizes the conditional portfolio variance subject to a constant target expected return. Della Corte et al. (2009) maximize the conditional expected return subject to a constant target volatility. Ackermann et al. (2016) and Daniel et al. (2017) maximize the conditional Sharpe ratio and rescale the portfolio such that the notional value is equal to one. Note that the value of the conditional Sharpe ratio varies over time, and these portfolios differ with regard to the time-variation in the notional value (or leverage). This means that they place different weights on the conditional Sharpe ratio. As a result, they have different unconditional Sharpe ratios.

Our main focus is the strategy MV that maximizes $\mu_p - \frac{\vartheta}{2}\sigma_p^2$, where μ_p and σ_p^2 are the conditional expected excess return and variance of the currency portfolio, and ϑ is a time-invariant parameter. The portfolio weights of MV are $\theta_t^{MV} = \vartheta\Omega_t^{-1}\mu_t$, where μ_t and Ω_t are the vector of expected excess returns and the covariance matrix at time t. MV delivers not only the optimal conditional Sharpe ratio but also a time-varying notional value that is high (low) when the conditional Sharpe ratio is high (low). This adjustment increases the contribution of the future Sharpe ratio when the risk adjusted FX market return is expected to be high and suppresses the contribution when the risk adjusted return is expected to be low.

Opposite to MV the approach of Baz et al. (2001) implies a low (high) notional value when the conditional expected return is high (low) and the conditional Sharpe ratio tends to be high (low). Moreover, the notional value is insensitive to changes in the conditional volatility. Accordingly, the strategy invests less (more) aggressive when conditional prices of risk are high (low), and we expect a lower unconditional Sharpe ratio compared to MV. The strategies of Ackermann et al. (2016) and Daniel et al. (2017) have no market timing by design (i.e., constant notional value), and we expect a lower unconditional Sharpe ratio compared to MV. In the approach of Della Corte et al. (2009) the notional value only varies with the conditional return volatility, but not the conditional expected return. We expect that the unconditional Sharpe ratio is higher than that of the strategy with a constant notional value due to volatility timing, but lower than that of MV as MV also uses information about expected returns for the market timing. We verify these conjectures in the data.

The theoretical properties of MV are well-known. However, the empirical OOS performance is not documented in FX markets. Moreover, the empirical literature has not analyzed the importance of market timing with regard to MV versus other mean-variance strategies. In stock markets it is documented that MV can feature extreme portfolio weights due to parameter uncertainty. In turn, this can lead to a bad OOS performance. Accordingly, portfolio constraints (as a kind of shrinkage) can be beneficial to ensure stability in mean-variance optimized portfolios, and improve the OOS performance. In this sense, portfolio constraints have offsetting effects, they are beneficial to address estimation error concerns but costly in terms of market timing. An advantage of FX markets is that estimation errors are not a major concern (Baz et al., 2001; Ackermann et al., 2016; Daniel et al., 2017). Using FX data we document that portfolio constraints provide no value in terms of stability, but there are large costs due to the suboptimal market timing.

Ferson and Siegel (2001), Penaranda (2016) and Penaranda and Wu (2020) discuss theoretical properties of mean-variance optimized portfolios and the unconditional return distribution. These are theory papers with interesting qualitative results but the quantitative effect in the data is not studied. Our paper is the first to analyze this difference and quantify it using FX market data.

A common concern is that a mean-variance optimization ignores jump risks and may be subject to large crash or downside risk (Brunnermeier et al., 2008; Lettau et al., 2014; Dobrynskaya, 2014, 2015; Chernov et al., 2018). For instance, the high-minus-low forward discount sorted carry trade strategy of Lustig and Verdelhan (2007) is subject to significant crashes, exhibiting a negative skewness and significant maximum loss. In contrast, MV's return distribution features a positive skewness and significantly lower maximum loss per 1% expected excess return. Thus, MV's high average excess returns do not appear to be a compensation for downside risk. Since a mean-variance optimization with market timing increases average returns and reduces the downside risk, it is unlikely that crash risks are of first order importance to explain risk premia in FX markets. This finding is consistent with the evidence provided by Bekaert and Panayotov (2018), Daniel et al. (2017), and Maurer et al. (2021).

We further show that the notional value of our strategy predicts future realized returns and the volatility of mean-variance optimized portfolios. This confirms that MV invests more (less) aggressively when the risk-return trade-off is more (less) favorable. We find the predictability in both the first and the second moments of returns, which sets the market timing of our strategy apart from volatility managed portfolios (Fleming et al., 2001; Della Corte et al., 2009; Moreira and Muir, 2017; Cederburg et al., 2020). Our asset pricing tests on MV's excess return using popular FX pricing factors produce economically and statistically significant abnormal returns, indicating that MV does not simply invest in a combination of well-known currency risk factors. This result concurs with the finding of Chernov et al. (2021) that unpriced risks constitute a major component of risks in currency markets. Finally, Maurer et al. (2021) and Chernov et al. (2021) find that mean-variance optimized currency portfolios correctly price the cross-section of FX returns, and other popular pricing factors are subsumed. These papers focus on the pricing properties, while our paper analyzes the economic value of market timing based on the time-variation in the conditional Sharpe ratio. We conclude that it is costly to follow suboptimal market timing policies, in particular imposing leverage or risk (i.e., conditional volatility) limits.

Our paper is organized as follows. Section 2 discusses the market timing intuition of our currency portfolio construction, and describes FX strategies and the data. Section 3 reports our main empirical results concerning the market timing of MV. Section 4 compares the performance of MV with other popular FX strategies. Section 5 discusses robustness checks. Section 6 concludes. The appendix and Online Appendix present further details about strategies and additional Tables.

2 Intuition of Market Timing and Currency Strategies

We first discuss the intuition and implications of our FX market timing approach. The intuition provides the motivation for studying various currency strategies.

2.1 Intuitive Aspects of Market Timing

Market timing exploits the time variation in conditional expected returns and risks. It focuses on the capital allocation between the conditionally mean-variance efficient tangency portfolio and the risk-free asset. Hence, market timing dynamically adjusts the leverage or notional value of the conditionally mean-variance efficient strategy with the aim to enhance the unconditional Sharpe ratio. Our specific approach to time the market sets the notional amount equal to the ratio of the conditional expected return and the conditional variance of the strategy. That is, the strategy takes a larger position in the tangency portfolio financed by a larger short position in the risk-free asset (i.e., a larger leverage) when the conditional Sharpe ratio of the tangency portfolio increases.

We now intuitively illustrate the idea of our market timing approach. Consider a currency strategy in a three-date setting $t \in \{\tau, \tau+1, \tau+2\}$. Let rx_t denote the strategy's return in excess of the risk-free rate, $\mu_t \equiv E_t\left[rx_{t+1}\right]$ the conditional expected return, and $\sigma_t \equiv Vol_t\left[rx_{t+1}\right]$ the conditional volatility. In the current illustration, the unconditional Sharpe ratio is represented by the two-period Sharpe ratio $SR_\tau^{(2)}$ of the investment from τ to $\tau+2$. Whereas, the conditional Sharpe ratio is represented by one-period Sharpe ratios SR_τ and $SR_{\tau+1}$. Our market timing is characterized by the time-varying notional values

$$NV_t = \frac{\mu_t}{\sigma_t^2}, \qquad t \in \{\tau, \tau+1\}. \tag{1}$$

Note that the two-period currency excess return realized at $t+2$ is obtained by aggregating the one-period returns scaled by the respective notional amounts $CT_{\tau+2}^{(2)} = NV_\tau\, rx_{\tau+1} + NV_{\tau+1}\, rx_{\tau+2}$,[1] implying an intuitive expression for the unconditional Sharpe ratio

$$SR_\tau^{(2)} = \frac{E_\tau\left[NV_\tau\, rx_{\tau+1} + NV_{\tau+1}\, rx_{\tau+2}\right]}{\sqrt{Var_\tau\left[NV_\tau\, rx_{\tau+1} + NV_{\tau+1}\, rx_{\tau+2}\right]}} = \frac{E_\tau\left[\frac{\mu_\tau}{\sigma_\tau}\frac{rx_{\tau+1}}{\sigma_\tau} + \frac{\mu_{\tau+1}}{\sigma_{\tau+1}}\frac{rx_{\tau+2}}{\sigma_{\tau+1}}\right]}{\sqrt{Var_\tau\left[\frac{\mu_\tau}{\sigma_\tau}\frac{rx_{\tau+1}}{\sigma_\tau} + \frac{\mu_{\tau+1}}{\sigma_{\tau+1}}\frac{rx_{\tau+2}}{\sigma_{\tau+1}}\right]}}. \tag{2}$$

Several observations rationalize the choice and the implications of the market timing $NV_t = \frac{\mu_t}{\sigma_t^2}$. First, while the choice of NV_t does not affect the one-period Sharpe ratio, it crucially

[1] The weight NV_t are determined at date t, which scales up the strategy's return rx_{t+1} realized at the next date $t+1$.

matters for the two-period Sharpe ratio. Two strategies may offer identical and optimal one-period Sharpe ratio but different two-period Sharpe ratios. Our market timing preserves the one-period (or conditional) mean-variance efficiency, and further increases the two-period (or unconditional) Sharpe ratio.

Second, in the two-period Sharpe ratio (2) the next-period risk-adjusted return $\frac{rx_{t+1}}{\sigma_t}$ is scaled by the current price of risk of the strategy $\frac{\mu_t}{\sigma_t}$, $t \in \{\tau, \tau+1\}$. Therefore, a larger position in the currency strategy is taken when its current price of risk is higher. Intuitively, if the risk-return trade-off is expected to be much more attractive in the first period than in the second, then the market timing strategy invests aggressively in the first period and reduces the risk exposure in the second period to achieve a higher two-period Sharpe ratio. That is, the adjustment helps to boost the two-period Sharpe ratio as it enhances the contribution of the future risk-adjusted return $\frac{rx_{\tau+1}}{\sigma_\tau}$ when this risk-adjusted return is expected to be high (i.e., large $\frac{\mu_t}{\sigma_t}$), and suppresses the contribution when it is expected to be low.

This enhancement and suppression effect can be seen in the following analogous but simplified inequality (which approximates (2) when the serial correlation of the return is insignificant and omitted),

$$\frac{x_1 + x_2}{y_1 + y_2} \leq \frac{x_1 \frac{x_1}{y_1} + x_2 \frac{x_2}{y_2}}{y_1 \frac{x_1}{y_1} + y_2 \frac{x_2}{y_2}}.$$

Note that this inequality always holds for any four positive real numbers x_1, x_2, x_3, x_4, because the difference between the right-hand and and left-hand sides, $(x_1 + x_2)^{-1}(y_1 + y_2)^{-1} \left(x_1 \sqrt{\frac{y_2}{y_1}} - x_2 \sqrt{\frac{y_1}{y_2}} \right)^2$, is always positive. Consider the right hand side: (i) when $\frac{x_1}{y_1} > \frac{x_2}{y_2}$, we are enhancing the dominant pairs x_1, y_1 by multiplying them with the larger ratio $\frac{x_1}{y_1}$, and suppressing the dominated pairs x_2, y_2 with the smaller ratio $\frac{x_2}{y_2}$, (ii) when $\frac{x_1}{y_1} < \frac{x_2}{y_2}$, we are enhancing the dominant pairs x_2, y_2 by multiplying them with the larger ratio $\frac{x_2}{y_2}$, and suppressing the dominated pairs x_1, y_1 with the smaller ratio $\frac{x_1}{y_1}$. As a result, the right hand side is always larger than the left hand side.

Finally, the market timing improvement of the two-period Sharpe ratio tends to be larger when the time variation in the strategy's market price is more significant, i.e., when the moments of FX market returns are more volatile. Intuitively, this is because the above enhancement and suppression of the return contribution to the two-period Sharpe ratio requires that the return moments be different across the two dates.[2] Quantitatively, the two-period Sharpe ratio is a convex function of the variation across dates of the strategy's return moments, featuring a significant gain when this variation is sufficiently large. An important testable implication is that the market timing leads to a larger improvement in the unconditional Sharpe ratio when changes in FX market conditions are larger. We discuss out-of-sample tests and present supportive empirical evidence for this key implication in Section 3.3.

2.2 Currency Strategies

Let rx_{t+1} denote the vector of excess returns against the USD of N foreign currencies for the holding period from month t to $t+1$, and θ_t^S the vector of portfolio weights of a currency strategy S at the end of month t. The sum of weights $\sum_i \theta_{i,t}^S$ does not need to be equal to 1 as rx_{t+1} are excess returns. The excess return of strategy S is $rx_{t+1}'\theta_t^S$, and the total USD risk exposure is measured by its notional value, $\sum_i |\theta_{i,t}^S|$. Next, we describe the currency strategies in our analysis based on their efficiency and market timing characteristics.

Conditionally Mean-Variance Efficient Strategies

We first introduce our principal strategy, which is conditionally mean-variance efficient and uses market timing based on the conditional Sharpe ratio. Second, we describe other conditionally mean-variance efficient strategies.

Conditional Mean-Variance Efficiency with Market Timing: Our principal currency strategy, MV, is defined by the portfolio weights

$$\theta_t^{MV} = \vartheta \Omega_t^{-1} \mu_t,$$

where Ω_t and μ_t are the conditional covariance matrix and vector of expected excess returns of the N currencies against the USD, ϑ is a time invariant scaling factor and can be interpreted as the inverse of the relative risk aversion. θ_t^{MV} maximizes $\mu_p - \frac{\vartheta}{2}\sigma_p^2$, where $\mu_p = \mu_t'\theta_t^{MV}$ and

[2] When return moments are similar at $\tau+1$ and $\tau+2$, the scaling parameters NV_τ and $NV_{\tau+1}$ do not matter for the two-period Sharpe ratio, offering no market-timing improvement to $SR_\tau^{(2)}$.

$\sigma_p^2 = \theta'^{MV}_t \Sigma_t \theta^{MV}_t$ are the conditional expected excess return and volatility of the portfolio. MV features both conditional mean-variance efficiency and market timing. As elaborated intuitively in (1), MV's market timing is characterized by its time-varying notional value, which depends on the size of the conditional expected excess returns and the conditional covariance matrix. Note that ϑ is constant, and thus, does not affect the market timing or the Sharpe ratio of the strategy. In our empirical analysis we assume the investor believes the average conditional Sharpe ratio of MV is equal to 1, and we choose ϑ such that MV targets an unconditional volatility of 10% .

A practical and important challenge of a mean-variance optimization is the estimation of the conditional covariance matrix and expected excess returns. Estimation errors may lead to a deceptively high in-sample but low out-of-sample Sharpe ratio. For instance, DeMiguel et al. (2009) show that mean-variance optimized portfolios in the US stock market earn low out-of-sample returns and are consistently outperformed by equally weighted portfolios. They attribute this finding to estimation errors. We address the estimation error problem for both expected returns and covariances. The realized excess return of currency i denominated in USD is composed of currency i's forward discount and realized exchange rate growth, $rx_{i,t+1} = f_{i,t} + \Delta s_{i,t+1}$. The empirical finding that short-term exchange rate movements $x_{i,t+1}$ are close to a random walk, $E_t[\Delta s_{i,t+1}] \approx 0$ (Meese and Rogoff, 1983) then implies that forward discounts are good predictors of conditional currency excess returns, $E_t[rx_{i,t+1}] \approx f_{i,t}$. Motivated by this observation, we employ the current forward discount $f_{i,t}$ to proxy for the conditional expected excess return $\mu_{i,t}$ of currency i. Baz et al. (2001), Della Corte et al. (2009), Ackermann et al. (2016) and Daniel et al. (2017) employ the same proxy to estimate expected excess returns in currency markets. Furthermore, Lustig et al. (2011) document that FX markets have a strong factor structure, and the first few PCs capture most of the covariation of FX returns. Accordingly, we employ PCA and retain PCs that explain at least 1% of the common variation in returns and construct a robust version of the covariance matrix Ω_t^{-1}.

In the first step, we use an exponentially weighted moving average (EWMA) popularized by J.P. Morgan's RiskMetrics to estimate daily covariances in an expanding window,

$$\widehat{\Omega}_{d,\tau,ij} = \delta\widehat{\Omega}_{d,\tau-1,ij} + (1-\delta)\Delta s_{d,i,\tau-1}\Delta s_{d,j,\tau-1},$$

where $\Delta s_{d,i,\tau}$ is the daily exchange rate growth of currency i against the USD on day τ, the initial value at $\tau = 1$ is $\widehat{\Omega}_{d,\tau,ij} = \Delta s_{d,i,\tau}\Delta s_{d,j,\tau}$, and EWMA weight $\delta = 0.94$, i.e., a half-life of 11 trading days (Fleming et al., 2001). Let τ_t be the last day of month t, and T_t the number of trading days in month t. We define the monthly estimate $\widehat{\Omega}_{t,ij} = T_t\widehat{\Omega}_{d,\tau_t,ij}$.

In the second (PCA) step, we diagonalize the covariance matrix $\widehat{\Omega}_t = W_t\Lambda_t W_t'$, with $\Lambda_t = Diag(\lambda_{1,t},\ldots,\lambda_{N,t})$. To generate a robust (inverse) of the covariance matrix $\widetilde{\Omega}_t^{-1} = \widetilde{W}_t\widetilde{\Lambda}_t^{-1}\widetilde{W}_t'$, we remove all K eigenvalues $\lambda_{k,t}$ for which $\frac{\lambda_{k,t}}{\sum_{h=1}^{N}\lambda_{h,t}} < \bar{\lambda} = 1\%$, i.e., the PCs which explain less than $\bar{\lambda} = 1\%$ of the common variation in exchange rates. $N \times (N-K)$ matrix $\widetilde{W}_t$ is obtained by removing the K columns of the $N \times N$ rotation matrix W_t associated with the K removed eigenvalues $\lambda_{k,t}$, and $(N-K)\times(N-K)$ matrix $\widetilde{\Lambda}_t$ by removing K eigenvalues from the $N \times N$ matrix Λ_t.

As a result, MV is a mean-variance optimized strategy that is linear in the $(N-K)$ dominant PCs of FX markets. Because the removed PCs explain only a small fraction of the exchange rate covariation, this procedure reduces the covariance matrix estimation errors and alleviates in-sample over-fitting and near-arbitrage opportunities arising from factors with unreasonably large in-sample Sharpe ratio (Ross, 1976; Kozak et al., 2018).

Other Conditionally Mean-Variance Efficient Strategies: For comparison, we consider several alternative currency strategies that also offer the optimal conditional Sharpe ratio but differ from MV with respect to the market timing, i.e., the time-variation in the notional value. All conditionally mean-variance efficient strategies, including MV, have proportional portfolio weights, i.e., invest in the same risky asset portfolio. They only differ with respect to the notional value. For the construction of the following strategies we define the conditional expected excess return and variance of MV, $\mu_t^{MV} = \mu_t'\theta_t^{MV}$ and $\sigma_t^{MV} = \sqrt{\theta_t^{MV\prime}\Omega_t\theta_t^{MV}}$.

MV_{CN} (or notional target) is a conditionally mean-variance efficient strategy whose portfolio weights are scaled from MV's weights to keep the notional value constant, $\theta_t^{MV_{CN}} = \frac{1}{\sum_i |\theta_{i,t}^{MV}|}\theta_t^{MV}$. We choose a notional value of 4 in order to generate an annualized volatility of roughly 10%. The constant notional value requirement of MV_{CN} precludes market timing. This strategy is examined by Ackermann et al. (2016) and Daniel et al. (2017). Note that both of these papers use forward discounts to proxy for expected excess returns but they do not employ our PCA approach to address estimation errors in the covariance matrix of currency returns.

MV_{CV} (or volatility target) with $\theta_t^{MV_{CV}} = \frac{\bar{\sigma}}{\sigma_t^{MV}}\theta_t^{MV}$ is a conditionally mean-variance efficient strategy with portfolio weights rescaled to keep the conditional volatility constant and equal to $\bar{\sigma}$. We choose $\bar{\sigma} = 10\%$. MV_{CV} is implemented by Della Corte et al. (2009) in FX markets.

MV_{CY} (or yield target) with $\theta_t^{MV_{CY}} = \frac{\bar{\mu}}{\mu_t^{MV}}\theta_t^{MV}$ is a conditionally mean-variance efficient strategy with portfolio weights rescaled to keep the conditional expected excess return constant and equal to $\bar{\mu}$. We choose $\bar{\mu} = 10\%$. Baz et al. (2001) analyze MV_{CY} in FX markets. The volatility and yield targets generate market timing patterns in MV_{CV} and MV_{CY} that are different from the market timing of MV. In the next section we show that the market timing of MV generates a more desirable unconditional return distribution.

MV_{FS} is a conditionally mean-variance efficient strategy with portfolio weights $\theta_t^{MV_{FS}} = \frac{\zeta\mu_t^{MV}}{\left(\mu_t^{MV}\right)^2+\left(\sigma_t^{MV}\right)^2}\theta_t^{MV}$ where ζ is a time-invariant constant parameter.[3] Ferson and Siegel (2001) and Penaranda (2016) introduce and discuss theoretical properties of MV_{FS}. Moreover, Chernov et al. (2021) test the empirical properties of MV_{FS} as a pricing factor in FX markets. MV_{FS} features a different market timing than MV. That is, MV_{FS} reduces its holdings uniformly across currencies when the conditional Sharpe ratio increases. A comparison of MV_{FS} and MV in our empirical section reveals that MV's market timing approach leads to a more attractive OOS performance.

[3]In theory, if MV is conditionally mean-variance efficient and for $\vartheta = 1$, then the conditional Sharpe ratio of MV is $SR_t^{MV} = \sigma_t^{MV} = \sqrt{\mu_t^{MV}}$, and thus, $\theta_t^{MV_{FS}} = \frac{\zeta}{1+\left(SR_t^{MV}\right)^2}\theta_t^{MV}$

Other Common Strategies

For completeness, we also consider other currency strategies that are commonly used in the literature. We briefly characterize some of the key strategies, and relegate details to Appendix 6. These strategies include the Dollar strategy DOL (Lustig et al., 2011), the Dollar carry strategy $DDOL$ (Lustig et al., 2014), the momentum strategy MOM (Burnside et al., 2011; Menkhoff et al., 2012b), the value strategy VAL (Bilson, 1984; Menkhoff et al., 2017), and the equally weighted high-minus-low forward discount carry strategy HML (Lustig and Verdelhan, 2007).

SW (or spread weighted carry strategy) is defined by portfolio weights that are set equal to the forward discounts, $\theta_t^{SW} = f_t$. The SW strategy is dynamic and features a market timing based on the absolute size of the forward discounts. SW enhances the strategy HML by taking into account the size and time variation in the forward discounts.

HML_{VM} (or the volatility managed portfolio of HML) and SW_{VM} (or the volatility managed portfolio of SW) are strategies with portfolio weights $\theta_t^{HML_{VM}} = \frac{1}{(\sigma_t^{HML})^2}\theta_t^{HML}$ and $\theta_t^{SW_{VM}} = \frac{1}{(\sigma_t^{SW})^2}\theta_t^{SW}$. The conditional volatilities σ_t^{HML} and σ_t^{SW} of the HML and SW are computed using daily returns of these factors over the past month (Moreira and Muir, 2017).

2.3 Data

We collect daily spot and 1-month forward exchange rates from 31 October 1983 to 30 January 2016 from Reuters via Datastream. We use quotes of the last day of the month to compute monthly returns.

In the main analysis, we focus on the data of 15 developed countries reported and analyzed in Lustig et al. (2011). Trading frictions are typically lower for currencies of developed countries (e.g., they have a large active trading volume, there are less capital controls, liquidity is higher, transaction costs are lower) than for currencies of emerging countries. Thus, our strategy may be easier and cheaper to implement using only developed currencies. We check the robustness of our results using the dataset of 29 developed and emerging currencies. The currencies included are the standard ones considered in the literature, see for example Lustig et al. (2011). All our results remain qualitatively the same. The results for this 29-currency set are available upon request.

3 Economic Value of Market Timing

Section 3.1 documents that the OOS performance of MV dominates other mean-variance optimized strategies. Section 3.2 examines how the market timing of MV, which is characterized by its time-varying notional value, predicts future returns and risks. Section 3.3 validates the key implication that market timing is more beneficial when market conditions and return moments are more volatile. Finally, section 3.4 compares market timing based on expected returns versus volatility.

3.1 Performance of Mean-Variance Efficient Strategies

Our empirical analysis starts with a performance evaluation of MV in comparison with four other conditionally mean-variance efficient strategies, MV_{CV}, MV_{CY}, MV_{CN}, and MV_{FS} (Section 2.2). Table 1 presents the out-of-sample (OOS) performance of these strategies. All strategies use information available at the end of month t to construct the portfolios that are held from end of t to end of $t+1$. Thus, there is no look-ahead bias. All reported statistics are annualized, and include the average excess returns (Mean), volatility (Vol), unconditional Sharpe ratio (SR), skewness (Skew), kurtosis (Kurt). We use the test proposed by Ledoit and Wolf (2008), which is robust to heteroskedasticity and autocorrelation, to test the hypothesis whether the differential Sharpe ratio (ΔSR) of MV versus other strategies is equal to zero, i.e., the Sharpe ratios are equal. We indicate a rejection of the test at the 10%, 5% and 1% significance level by 1, 2 and 3 stars.[4] We also report the maximum drawdown (MDD, in percentage points), which measures the maximum loss from peak to trough a strategy has experienced during the entire sample period, and the MDD per 1% expected excess return (|MDD|/Mean). |MDD|/Mean also measures the expected time in years to recover from the

[4] We choose a block size of 10 for block-bootstrapping to estimate p-values. This is a conservative value and our results are stronger if we use a smaller value closer to what is used by Ledoit and Wolf (2008) in their examples.

MDD. Since all strategies can be levered up or down, |MDD|/Mean is a more useful measure than MDD when we compare the downside or crash risks between strategies. We estimate the relative utility gain of MV with respect to other strategies (Utility) for investors with constant relative risk aversion of $\gamma = 1$ and $\gamma = 10$. That is, we follow Fleming et al. (2001) and define Utility as the annual percentage an investor is willing to pay to switch from a strategy to MV.

Table 1: **Performance of Mean-Variance Optimized Portfolios**

	MV	MV_{CV}	MV_{CY}	MV_{CN}	MV_{FS}
Mean (in %)	20.30	12.04	8.59	8.01	9.93
Vol (in %)	18.06	13.93	21.04	12.79	9.29
SR	1.12	0.86	0.41	0.63	1.07
ΔSR		0.26***	0.72***	0.50***	0.06**
(p-value)		(0.0078)	(0.0026)	(0.0008)	(0.0234)
Skew	0.67	-0.48	-0.07	-0.66	0.09
Kurt	6.63	3.96	8.52	4.93	5.80
MDD (in %)	-36.07	-58.73	-132.13	-74.22	-22.60
|MDD|/Mean	1.78	4.88	15.39	9.26	2.28
Utility gamma=1 (in %)		5.38	7.42	8.74	8.38
Utility gamma=10 (in %)		2.81	3.88	3.08	5.50

Notes: Statistics of monthly out-of-sample excess returns of mean-variance optimized portfolios. MV maximizes the expected return minus a constant times the variance of the portfolio. MV_{CV}, MV_{CY}, and MV_{CN} impose a constant volatility, constant yield, and constant notional value in the construction of MV. MV_{FS} is the mean-variance portfolio suggested by Ferson and Siegel (2001), and reduces the risk exposure of MV when the conditional Sharpe ratio increases. ΔSR is the difference between the Sharpe ratio of MV and that of other strategies. The reported p-value is the test for ΔSR = 0. Standard errors are robust to heteroskedasticity and autocorrelation (Ledoit and Wolf, 2008). All statistics are annualized.

The main findings in Table 1 are as follows. The MV strategy outperforms other conditionally mean-variance efficient strategies across various performance measures. MV offers the highest OOS unconditional Sharpe ratio of 1.12 per annum, which is statistically significantly higher than that of other strategies. MV is the only strategy that features a return distribution with both a positive and a large (OOS and monthly) skewness of 0.67. That is, MV frequently has high returns (i.e., a large upside potential), whereas it's low returns are relatively close to the mean (i.e., a limited downside risk). A large literature argues that the high returns of currency strategies are a compensation for crash risk (Brunnermeier et al., 2008; Dobrynskaya, 2014; Galsband and Nitschka, 2014; Lettau et al., 2014). The positive skewness and high average return of the MV indicates that it is not exposed to or explained by large crash risks. The maximum drawdown strengthens the finding that MV does not have much crash risk exposure. Among all reported strategies, MV has the least maximum loss of -36.07%. In comparison, the yield target mean-variance optimized strategy MV_{CY} has a maximum drawdown of -132.13%, i.e., losing all of its investment at some point during our sample. The MV also has the lowest maximum drawdown per 1% expected excess return (|MDD|/Mean) at 1.78, indicating its shortest expected time to recover from the maximum drawdown. MV also offers a large positive utility gain. That is, for a reasonable range of relative risk aversions ($\gamma = 1$ and $\gamma = 10$), investors are willing to pay more than 5.38% per year to switch from any of the alternative conditionally mean-variance efficient strategies to MV. Finally, note that the next best strategy is MV_{FS}. In theory, MV_{FS} is constructed to optimize the unconditional Sharpe ratio. However, empirically it underperforms MV out-of-sample.

In summary, as all strategies in Table 1 are conditionally mean-variance efficient, MV's unconditional outperformance is due to its market timing. The outperformance is statistically and economically significant. Our findings demonstrates the importance of the market timing of MV in real market data. It is costly to limit the leverage or risk of a portfolio (i.e., impose a constant notional value or volatility), or implement other suboptimal market timing policies. This finding is important as such limits are often imposed in practice.

3.2 Market Timing and FX Market Predictability

The outperformance of MV due to its market timing indicates that the strategy has predictive power in FX markets. It is important to understand whether properties of MV's OOS return distribution, such as the positive skewness and low relative maximum drawdown (|MDD|/Mean), can be explained by MV's ability to predict adverse events in FX markets. We examine this hypothesis by testing whether MV's market timing can predict future FX market returns or moments at either the portfolio level (Table 2) or individual currency level (Table 3).

We first test whether the notional value of MV $\sum_i |\theta_{i,t}^{MV}|$ can predict returns and risks of the conditionally mean-variance efficient portfolio MV_{CN} without market timing. Specifically, we predict the future (next-month) realized Sharpe ratio (i.e., realized excess return divided by the volatility), realized excess return, and volatility of strategy MV_{CN}. The volatility is estimated using daily returns within the month. Since the weights of MV are equal to the notional value of MV multiplied by the weights of MV_{CN}, successful market timing implies that the notional value predicts expected returns or risks of MV_{CN}. Table 2 reports the estimation results and R^2 of the predictive regressions. We control for the conditional expected excess return and volatility of MV_{CN} in our regressions. This provides insights to what extent the first two moments of MV_{CN} capture the information content of MV's notional value.

We find that MV's notional value $\sum_i |\theta_{i,t}^{MV}|$ by itself is able to predict the future realized excess return, volatility and Sharpe ratio of strategy MV_{CN}. The regression slope coefficients are significant at the 1% level, and the adjusted R^2 is 2.67%, 19.57% and 5.72%, respectively. These results confirm MV's ability to time the market. The relatively high R^2 in the prediction of the volatility indicates that there is particularly much value to time the market based on changes in risks. The signs of slope coefficients are as expected. An increase in MV's current exposure to FX markets is associated with a higher return, lower volatility and higher Sharpe ratio of the strategy MV_{CN} in the subsequent month. That is, MV trades more aggressively when future returns are higher, risks are lower, and the expected return-risk trade-off is more attractive.

Panel A shows that the conditional mean $\mu_t^{MV_{CN}}$ of strategy MV_{CN} by itself does not have much predictive power. When we include both $\sum_i |\theta_{i,t}^{MV}|$ and $\mu_t^{MV_{CN}}$ as predictors in the predictive regression (Panel A), MV's notional value remains a statistically significant predictor of MV_{CN}'s future realized return, volatility and Sharpe ratio. Panel B shows that

Table 2: **Portfolio Level Predictive Regressions**

Panel A	MV_{CN}			$Vol\,(MV_{CN})$			$SR\,(MV_{CN})$		
$\sum_i \lvert\theta_{i,t}^{MV}\rvert$	0.126***		0.122**	-0.430***		-0.450***	0.060***		0.059***
(t-stat)	(2.649)		(2.281)	(-6.651)		(-6.839)	(3.512)		(3.342)
$\mu_t^{MV_{CN}}$		0.057	0.036		0.082	0.158**		0.016	0.007
(t-stat)		(0.750)	(0.447)		(1.090)	(2.388)		(0.922)	(0.346)
$Adj.R^2$ (%)	2.67	1.01	2.53	19.57	6.66	20.70	5.72	1.43	5.51

Panel B	MV_{CN}			$Vol\,(MV_{CN})$			$SR\,(MV_{CN})$		
$\sum_i \lvert\theta_{i,t}^{MV}\rvert$	0.126***		0.028	-0.430***		-0.126**	0.060***		0.041**
(t-stat)	(2.649)		(0.581)	(-6.651)		(-2.330)	(3.512)		(2.000)
$\sigma_t^{MV_{CN}}$		-0.203***	-0.180*		0.658***	0.557***		-0.067***	-0.034
(t-stat)		(-2.720)	(-1.939)		(9.058)	(5.786)		(-3.661)	(-1.498)
$Adj.R^2$ (%)	2.67	4.12	3.93	19.57	27.38	27.84	5.72	5.10	6.05

Notes: 1-month ahead predictive regressions: $Y_{t,t+1} = c_{const} + c_{trend}t + \sum_i c_i X_{i,t} + \epsilon_t$. $Y_{t,t+1}$: realized Sharpe ratio ($SR\,(MV_{CN})$), realized return (MV_{CN}, in %) or realized volatility ($Vol\,(MV_{CN})$, in %) of strategy MV_{CN}. $X_{i,t}$: MV dollar risk exposure ($\lvert\theta_{i,t}^{MV}\rvert$), conditional mean ($\mu_t^{MV_{CN}}$, in %), conditional risk ($\sigma_t^{MV_{CN}}$, in %). c_{const} is a constant term, $c_{trend}t$ controls for any time-series trend in $Y_{t,t+1}$, c_i are the slope coefficients associated with the predictors $X_{i,t}$; their corresponding t-statistics are reported in parentheses. Significance at the 1%, 5% or 10% level are indicated by ***, ** or *, where standard errors are calculated using Newey and West (1987) to account for heteroskedasticity and auto-correlation.

the conditional volatility $\sigma_t^{MV_{CN}}$ of strategy MV_{CN} by itself is able to predict the future performance of MV_{CN} at the 1% level of significance. When we include both $\sum_i |\theta_{i,t}^{MV}|$ and $\sigma_t^{MV_{CN}}$ as predictors in the predictive regression (Panel B), MV's notional value also remains a statistically significant predictor of MV_{CN}'s volatility and Sharpe ratio. These results show that MV_{CN}'s conditional mean and conditional volatility do not fully subsume the predictive power of MV's notional value.

In summary, Table 2 provides evidence in favor of MV's ability to time the market. In particular, we find that the notional value of MV increases when MV_{CN}'s realized returns are high, risk is low, and its expected return-risk trade-off is more attractive in the subsequent month.

Next, we consider changes in MV's portfolio weights as a predictor of returns and risks of individual currencies (Table 3). These more granular regressions shed light on both the market timing ability of MV and the conditional efficiency of mean-variance optimized portfolio in general. That is, changes in the portfolio weights of MV should reflect a time-variation in the expected return-risk trade-off of the portfolio as well as of the individual currency relative to the other currencies. Table 3 reports the estimated regression coefficients and R^2 of predictive panel regressions. We predict changes in future returns, volatility, and skewness of all individual currencies. All predictive panel regressions of Table 3 include the currency specific forward discount $\mu_{i,t} = f_{i,t}$, the conditional volatility $\sigma_{i,t} = \Omega_{i,i,t}$, the time lag of the predicted quantity, and a time trend as controls.

In the panel data of individual currencies, the change $\Delta\theta_{i,t}^{MV}$ in MV's portfolio weight in currency i over past months is able to predict changes in the return and skewness of currency i's return against the USD in the subsequent month. The regression slope coefficients are significant at the 1% level.[5] Similarly, a change in MV's exposure to currency i in the past, $\Delta|\theta_{i,t}^{MV}|$, is able to predict changes in the volatility of individual currency returns in the subsequent month. The regression slope coefficient is significant at the 1% level. This confirms that MV invests more in a currency when the future excess return is higher and

[5]Because both return and skewness are signed quantities (i.e., both their sign and magnitude are meaningful), we employ the original (but not absolute) change $\Delta\theta_{i,t}^{MV}$ in portfolio weight as a signed predictor. For unsigned volatility prediction, we employ the unsigned exposure $\Delta|\theta_{i,t}^{MV}|$ as an unsigned predictor.

Table 3: **Individual Currencies Predictive Panel Regression**

$Y_{i,t+1}$	$\Delta rx_{i,t+1}$	$\Delta\text{Vol}(rx_{i,t+1})$	$\Delta\text{Skew}(rx_{i,t+1})$
$\Delta\theta^{MV}_{i,t}$	0.0006***		0.0035***
(t-stat)	(3.08)		(2.67)
$\Delta\lvert\theta^{MV}_{i,t}\rvert$		-0.0012***	
(t-stat)		(-6.93)	
$\mu_{i,t}$	0.0680**	-0.0353***	0.0111
(t-stat)	(2.12)	(-3.77)	(0.08)
$\sigma_{i,t}$	-0.5609	2.2488***	0.9527
(t-stat)	(-0.52)	(5.56)	(0.28)
R^2 (in %)	26.46%	28.33%	27.00%

Notes: 1-month ahead predictive regressions: $Y_{i,t+1} = c_{const} + c_{trend}t + c_\theta X_{i,t} + c_\mu \mu_{i,t} + c_\sigma \sigma_{i,t} + c_{lag} Y_{i,t} + \epsilon_{i,t}$. c_{const} is a constant term, $c_{trend}t$ controls for any time-series trend, and c_{lag} controls for the lag of the LHS variable. $Y_{i,t+1} = \Delta rx_{i,t+1} = (rx_{i,t+1} - rx_{i,t})$ is the change in realized monthly return of currency i against the USD. $Y_{i,t+1} = \Delta\text{Vol}(rx_{i,t+1}) =$ (Volatility of $rx_{i,t+1}$−Volatility of $rx_{i,t}$) is the change in realized monthly volatility (i.e., volatility estimated from daily data within the month) of the returns of currency i against the USD. $Y_{i,t+1} = \Delta\text{Skew}(rx_{i,t+1}) =$ (Skewness of $rx_{i,t+1}$ - Skewness of $rx_{i,t}$) is the change in realized monthly skewness of the returns of currency i against the USD. $\Delta\theta^{MV}_{i,t} = (\theta^{MV}_{i,t} - \theta^{MV}_{i,t-1})$ is the change in portfolio weight of MV in currency i. $\Delta|\theta^{MV}_{i,t}| = (|\theta^{MV}_{i,t}| - |\theta^{MV}_{i,t-1}|)$ is the change in the exposure of MV to currency i. The panel regressions include currency fixed effects. Significance at the 1%, 5% or 10% level are indicated by ***, ** or *. Standard errors are calculated to account for cross-currency correlations, heteroskedasticity, and within currency autocorrelations (Newey and West, 1987).

the crash risk lower. Moreover, MV reduces its risk exposure to currencies which are more volatile in the subsequent month. That is, MV can time currency markets and increases loadings on desirable (high return, low volatility, positive skewness) currencies. Compared to Table 2, the current predictive regressions tend to offer higher R^2. These may be due to several reasons. First, we control for the conditional mean $\mu_{i,t}$ and conditional volatility $\sigma_{i,t}$ of individual currency returns. Second, the panel regression does not only focus on market timing but also on the relative importance of each currency in the composition of the MV portfolio.

In summary, Table 3 strengthens the evidence for MV's market timing and efficiency. We show that changes in MV's portfolio weights and exposures predict future changes in the performance of individual currencies, even after controlling for the currency specific forward discount and volatility.

For completeness, the Online Appendix presents the slope coefficient estimates and R^2 of predictive regressions for all individual currencies. Tables 8 reports the results of the regressions of the change $\Delta rx_{i,t+1}$ in currency i's realized return on the change $\Delta\theta^{MV}_{i,t}$ in MV's portfolio weight in currency i. Table 9 shows the regressions of the change $\Delta\text{Vol}(rx_{i,t+1})$ in currency i's realized return volatility on the change $\Delta|\theta^{MV}_{i,t}|$ in MV's exposure to currency i. Table 10 provides results for the regressions of the change $\Delta\text{Skew}(rx_{i,t+1})$ in currency i's realized return skewness on $\Delta\theta^{MV}_{i,t}$.

3.3 Market Timing and Changes in Economic Conditions

Having demonstrated the effect of the market timing on the performance and predictive power of MV, we now examine an important testable economic implication of MV's market timing. As suggested by the intuitive analysis in Section 2.1, the outperformance of MV over other conditionally mean-variance efficient strategies is facilitated by the time variation in economic conditions. That is, market timing delivers value in a market environment in which the timing is relevant. To test this implication we consider various conditioning variables that proxy for the state of the economy and financial markets. We examine whether periods of large versus small absolute changes in these variables coincide with periods of significant versus less significant outperformance of strategy MV over other conditionally mean-variance efficient strategies.

Table 4 reports the outperformance of MV over the strategies MV_{CV}, MV_{CY}, MV_{CN}, MV_{FS} while conditioning on small versus large changes in five alternative state variables. The five conditioning variables are the conditional expected excess return of MV (μ_t^{MV}; Panel A), conditional volatility of MV (σ_t^{MV}; Panel B), 3-month T-bill rate (Panel C), term spread (the difference between 10-year and 3-month Treasury yields, Panel D), and default spread (the difference between Moody's Aaa corporate bond yield and 3-month Treasury yield, Panel E). These variables either characterize MV's expected return-risk trade-off, or are known indicators of the economy and financial markets. The relative performance is measured by the difference in Sharpe ratios ΔSR of MV versus an alternative strategy. We divide our data into two (quantile) subsamples, which contain months of respectively small (below median) and large (above median) absolute changes in the five alternative conditioning variables. For each conditioning variable, we report the difference in Sharpe ratios ΔSR for the two subsamples.

Across all conditioning variables, the Sharpe ratio differential between the MV and all alternative strategies is consistently higher in subsmaples of large absolute changes in the conditioning variables compared to subsamples of small absolute changes. In fact, when changes in the conditioning variables are large, then the Sharpe ratio of MV is always statistically significantly higher than the Sharp ratio of all other strategies. In contrast, when changes in the conditioning variables are small, then the difference in Sharpe ratios is in most cases statistically insignificant. As absolute changes in these conditioning variables represent the time variation in market conditions, this finding supports the implication that MV's market timing is relatively more beneficial when return moments are more volatile.

Furthermore, in the cross section of all strategies and for both subsamples of small and large changes in the conditioning variables, MV always offers the highest Sharpe ratio among all conditionally efficient mean-variance strategies. This is reflected by $\Delta SR > 0$ everywhere in Table 4. This emphasizes MV's steady outperformance over other strategies across different economy conditions. The Sharpe ratio differential (i.e., outperformance of MV) ranges

Table 4: **Economic Conditions and** MV

	$\Delta SR(MV_{CV})$	$\Delta SR(MV_{CY})$	$\Delta SR(MV_{CN})$	$\Delta SR(MV_{FS})$
	Panel A. Conditional Mean μ_t^{MV}			
Small changes	0.214	0.429*	0.407*	0.034
Large changes	0.231**	0.755**	0.472**	0.068*
	Panel B. Conditional Volatility σ_t^{MV}			
Small changes	0.222	0.587*	0.437*	0.032
Large changes	0.256**	0.721**	0.519**	0.069*
	Panel C. 3-month T-bill			
Small changes	0.168	0.504*	0.273	0.036
Large changes	0.340***	0.935*	0.698***	0.070**
	Panel D. Term Spread			
Small changes	0.128	0.462	0.242	0.028
Large changes	0.375***	0.941**	0.711***	0.077**
	Panel E. Default Spread			
Small changes	0.159	0.547**	0.257	0.032
Large changes	0.349**	0.865**	0.676***	0.075**

Notes: Differences between the Sharpe ratio of MV and those of other conditionally efficient mean-variance portfolios conditional on small (below median) versus large (above median) changes in state variables. Significance at the 1%, 5% or 10% level are indicated by ***, ** or *. Each panel reports the results for one conditioning variable.

from 0.941 for MV versus MV_{CY} in the subsample of large changes in the Term Spread, to 0.028 for MV versus MV_{FS} in the subsample of small changes in the Term Spread. Among the alternative strategies, MV_{FS} is closest to MV, and MV_{CY} consistently performs the worst. This finding fits the fact that MV_{FS} features market timing to improve the unconditional Sharpe ratio, while MV_{CY} basically follows the opposite timing pattern of MV.

In summary, Table 4 provides evidence to support the relative advantage of MV's market timing over other conditionally mean-variance efficient strategies. Market timing is most beneficial when market conditions are rapidly changing from month to month. In turn, it is especially costly to impose leverage or risk (i.e., conditional volatility) limits during these times.

3.4 Market Timing Based on Expected Returns versus Volatility

To further evaluate the economic value of timing with respect to the conditional expected excess return and the conditional volatility, we construct alternative strategies that focus on the expected excess return, the return volatility, and combinations of the two. We compare MV to four alternative strategies: HML, SW, HML_{VM}, SW_{VM}. Unlike the strategies in Table 1, these four strategies are (in general) not conditionally mean-variance efficient. HML is the standard high-minus-low interest rate sorted currency carry strategy (Lustig and Verdelhan, 2007). SW is a spread weighted currency carry strategy with portfolio weights $\theta_t^{SW} = f_t$. The subscript VM (volatility managed) indicates a scaling of the portfolio weights based on the current return volatility of the strategy (see Section 2.2 for details). Table 5 presents the OOS performance of these strategies. The performance metrics are the same as in Table 1.

First, we make comparisons between the four alternative strategies to get insights into the value of market timing based on expected returns versus volatility. In contrast to HML, SW is not equally weighted and its notional value increases (decreases) when the average magnitude of forward discounts increases (decreases). Thus, similar to MV, SW features some fine-tuning in its portfolio weights with respect to $\mu_t = f_t$, and it uses market timing based on the magnitude of conditional expected excess returns. SW has a higher Sharpe ratio,

Table 5: **Market Timing: Expected Returns versus Volatility**

	MV	HML	SW	HML_{VM}	SW_{VM}
Mean	20.30	5.23	5.67	7.93	11.11
Vol	18.06	9.66	9.00	10.57	13.20
SR	1.12	0.54	0.63	0.75	0.84
ΔSR		0.59**	0.49**	0.37**	0.28
(p-value)		(0.012)	(0.030)	(0.043)	(0.202)
Skew	0.67	-0.89	-0.61	2.56	-0.02
Kurt	6.63	5.59	11.92	26.21	13.66
MDD	-36.07	-42.78	-43.78	-22.09	-39.90
|MDD|/Mean	1.78	8.18	7.72	2.79	3.59

Notes: Statistics of monthly out-of-sample excess returns. MV maximizes the expected return minus a constant times the variance of the portfolio. HML is an equally weighted carry trade strategy. SW is the spread-weighted carry trade strategy. HML_{VM} and SW_{VM} are volatility managed portfolio of HML, and SW. ΔSR is the difference between the Sharpe ratio of MV and that of other strategies. The reported p-value is the test for $\Delta\text{SR} = 0$. Standard errors are robust to heteroskedasticity and autocorrelation (Ledoit and Wolf, 2008). All statistics are annualized.

less negative skewness and smaller maximum drawdown per 1% expected return compared to HML. This suggests that there is value in fine-tuning the portfolio weights and timing the market based on the magnitude of expected excess returns. However, the improvement in OOS performance is relative moderate. That is, SW still delivers less attractive OOS returns than MV.

The comparison between HML and HML_{VM} as well as SW and SW_{VM} provides insights about the benefit of market timing with respect to volatility while keeping other characteristics of the strategy constant. The volatility adjusted portfolios are in the spirit of Fleming et al. (2001) and Moreira and Muir (2017). The volatility managed strategies have higher Sharpe ratios (an increase of about 0.2). More strikingly there is a substantial improvement with respect to the downside risk. The skewness turns from negative to positive (in the case of HML) or zero (in the case of SW). Moreover, the maximum drawdown per 1% expected return is less than half after market timing based on volatility. Therefore, market timing based on volatility is important, in particular in terms of reducing crash risks.

Finally, we emphasize that MV outperforms all four strategies across various performance measures. MV offers an OOS unconditional Sharpe ratio of 1.12, which is statistically significantly higher than that of HML (0.54), SW (0.63), and HML_{VM} (0.75). It also dominates the Sharpe ratio of SW_{VM} (0.84), though the difference is not statistically significantly. Furthermore, MV features an unconditional return distribution with a positive skewness, and it has the most desirable maximum drawdown per 1% expected excess return, $|\text{MDD}|/\text{Mean} = 1.78$. In comparison, $|\text{MDD}|/\text{Mean}$ for HML_{VM} is 2.79, for SW_{VM} is is 3.59, and for HML and SW they are 8.18 and 7.72. Accordingly, MV features substantially less crash risks.

In summary, we find that market timing based on volatility delivers relatively more value than market timing based on expected returns. However, the two are not mutually exclusive and both dimensions are important. A combination of both dimensions significantly improves the OOS performance.

4 MV versus Common Benchmarks in the Literature

In section 4.1 we first compare the performance of MV and various benchmark portfolios. In section 4.2 we show that MV is not spanned by common FX risk factors.

4.1 Performance of MV versus Benchmarks

We now compare MV to popular FX strategies in the literature, namely, DOL, $DDOL$, HML, MOM, and VAL. In our analysis we consider these five strategies individually as benchmarks, and further construct various portfolios of these strategies. This empirical exercise aims to position MV against a broad set of common benchmarks in the FX literature. Panel A in Table 6 reports the OOS performance of MV, while Panel B provides statistics of the five benchmark strategies. Panels C considers equally weighted portfolios of the five benchmark strategies, and Panels D and E construct global minimum variance and mean-variance optimized portfolios.[6] For the global minimum variance and the mean-variance optimized portfolios we require the covariance matrix and the expected excess returns of the five strategies. For simplicity, we use sample estimates using our entire sample. In that sense, these are static and unconditionally optimized portfolios. Moreover, there is a look-ahead bias, and the performance is not out-of-sample. Accordingly, an outperformance of MV (which does not suffer from a look ahead bias and its performance is OOS) is even more impressive. The portfolios are constructed using diverse subsets of the five strategies.[7]

The main findings of Table 6 are as follows. MV offers returns with a higher OOS unconditional Sharpe ratio than all benchmarks. In many cases the difference in Sharpe ratios is statistically significant. In addition, MV has a positive skewness, while the skewness is negative for all other strategies, except for MOM. In the same spirit MV has the most desirable maximum drawdown per 1% expected excess return |MDD|/Mean. Thus, MV is more attractive in terms of the Sharpe ratio as well as in terms of crash risks. This finding emphasizes the importance of MV in comparison to popular benchmarks in the literature.

[6] Global-minimum-variance portfolio is the left-most portfolio on the efficient frontier in the variance-mean coordinate plane.

[7] The subsets are $\{HML, DDOL\}$, $\{HML, DDOL, MOM\}$, $\{HML, DDOL, VAL\}$, $\{HML, DDOL, DOL\}$, $\{HML, DDOL, MOM, VAL\}$, and $\{HML, DDOL, MOM, VAL, DOL\}$.

Table 6: **Performance of MV and Popular Currency Portfolios in the Literature**

	Mean (%)	Vol (%)	SR	ΔSR	Skew	Kurt	$\frac{\lvert MDD \rvert}{Mean}$
Panel A. MV	20.30	18.06	1.12		0.67	6.63	1.78
Panel B. Long-short							
DOL	1.55	8.63	0.18	0.94***	-0.17	3.60	27.21
DDOL	5.15	8.52	0.61	0.52**	-0.16	3.71	4.68
HML	5.23	9.66	0.54	0.58**	-0.89	5.59	8.18
MOM	2.34	13.79	0.17	0.95***	0.40	6.91	12.98
VAL	4.55	8.68	0.52	0.60**	-0.01	4.18	5.01
Panel C. 1/N							
HML,DDOL	5.19	6.85	0.76	0.37*	-0.61	4.44	4.80
HML,DDOL,MOM	4.24	6.63	0.64	0.49**	-0.58	4.03	3.67
HML,DDOL,VAL	4.98	5.64	0.88	0.24	-0.49	5.00	2.91
HML,DDOL,DOL	3.98	6.23	0.64	0.49**	-0.45	5.15	5.56
HML,DDOL,MOM,VAL	4.32	5.36	0.81	0.32	-0.41	4.45	2.50
HML,DDOL,MOM,VAL,DOL	3.76	4.70	0.80	0.32*	-0.56	4.19	2.57
Panel D. Global Minimum Variance							
HML,DDOL	5.19	6.80	0.76	0.36*	-0.55	4.33	4.30
HML,DDOL,MOM	4.67	6.23	0.75	0.38*	-0.70	3.87	3.62
HML,DDOL,VAL	4.93	5.45	0.91	0.22	-0.28	4.72	2.17
HML,DDOL,DOL	4.03	6.23	0.65	0.48**	-0.45	5.14	5.41
HML,DDOL,MOM,VAL	4.54	5.01	0.91	0.22	-0.16	5.08	1.82
HML,DDOL,MOM,VAL,DOL	3.43	4.26	0.81	0.32	-0.19	4.57	3.21
Panel E. Mean-Variance Optimized							
HML,DDOL	5.29	6.94	0.76	0.36*	-0.56	4.34	4.34
HML,DDOL,MOM	5.43	7.03	0.77	0.35*	-0.64	4.02	3.90
HML,DDOL,VAL	7.48	8.25	0.91	0.22	-0.31	4.67	2.14
HML,DDOL,DOL	5.45	7.04	0.77	0.35*	-0.54	4.15	4.06
HML,DDOL,MOM,VAL	7.81	8.43	0.93	0.20	-0.29	4.42	1.97
HML,DDOL,MOM,VAL,DOL	7.85	8.45	0.93	0.20	-0.29	4.40	1.94

Notes: Panel A shows the performance of MV. Panel B shows the performance of popular long-short currency portfolios. Panel C, D, and E show the performance of equally-weighted, global minimum variance, and mean-variance optimized portfolios of subsets of the strategies in Panel B. All statistics are annualized.

In summary, Table 6 provides additional evidence for the importance of MV, and its outperformance over a broad mix of well-known benchmarks.

4.2 Pricing

This section empirically studies MV's performance from an asset pricing perspective. Specifically, we address the question whether MV's excess return and Sharpe ratio can be explained by, and hence is a compensation for MV's loadings on well-known risk factors in the FX literature.

Table 7 reports MV's abnormal return estimates in twelve different linear factor models. We consider both traded factors (DOL, HML, $DDOL$, MOM, VAL, SW, HML_{VM}, SW_{VM}, MKT, and INT) and non-traded factors (VOL, ILL, and $SKEW$). For each model, the pricing of MV's returns is evaluated based on its model-implied abnormal returns under two alternative approaches. The factor risk premia are estimated as the time-series average of the factor returns in the first approach (provided that the factors are traded portfolios), and as the implied premia in a cross-sectional pricing regression in the second approach.[8] The abnormal returns are referred to as α_{MV} (in the time-series estimation approach), and α^*_{MV} (in the cross-sectional estimation approach) in Table 7. For completeness, the corresponding estimates of MV's loadings (i.e., betas) on various pricing factors are reported in Tables 11 and 12 in Appendix 6.

Table 7 shows that across all linear factor models, strategy MV earns large positive and statistically significant abnormal returns, ranging from 9% to 20% for both time-series and cross-sectional estimations. The positive signs of the abnormal returns indicate that MV's excess returns are not explained by popular FX market risks. In other words, MV is not spanned by well-known currency strategies or factors. In summary, Table 7 establishes that MV's return is an anomaly with respect to popular FX risk factors, and indicates the existence of new risk sources to rationalize MV's return.

[8] Risk premia of factors that are not traded need to be estimated in the cross-sectional pricing regression using the generalized method of moments (GMM). To estimate risk premia in the cross-sectional pricing regression we use the following test assets: 5 interest sorted, 5 momentum sorted, 5 value sorted portfolios, $DDOL$, SW, SW_{CN}, HML_{VM}, SW_{VM}, and various mean-variance optimized portfolios.

Table 7: **Abnormal Returns of the MV**

Factors	α^*_{MV} (in %)	(t-stat α^*)	α_{MV} (in %)	(t-stat α)	R^2 (%)
DOL, HML	16.361***	(4.637)	15.880***	(4.743)	20
DOL, $DDOL$	18.453***	(4.718)	18.117***	(4.768)	5
DOL, SW	16.236***	(4.724)	14.690***	(4.859)	24
DOL, HML_{VM}	15.840***	(4.505)	12.510***	(4.073)	30
DOL, SW_{VM}	15.116***	(4.289)	12.268***	(4.005)	27
DOL, VOL	19.068***	(5.040)			4
DOL, $SKEW$	17.412***	(4.819)			3
DOL, ILL	19.120***	(4.941)			3
MKT, INT	19.575***	(5.033)	19.945***	(5.049)	0
Set A (8 assets)	12.588***	(3.909)	9.163***	(3.579)	43
Set B (11 assets)	12.068***	(3.879)			44
Set C (13 assets)	9.546***	(3.308)			45

Notes: Abnormal returns of MV according to linear factor models. Time-series regression: $MV_t = \alpha_{MV} + \sum_i \beta_{MV,i} F_{i,t} + \varepsilon_t$. Cross-sectional relationship: $E[MV_t] = \alpha^*_{MV} + \sum_i \beta_{MV,i}\gamma_i$. $E[x_t]$ is the time-series average of x_t, α_{MV} the abnormal return in the time-series equation, α^*_{MV} the abnormal return in the cross-sectional equation, $\beta_{MV,i}$ is the factor loading of the MV on factor $F_{i,t}$, and γ_i is the risk premium (estimated in the cross-section) of factor $F_{i,t}$. Set A contains factors $DOL, HML, DDOL, MOM, VAL, SW, HML_{VM}$ and SW_{VM}. Set B contains all factors in Set A plus $VOL, SKEW$ and ILL. Set C contains all factors in Set B plus MKT and INT. The reported α_{MV} and α^*_{MV} are annualized. R^2 measures the fit of the time-series relationship. Newey and West (1987) robust t-statistics are reported in parentheses next to the coefficient estimates. Significance at the 1%, 5% or 10% level are indicated by ***,** or *.

5 Robustness

This section carries out various robustness checks to strengthen our empirical analysis. We briefly describe these robustness checks here, and relegate details to the Online Appendix. *MV*'s construction is based on exchange rate forward discounts as proxies for the conditional expected excess returns and a robust PCA-based estimation of the covariance matrix (Section 2.2). We verify the robustness of these estimations.

For alternative estimates of the conditional expected excess returns we either use a simple historical average of returns, or forward discounts together with an elastic net prediction of exchange rate changes $\Delta s_{i,t+1}$. For the exchange rate predictions we use 18 predictors related to past exchange rate changes, volatility, liquidity, skewness and intermediary capital factors. The OOS R^2 to predict 1-month ahead exchange rate changes are positive but relatively small (0.36% on average). We find that the historical average of returns is a bad proxy. Moreover, the elastic net predictions also do not provide any economic value. That is, the OOS performance of *MV* is strongest in our baseline construction using the random walk assumption for the exchange rates (Table 13 in the Online Appendix). The reasons is that the predictions are close to zero and the prediction noise is relatively large. The noise has a negative effect on the OOS performance of *MV*.

For the exchange rate covariance matrix, we employ 30 alternative estimates, input them into *MV*, and compare the resulting performances with the original *MV* performance that is based on the robust estimation of the covariance matrix using PCA. The results suggest that our specification of *MV* in the main text remains the best choice (Table 14 in the Online Appendix).

In addition, we test the sensitivity of *MV* to changes in the threshold value $\bar{\lambda}$ when we decide how many PCs to remove in our PCA. In the main text we remove all PCs that explain less than $\bar{\lambda} = 1\%$ of the common variation in exchange rate changes. In Figure 1 in the Online Appendix we construct *MV* and report the unconditional Sharpe ratio of *MV* for various threshold values $\bar{\lambda}$. We further provide a brief intuitive motivation of our PCA and argue that our results can be motivated by a limits to arbitrage explanation.

Furthermore, to examine the sensitivity of *MV*'s performance to the set of currencies, we repeat our analysis for various subsets of currencies. We show that MV's outperformance is resilient with respect to different subsets of currencies (Table 15, Online Appendix).

Finally, we extend the robustness checks of our analysis to a larger set of 29 developed and emerging currencies and taking into account transaction costs. The results of these robustness checks broadly uphold our main findings that MV outperforms alternative strategies and market timing is economically important. The results for 29 developed and emerging currencies are repetitive and we not reported them. These results are available upon request.

6 Conclusion

We study the importance of market timing in FX markets. We find that MV (mean-variance optimized portfolio that maximizes unconstraint mean-variance preferences) outperforms alternative mean-variance optimized currency portfolios with leverage, risk (i.e., conditional volatility), or yield targets in our out-of-sample analysis. We conclude that it is costly to impose leverage and risk (i.e., conditional volatility) limits in the portfolio construction, or follow portfolio constructions with suboptimal market timing policies, even if these strategies are conditionally mean-variance efficient. Such suboptimal policies deliver is a significant reduction in the unconditional Sharpe ratio and increase in crash risks.

We further document that the market timing of MV has useful information to predict returns and risks in FX markets. Moreover, MV is not spanned by well-known currency pricing factors, and thus, it poses a new challenge for popular pricing models in the literature.

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Appendix: Description of Additional Strategies

This appendix briefly discuss FX strategies whose definitions are omitted in the main text but are used in the paper.

DOL, $DDOL$, HML: The Dollar DOL is a traded factor that invests equally in all currencies (Lustig et al., 2011), i.e., $\theta_{i,t}^{DOL} = \frac{1}{N}$. The dynamic or carry Dollar $DDOL$ takes a long (short) position in the DOL when the median forward discount (against the USD) across all exchange rates is positive (negative) (Lustig et al., 2014), $\theta_t^{DDOL} = sign(median(\{f_{j,t}\}_{j=1}^N))\theta_t^{DOL}$. HML is an equally weighted high-minus-low forward discount carry portfolio introduced in Lustig and Verdelhan (2007). Currencies are sorted into quintiles based on their forward discount $f_{i,t}$ (against the USD). The HML takes a long position in the equally weighted portfolio of currencies in the top quintile and a short position in the equally weighted portfolio of currencies in the bottom quintile.

SW, SW_{CN}: Portfolio SW has weights equal to the forward discounts (against the USD) $\theta_t^{SW} = f_t$, which refines the strategy HML by taking into account the size and time variation in the forward discounts. Thus, the SW is dynamic and uses market timing based on the absolute size of the forward discounts. Portfolio SW_{CN} is a rescaled version of SW so that the notional value (i.e., total exposure to the USD) does not change through time.

HML_{VM}, SW_{VM}: These are respectively the volatility managed portfolios of HML and SW, following Fleming et al. (2001) and Moreira and Muir (2017). We compute the conditional volatility of the HML and SW denoted by σ_t^{HML} and σ_t^{SW} using daily returns of these factors over the past month and define the volatility managed factors as $\theta_t^{HML_{VM}} = \frac{\theta_t^{HML}}{(\sigma_t^{HML})^2}$ and $\theta_t^{SW_{VM}} = \frac{\theta_t^{SW}}{(\sigma_t^{SW})^2}$.

MOM: Momentum MOM strategies are popular in equity and FX markets (Burnside et al., 2011; Menkhoff et al., 2012b). Currencies are sorted on their past 12-month performance into quintiles. The top quintile contains the winner currencies and the bottom quintile the loser currencies. We build equally weighted currency portfolios for each quintile. We denote these five portfolios by $MomP_i$ $\forall i \in \{1, \ldots, 5\}$. MOM takes a long position in the equally weighted portfolio of winner currencies and a short position in the equally weighted portfolio of loser currencies.

VAL: The value VAL strategy assumes that in the long run undervalued currencies (with low real exchange rates) appreciate against overvalued currencies (with high real exchange rates) (Bilson, 1984; Menkhoff et al., 2017). Currencies are sorted on their past real exchange rates against the USD into quintiles (the top quintile contains overvalued, the bottom quintile undervalued currencies). We proxy the real exchange rate of currency i against USD using the PPP at time t (i.e., the ratio of the prices in currency i and the USD of a representative consumption bundle) multiplied by the nominal exchange rate $X_{i,t}$. VAL takes a long position in the equally weighted portfolio of currencies in the bottom quintile and a short position in the equally weighted portfolio of currencies in the top quintile.

VOL, ILL, $SKEW$: First, the global FX market volatility at the end of month t is constructed as,

$$\widehat{VOL}_t = \frac{1}{T_t \times N} \sum_{\tau=1}^{T_t} \sum_{I=1}^{N} \left| \Delta s_{d,i,\tau} \right|,$$

where $\Delta s_{d,i,\tau}$ is the daily exchange rate growth of currency i against the USD on day τ in month t, T_t is the number of trading days in month t. The VOL_t index is the time series of residuals after estimating an AR(1) process for the $\widehat{VOL}_t$, i.e., $\widehat{VOL}_t = \rho_v \widehat{VOL}_{t-1} + VOL_t$. Thus, VOL captures unexpected changes in the volatility (Menkhoff et al., 2012a). Second, a monthly systematic FX illiquidity measure $\widehat{ILL}$ is constructed as the average of standardized daily relative bid-ask spread and Corwin and Schultz (2012) liquidity estimate within a month and across all currencies. The ILL_t index is the time series of residuals after estimating an AR(1) process for the $\widehat{ILL}_t$, i.e., $\widehat{ILL}_t = \rho_{ILL}\widehat{ILL}_{t-1}+ILL_t$. Thus, ILL captures unexpected changes in the illiquidity (Karnaukh et al., 2015). Third, $SKEW$ is constructed to capture the average skewness of exchange rate growths of investment (i.e., positive forward discount) net of funding (i.e., negative forward discount) currencies (Rafferty, 2012),

$$SKEW_t = \frac{1}{N}\sum_i sign(f_{i,t-1})\frac{\frac{1}{T_t}\sum_{\tau}^{T_t}\left(\Delta s_{d,i,\tau} - \overline{\Delta s}_{d,i,\tau}\right)^3}{\left(\frac{1}{T_t}\sum_{\tau}^{T_t}\left(\Delta s_{d,i,\tau} - \overline{\Delta s}_{d,i,\tau}\right)^2\right)^{\frac{3}{2}}},$$

where $\Delta s_{d,i,\tau}$ is the daily exchange rate growth of currency i against the USD on day τ in month t, $\overline{\Delta s}_{d,i,\tau} = \frac{1}{T_t}\sum_{h=1}^{T_t}\Delta s_{d,i,h}$ is the sample average of the daily exchange rate growth $\Delta s_{d,i,\tau}$ in month t, and T_t is the number of trading days in month t.

Finally, we use two stock market factors, the value weighted US stock market index MKT and the intermediary capital risk factor INT of He et al. (2017).

Online Appendix

This online appendix presents (i) additional empirical results of the main text, and (ii) various robustness checks concerning strategy MV that are described in Section 5 in the main text.

A Additional Empirical Results

Extending the results of Table 3 in the main text, Tables 8-10 below list the slope coefficient estimates and R^2 of predictive regressions at the individual currency level. Table 8 concerns the regressions of the change $\Delta rx_{i,t+1}$ in currency i's realized return on the change $\Delta\theta_{i,t}^{MV}$ in MV's portfolio weight in currency i, Table 9 the regressions of the change $\Delta Vol(rx_{i,t+1})$ in currency i's realized return volatility on the change $\Delta|\theta_{i,t}^{MV}|$ in MV's exposure to currency i, and Table 10 the regressions of the change $\Delta Skew(rx_{i,t+1})$ in currency i's realized return skewness on $\Delta\theta_{i,t}^{MV}$. These tables confirm the predictability of the future performance of individual currency returns.

Table 8: **Predictive Regressions: Regression of $\Delta rx_{i,t+1}$ on $\Delta\theta^{MV}_{i,t}$**

$\Delta rx_{i,t+1}$	$\Delta\theta^{MV}_{i,t}$	(t-stat)	$\mu_t^{MV_{t+1}}$	(t-stat)	$\sigma_t^{MV_{t+1}}$	(t-stat)	$Adj.R^2$ (%)
Australia	0.0038**	(2.29)	-0.0276	(-0.39)	-0.1896	(-0.14)	22.32%
Belgium	0.0003	(0.24)	0.1169	(1.39)	-0.9622	(-0.37)	27.95%
Canada	0.0001	(0.22)	0.0206	(0.44)	-1.4278	(-0.99)	30.06%
Denmark	0.0022**	(2.09)	0.0123	(0.21)	0.1134	(0.07)	26.93%
Euro	0.0011	(0.11)	-0.6297	(-0.95)	0.1335	(0.02)	34.27%
France	0.0009	(0.40)	0.1428**	(2.03)	-3.6455	(-1.62)	27.41%
Germany	0.0030	(1.25)	0.1524*	(1.79)	-3.6846	(-1.48)	27.50%
Italy	-0.0016	(-1.65)	0.3130***	(3.11)	-2.9814	(-0.99)	29.40%
Japan	0.0020	(1.47)	0.0851	(1.46)	1.7387	(1.02)	27.68%
Netherlands	0.0007	(0.38)	0.1820**	(2.01)	-3.7457	(-1.25)	28.19%
New Zealand	0.0015	(1.17)	0.1130**	(1.98)	-2.0373	(-1.47)	26.16%
Norway	0.0007	(0.89)	0.0183	(0.43)	-0.0994	(-0.07)	28.09%
Sweden	0.0007	(0.95)	0.0016	(0.03)	0.8588	(0.52)	20.27%
Switzerland	0.0000	(0.00)	0.0857	(1.31)	0.3815	(0.20)	25.90%
United Kingdom	0.0000	(0.06)	0.0771	(0.87)	-0.2163	(-0.13)	21.41%
Panel	0.0006***	(3.08)	0.0680**	(2.12)	-0.5609	(-0.52)	26.46%

Notes: 1-month ahead predictive regressions: $\Delta rx_{i,t+1} = c_{const} + c_{trend}t + c_\theta\hat{\theta} + c_\mu\mu_t^{rx_{i,t+1}} + c_\sigma\sigma_t^{rx_{i,t+1}} + c_{lag}\Delta rx_{i,t} + \epsilon_{i,t}$. c_{const} is a constant term, $c_{trend}t$ controls for any time-series trend, and c_{lag} controls for the lag of the LHS variable. $\Delta rx_{i,t+1} = (rx_{i,t+1} - rx_{i,t})$ is the change in realized monthly return of currency i against the USD. $\hat{\theta} = \Delta\theta^{MV}_{i,t} = (\theta^{MV}_{i,t} - \theta^{MV}_{i,t-1})$ is the change in portfolio weight of MV in currency i. Each row represents the regression of one currency. The last row is a panel regression including all above currencies and currency fixed effects. Significance of predictor at the 1%, 5% or 10% level are indicated by ***, ** or *, where standard errors are calculated using Newey and West (1987) to account for heteroskedasticity and auto-correlation (and cross-currency correlation in the panel regression).

Table 9: **Predictive Regressions: Regression of ΔVol$(rx_{i,t+1})$ on $\Delta|\theta_{i,t}^{MV}|$**

ΔVol$(rx_{i,t+1})$	$\Delta\|\theta_{i,t}^{MV}\|$	(t-stat)	$\mu_t^{MV_{t+1}}$	(t-stat)	$\sigma_t^{MV_{t+1}}$	(t-stat)	$Adj.R^2$ (%)
Australia	-0.0021***	(-4.58)	-0.0419*	(-1.83)	2.1776**	(2.35)	23.31%
Belgium	-0.0018***	(-4.96)	-0.0500**	(-2.03)	3.5749***	(6.89)	33.81%
Canada	-0.0007***	(-3.29)	0.0039	(0.26)	1.4214**	(2.47)	14.03%
Denmark	-0.0021***	(-4.04)	-0.0454***	(-2.72)	2.5839***	(5.60)	30.53%
Euro	0.0017	(0.84)	-0.0520	(-0.91)	2.1090*	(2.03)	-0.59%
France	-0.0023***	(-4.07)	-0.0862***	(-2.97)	4.2739***	(6.25)	39.85%
Germany	-0.0022***	(-5.02)	-0.0782***	(-2.97)	4.1336***	(6.14)	38.79%
Italy	-0.0007***	(-2.97)	-0.1169***	(-2.67)	3.4597***	(6.10)	40.78%
Japan	-0.0013***	(-4.13)	-0.0029	(-0.16)	2.8229***	(6.01)	30.64%
Netherlands	-0.0015***	(-3.71)	-0.1198***	(-5.20)	5.4053***	(9.61)	41.74%
New Zealand	-0.0021***	(-4.26)	-0.0371	(-1.51)	1.7602***	(3.74)	23.84%
Norway	-0.0010***	(-5.35)	-0.0354**	(-2.08)	2.0967***	(3.85)	31.01%
Sweden	-0.0006**	(-2.14)	-0.0423**	(-2.27)	1.8803***	(3.30)	26.24%
Switzerland	-0.0015***	(-6.43)	-0.0484***	(-2.63)	3.4903***	(4.30)	36.98%
United Kingdom	-0.0010***	(-4.21)	-0.0228	(-1.10)	1.8570***	(3.41)	25.57%
Panel	-0.0012***	(-6.93)	-0.0353***	(-3.77)	2.2488***	(5.56)	28.33%

Notes: 1-month ahead predictive regressions: $\Delta\text{Vol}(rx_{i,t+1}) = c_{const} + c_{trend}t + c_\theta\hat{\theta} + c_\mu\mu_t^{rx_{i,t+1}} + c_\sigma\sigma_t^{rx_{i,t+1}} + c_{lag}\Delta\text{Vol}(rx_{i,t}) + \epsilon_{i,t}$. c_{const} is a constant term, $c_{trend}t$ controls for any time-series trend, and c_{lag} controls for the lag of the LHS variable. $\Delta\text{Vol}(rx_{i,t+1})$ = (Volatility of $rx_{i,t+1}$−Volatility of $rx_{i,t}$) is the change in realized monthly volatility of the returns of currency i against the USD. $\hat{\theta} = \Delta|\theta_{i,t}^{MV}| = (|\theta_{i,t}^{MV}| - |\theta_{i,t-1}^{MV}|)$ is the change in the exposure of MV to currency i, and on other regressors. Each row represents the regression of one currency. The last row is a panel regression including all above currencies and currency fixed effects. Significance of predictor at the 1%, 5% or 10% level are indicated by ***, ** or *, where standard errors are calculated using Newey and West (1987) to account for heteroskedasticity and auto-correlation (and cross-currency correlation in the panel regression).

Table 10: **Predictive Regressions: Regression of ΔSkew$(rx_{i,t+1})$ on $\Delta\theta_{i,t}^{MV}$**

ΔSkew$(rx_{i,t+1})$	$\Delta\theta_{i,t}^{MV}$	(t-stat)	$\mu_t^{MV_{t+1}}$	(t-stat)	$\sigma_t^{MV_{t+1}}$	(t-stat)	$Adj.R^2$ (%)
Australia	0.0072	(0.82)	0.0962	(0.36)	-4.0728	(-1.21)	18.47%
Belgium	0.0111	(1.55)	-0.3026	(-1.07)	6.9059	(0.70)	25.54%
Canada	-0.0074	(-1.34)	0.7399*	(1.71)	-3.4030	(-0.78)	28.27%
Denmark	0.0069	(1.09)	-0.3328	(-1.20)	10.2398	(1.44)	28.85%
Euro	0.0691**	(2.08)	-6.8235***	(-4.41)	20.5388	(1.09)	20.45%
France	0.0025	(0.22)	-0.1364	(-0.30)	10.9514	(0.73)	29.07%
Germany	0.0401***	(2.66)	-0.1714	(-0.47)	7.3713	(0.55)	35.32%
Italy	0.0030	(0.65)	0.2744	(0.56)	-3.0514	(-0.21)	24.02%
Japan	0.0183**	(2.47)	0.2284	(0.84)	7.5808	(1.36)	26.35%
Netherlands	-0.0076	(-0.62)	-0.5752	(-1.05)	7.6998	(0.40)	35.06%
New Zealand	0.0141**	(2.11)	0.4276	(1.15)	-2.7049	(-0.68)	24.50%
Norway	-0.0035	(-0.73)	-0.0927	(-0.31)	-4.8343	(-0.74)	28.26%
Sweden	-0.0078**	(-1.98)	-0.2954	(-0.99)	-0.0975	(-0.02)	25.27%
Switzerland	0.0036	(0.58)	0.0051	(0.02)	6.2590	(0.67)	24.69%
United Kingdom	0.0050	(0.63)	0.1465	(0.36)	1.6728	(0.32)	26.74%
Panel	0.0035***	(2.67)	0.0111	(0.08)	0.9527	(0.28)	27.00%

Notes: 1-month ahead predictive regressions: $\Delta\text{Skew}(rx_{i,t+1}) = c_{const} + c_{trend}t + c_\theta\hat{\theta} + c_\mu\mu_t^{rx_{i,t+1}} + c_\sigma\sigma_t^{rx_{i,t+1}} + c_{lag}\Delta\text{Skew}(rx_{i,t}) + \epsilon_{i,t}$. c_{const} is a constant term, $c_{trend}t$ controls for any time-series trend, and c_{lag} controls for the lag of the LHS variable. $\Delta\text{Skew}(rx_{i,t+1})$ = (Skewness of $rx_{i,t+1}$ - Skewness of $rx_{i,t}$) is the change in realized monthly skewness of the returns of currency i against the USD. $\hat{\theta} = \Delta\theta_{i,t}^{MV} = (\theta_{i,t}^{MV} - \theta_{i,t-1}^{MV})$ is the change in portfolio weight of MV in currency i. Each row represents the regression of one currency. The last row is a panel regression including all above currencies and currency fixed effects. Significance of predictor at the 1%, 5% or 10% level are indicated by ***, ** or *, where standard errors are calculated using Newey and West (1987) to account for heteroskedasticity and auto-correlation (and cross-currency correlation in the panel regression).

Extending the results of Table 7 of the main text, Tables 11 and 12 below list the corresponding estimates of MV's loadings (i.e., betas) on various pricing factors.

Table 11: **Alpha and Betas of MV, Part 1**

Factors	MV					
α^*_{MV} (in %)	16.361***	18.453***	16.236***	15.840***	15.116***	12.588***
	(4.637)	(4.718)	(4.724)	(4.505)	(4.289)	(3.909)
α_{MV} (in %)	15.880***	18.117***	14.690***	12.510***	12.268***	9.163***
	(4.743)	(4.768)	(4.859)	(4.073)	(4.005)	(3.579)
$\beta_{MV,DOL}$	0.155	0.133	-0.161	0.404***	0.267**	0.264*
	(1.194)	(1.253)	(-1.432)	(3.660)	(2.441)	(2.104)
$\beta_{MV,HML}$	0.800***					0.019
	(6.259)					(0.176)
$\beta_{MV,DDOL}$		0.384***				-0.331**
		(2.916)				(-2.193)
$\beta_{MV,MOM}$						0.086
						(1.329)
$\beta_{MV,VAL}$						0.365**
						(2.760)
$\beta_{MV,SW}$			1.034***			0.740***
			(6.578)			(5.294)
$\beta_{MV,HML_{VM}}$				0.904***		0.417**
				(5.351)		(2.269)
$\beta_{MV,SW_{VM}}$					0.686***	0.267*
					(5.984)	(2.096)
R^2 (in %)	20	5	24	30	27	43

Notes: Estimation of linear factor model. Time-series relationship: $E[MV_t] = \alpha_{MV} + \sum_i \beta_{MV,i} E[F_{i,t}]$. Cross-sectional relationship: $E[MV_t] = \alpha^*_{MV} + \sum_i \beta_{MV,i}\gamma_i$. $E[x_t]$ is the time-series average of x_t, α_{MV} the abnormal return in the time-series equation, α^*_{MV} the abnormal return in the cross-sectional equation, $\beta_{MV,i}$ is the factor loading of the MV on factor $F_{i,t}$, and γ_i is the risk premium (estimated in the cross-section) of factor $F_{i,t}$. The reported α_{MV} and α^*_{MV} are annualized. R^2 measures the fit of the time-series relationship. Newey and West (1987) robust t-statistics are reported in parentheses below coefficient estimates. Significance at the 1%, 5% or 10% level are indicated by ***,** or *.

Table 12: **Alpha and Betas of MV, part 2**

Factors	MV					
α^*_{MV} (in %)	19.068*** (5.040)	17.412*** (4.819)	19.120*** (4.941)	12.068*** (3.879)	19.575*** (5.033)	9.546*** (3.308)
α_{MV} (in %)	20.365*** (5.538)	21.121*** (5.377)	19.448*** (4.965)	8.907*** (3.094)	19.945*** (5.049)	9.554*** (3.316)
$\beta_{MV,DOL}$	0.259** (2.210)	0.287** (2.493)	0.277** (2.360)	0.251* (2.053)		0.301** (2.534)
$\beta_{MV,HML}$				-0.017 (-0.137)		0.062 (0.472)
$\beta_{MV,DDOL}$				-0.317* (-2.089)		-0.334* (-2.172)
$\beta_{MV,MOM}$				0.088 (1.331)		0.090 (1.413)
$\beta_{MV,VAL}$				0.380** (2.688)		0.396** (2.874)
$\beta_{MV,SW}$				0.760*** (5.433)		0.743*** (5.329)
$\beta_{MV,HML_{VM}}$				0.429** (2.329)		0.408* (2.140)
$\beta_{MV,SW_{VM}}$				0.254* (1.968)		0.269* (2.152)
$\beta_{MV,VOL}$	-0.261** (-2.368)			-0.067 (-0.784)		-0.081 (-0.957)
$\beta_{MV,SKEW}$		-0.216** (-2.501)		0.043 (0.467)		0.051 (0.561)
$\beta_{MV,ILL}$			-0.173 (-1.344)	-0.049 (-0.480)		-0.075 (-0.763)
$\beta_{MV,MKT}$					-0.052 (-0.494)	-0.206** (-2.386)
$\beta_{MV,INT}$					0.060 (0.973)	0.033 (0.602)
R^2 (in %)	4	3	3	44	0	45

Continued on next page

Notes: Estimation of linear factor model. Time-series relationship: $E[MV_t] = \alpha_{MV} + \sum_i \beta_{MV,i} E[F_{i,t}]$. Cross-sectional relationship: $E[MV_t] = \alpha^*_{MV} + \sum_i \beta_{MV,i}\gamma_i$. $E[x_t]$ is the time-series average of x_t, α_{MV} the abnormal return in the time-series equation, α^*_{MV} the abnormal return in the cross-sectional equation, $\beta_{MV,i}$ is the factor loading of the MV on factor $F_{i,t}$, and γ_i is the risk premium (estimated in the cross-section) of factor $F_{i,t}$. The reported α_{MV} and α^*_{MV} are annualized. R^2 measures the fit of the time-series relationship. Newey and West (1987) robust t-statistics are reported in parentheses below coefficient estimates. Significance at the 1%, 5% or 10% level are indicated by ***,** or *.

B Robustness Checks

We examine the robustness of MV with respect to parameter estimations. Table 13 considers MV versus (i) $MV_{\widehat{\Delta s}}$, which attempts to predict exchange rate changes to improve our proxies of conditional expected excess returns, and (ii) MV_{AveRet}, which estimates the conditional expected excess returns using the average realized excess returns over the past 6 months for all currencies.

Recall that $\mu_{i,t} = f_{i,t} + \Delta s_{i,t+1}$, and in the main text we assume $\Delta s_{i,t+1} = 0$. For $MV_{\widehat{\Delta s}}$, we attempt to predict 1-month ahead exchange rate changes $\Delta s_{i,t+1}$ to potentially improve the estimation of μ_t. We use 18 predictors to predict 1-month ahead exchange rate changes $\Delta s_{i,t+1}$. We use the following predictors that are specific to currency i: current forward discount, past 1-month, 3-month, 6-month and 12-month returns, current PPP adjusted exchange rate, and a currency specific liquidity measure based on the average bid-ask spread and Corwin and Schultz (2012) estimated within the most recent month. Moreover, we use the following FX market wide (non-currency i specific) predictors: FX market volatility $\widehat{VOL}_t$ and its most recent innovation VOL_t, FX market illiquidity $\widehat{ILL}_t$ and its most recent innovation ILL_t, most recent skewness measure $SKEW_t$, average correlation across all exchange rate changes within the most recent month, most recent level and change in the intermediary capital risk factor INT_t, most recent stock market return MKT_t, and most recent level and change in the Pastor and Stambaugh (2003) stock market illiquidity measure. To predict the exchange rate change $\Delta s_{i,t+1}$ from the end of month t to $t+1$, we train our prediction model in an expanding window using information until the end of month t, and we employ an elastic net estimation. Therefore, our predictions are out-of-sample.

Table 13: **Alternative Estimates of μ_t**

	MV	$MV_{\widehat{\Delta s}}$	MV_{AveRet}
Mean	20.30	17.61	27.42
Vol	18.06	18.00	44.50
SR	1.12	0.98	0.62
ΔSR		0.14	0.51**
(p-value)		(0.248)	(0.017)
Skew	0.67	1.21	0.40
Kurt	6.63	7.39	3.90
MDD	-36.07	-31.32	-92.72
|MDD|/Mean	1.78	1.86	3.38

Notes: Statistics of monthly out-of-sample excess returns. All portfolios are mean-variance efficient: $MV_{\widehat{\Delta s}}$ attempts to predict exchange rate changes $\Delta s_{i,t+1}$ to improve the estimate of conditional expected excess returns μ_t. MV_{AveRet} estimates conditional expected excess returns μ_t using average realized excess returns over the past 6 months. ΔSR is the difference between the Sharpe ratio of our MV and that of other strategies. The reported p-value is the test for $\Delta\text{SR} = 0$. Standard errors are robust to heteroskedasticity and autocorrelation (Ledoit and Wolf, 2008). All statistics are annualized.

We evaluate the OOS R^2 of our predictions with respect to the random walk benchmark model, i.e., $\Delta s_{i,t+1} = 0$ (Meese and Rogoff, 1983). The average OOS R^2 of our predictions is 0.36%, where the Italian Lira is most predictable with an OOS R^2 of 1.55%, and the Euro is not predictable with an OOS R^2 of -2.55%. Note that the Euro is only available since 1999 and predictability appears to be more difficult in more recent times. Finally, we observe that the Euro is the only currency with a negative OOS R^2. Positive OOS R^2 is important to verify that our model is able to provide useful predicitons. Details about our estimation are available upon request.

Given our OOS predictions of $\widehat{\Delta s}_{i,t+1}$, we estimate the conditional expected return as $\mu_t = f_t + \widehat{\Delta s}_{t+1}$ and construct $MV_{\widehat{\Delta s}}$. First, Table 13 shows that historical average returns are a poor proxy for conditional expected excess returns. MV_{AveRet} significantly underperforms MV in terms of the unconditional Sharpe ratio and maximum drawdown per 1% expected excess return. Moreover, we find that MV has an unconditional Sharpe ratio of 1.12 in our main analysis but it drops to 0.98 for $MV_{\widehat{\Delta s}}$ when we attempt to predict $\Delta s_{i,t+1}$. The maximum drawdown per 1% expected return is virtually identical. This is interesting as it suggests that our predictions of exchange rate changes have no economic value although the predictive regressions deliver positive OOS R^2. The reason is that the OOS R^2 is relatively small while the predictions introduce additional noise, which negatively affects the stability and OOS performance of MV.

In Table 14 we explore alternative estimations of the covariance matrix, and report the OOS performance of MV corresponding to the various estimates. For the conditional expected excess returns, we impose again the random walk assumption of exchange rate changes $\Delta s_{i,t+1} = 0$ as in the main text of the paper. For the exchange rate covariance matrix, we employ 30 alternative estimates. We use sample covariance estimates, the shrinkage approach of Ledoit and Wolf (2003), and our PCA approach to obtain a robust inverse of the matrix. For these three methods we consider 1-year, 5-year, 10-year rolling windows, and expanding windows for our estimation. For the expanding windows we consider equal weights and exponentially decreasing weights (EWMA) of the data. Finally, we consider diagonalized versions of all these 15 covariance matrices. Row 15 in Table 14 reports the performance of MV corresponding to the estimation approach in the main text of the paper. Overall, MV

Table 14: **Alternative Estimations of Ω_t**

	Mean (%)	Vol (%)	SR	ΔSR	Skew	Kurt	$\frac{\lvert MDD \rvert}{Mean}$
1-year sample	32.75	52.67	0.62	0.50	9.29	180.01	4.60
1-year shrink	29.51	39.80	0.74	0.38	3.66	108.72	4.80
1-year PCA	13.54	12.91	1.05	0.08	-0.65	6.62	4.04
5-year sample	18.73	31.12	0.60	0.52	-7.89	104.28	11.52
5-year shrink	18.36	30.74	0.60	0.53	-7.99	106.23	11.63
5-year PCA	11.84	12.82	0.92	0.20	-0.23	8.58	4.84
10-year sample	18.92	27.11	0.70	0.43	-5.85	67.72	9.47
10-year shrink	18.65	26.86	0.69	0.43	-5.92	68.67	9.56
10-year PCA	12.73	13.84	0.92	0.20	0.77	13.54	3.79
Expanding sample	18.20	26.85	0.68	0.45	-6.07	70.06	9.85
Expanding shrink	17.97	26.61	0.68	0.45	-6.13	70.91	9.92
Expanding PCA	13.31	13.75	0.97	0.16	0.46	11.52	3.64
Expanding EWMA	74.72	94.13	0.79	0.33	11.16	180.94	2.43
Expanding EWMA shrink	22.27	21.85	1.02	0.11	2.89	35.55	2.76
Expanding EWMA PCA	20.30	18.06	1.12		0.67	6.63	1.78
Diagonalized Covariance Matrix:							
1-year sample	142.48	282.10	0.51	0.62***	2.06	29.90	8.51
1-year shrink	120.76	245.85	0.49	0.63***	0.79	19.14	9.19
1-year PCA	25.34	39.94	0.63	0.49**	-1.54	11.80	5.70
5-year sample	79.38	239.73	0.33	0.79***	-0.56	24.31	16.60
5-year shrink	75.78	235.52	0.32	0.80***	-0.56	24.98	17.18
5-year PCA	17.82	39.33	0.45	0.67***	-1.50	12.43	11.74
10-year sample	75.60	211.31	0.36	0.77***	-1.04	22.60	15.61
10-year shrink	72.54	207.73	0.35	0.78***	-1.08	23.23	16.13
10-year PCA	25.30	37.22	0.68	0.44**	-0.16	11.93	6.31
Expanding sample	74.65	210.61	0.35	0.77***	-1.04	22.89	15.81
Expanding shrink	71.32	207.01	0.34	0.78***	-1.09	23.54	16.41
Expanding PCA	24.45	37.43	0.65	0.47**	-0.55	10.04	6.54
Expanding EWMA	406.60	717.21	0.57	0.56**	2.19	24.21	5.28
Expanding EWMA shrink	61.51	197.28	0.31	0.81**	-2.46	47.17	11.42
Expanding EWMA PCA	46.84	42.78	1.10	0.03	0.83	6.52	1.88

Notes: The table reports the OOS performance of MV using 15 alternative estimates of the exchange rate covariance matrix. We use sample covariance estimates (indicated by "sample"), the shrinkage approach of Ledoit and Wolf (2003) (indicated by "shrink"), and our PCA approach to obtain a robust inverse of the matrix (indicated by "PCA"). For these three methods we consider 1-year, 5-year, 10-year rolling windows (indicated by "1-year", "5-year", "10-year"), and expanding windows (indicated by "Expanding") for our estimation. For the expanding windows we consider equal weights and exponentially decreasing weights of the data (indicated by "EWMA").

using an expanding window with the EWMA and PCA approach outperforms all alternatives in terms of unconditional Sharpe ratio and maximum drawdown per 1% expected return. In particular, the improvement with respect to crash risks is striking. Moreover, we find that the PCA approach dominates sample and shrinkage estimations for a given sample window. Finally, we notice that the diagonalized version of the covariance matrix generally delivers poor results. Accordingly, we conclude that the choice of our PCA approach to obtain a robust estimate of the inverse covariance matrix is of first order importance.

In the main text we exclude PCs that explain less than $\bar{\lambda} = 1\%$ of the common variation in exchange rate changes to construct a robust version of the inverse of the covariance matrix. In Figure 1 we show the sensitivity of MV to changes in $\bar{\lambda}$. That is, we construct MV and report its unconditional Sharpe ratio for various threshold values $\bar{\lambda}$. We notice a sharp increase in the unconditional Sharpe ratio as we increase $\bar{\lambda}$ from 0 to 0.1%. This illustrates that it is beneficial to remove PCs which capture a tiny part of the common variation in exchange rate changes. On the other side, we see a decrease in the Sharpe ratio as for $\bar{\lambda} > 1.2\%$. If the threshold value becomes large, then we exclude all PCs, the estimated covariance matrix is undefined, and MV has zero weights in many or even all months. We conclude that PCs are important even if they explain only a moderate amount of the common variation in exchange rates.

To rationalize this empirical pattern, we provide some theoretical insights that motivate our PCA and our choice of a relatively moderate $\bar{\lambda} \approx 1\%$. Suppose the exchange rate is equal to the ratio of country-specific (minimum variance) stochastic discount factors (SDFs). This is the case under relatively general conditions, e.g., in a pure diffusion model or when jump risks in FX markets are spanned (Maurer and Tran, 2021). Then, any exchange rate fluctuation must stem from a shock to one of the SDFs, and thus, it is priced risk. Accordingly, all exchange rate shocks are compensated by a premium and we should consider all shocks in the construction of our portfolio. However, if there are limits to arbitrage (e.g., transaction costs), then it is possible that there exists mispricing within these limits. Such mispricing means that there are non-fundamental or unpriced exchange rate changes. We would like to filter out such unpriced shocks in our analysis. Since frictions are relatively small in FX markets (especially for developed currencies), limits to arbitrage impose relatively

Sensitivity of MV to Changes in $\bar{\lambda}$

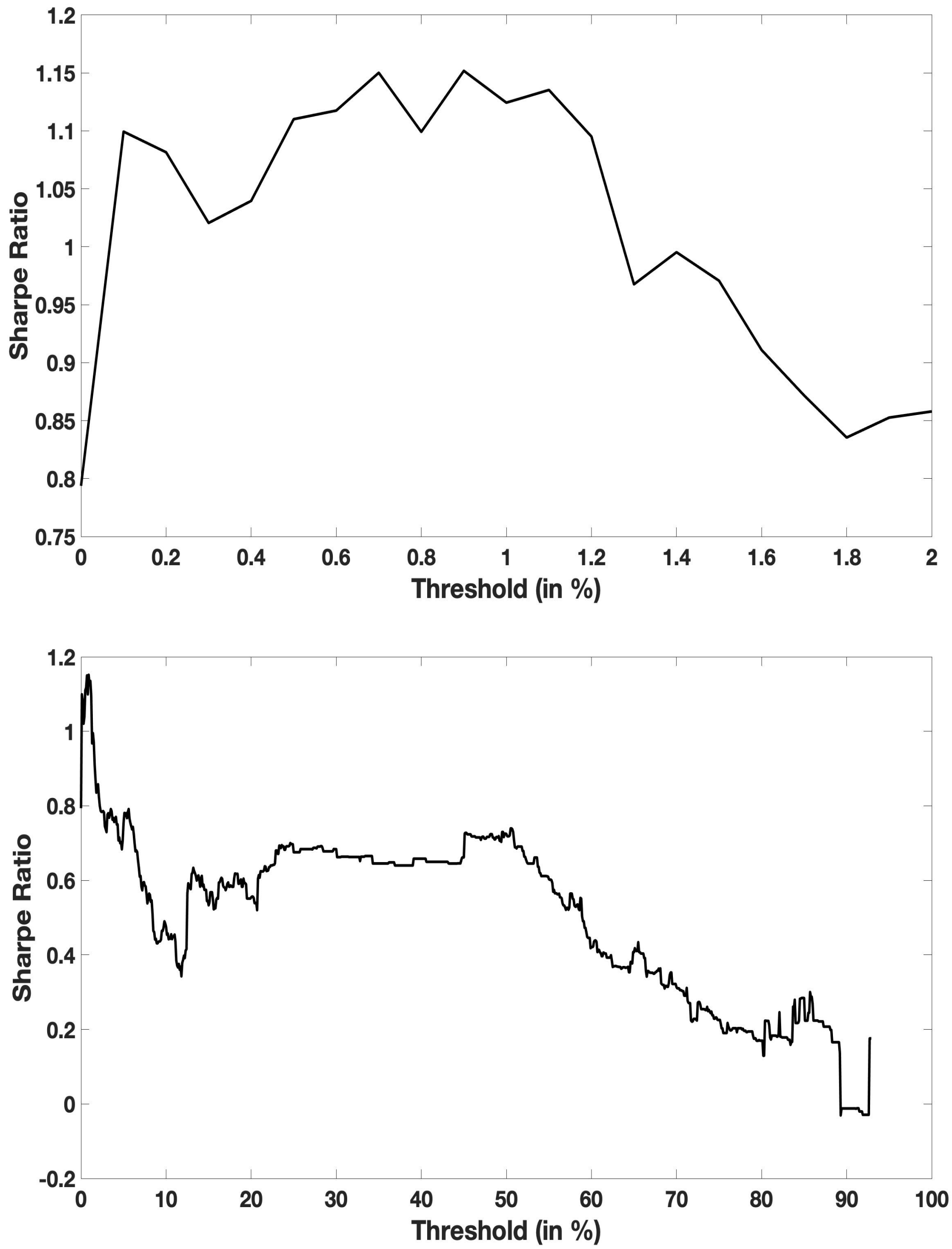

Figure 1: The solid black line shows the unconditional Sharpe ratio of MV for $\bar{\lambda} \in [0, 100\%]$. The first figure zooms in on the interval $\bar{\lambda} \in [0, 2\%]$.

narrow bounds and we only want to label relatively small exchange rate shocks as unpriced. This is the rational for using our PCA approach and the choice of a relatively small threshold value $\bar{\lambda}$.

Finally, we examine the robustness of MV's performance to the choice of the set of currencies. We remove one currency at a time from the original set of 15 currencies and repeat our analysis. When we remove only one currency, i.e., we repeat our analysis for a subset 14 currencies, then we report summary statistics of the results for all 15 possible subsets (Panel 1 in Table 15). When we remove 2 or more currencies and work with subsets of 13 or less currencies, there are too many possible subsets and we choose 100 random subsets to report summary statistics of the results of our analysis (Panel 2-5 in Table 15). We focus on the following performance measures: average returns, volatility, Sharpe ratio, skewness, kurtosis, and maximum drawdown per 1% expected return. We report the average, minimum and maximum of these measures across all simulations. The robustness check results show that MV's outperformance is resilient to different subsets of currencies.

Table 15: **Sensitivity to Set of Currencies**

	MV	MV_{CV}	MV_{CY}	MV_{CN}	MV_{FS}
Remove 1 currency					
Mean (%)	20.18	12.00	8.92	8.35	9.81
	[16.61,22.76]	[10.59,14.03]	[6.99,11.75]	[7.57,9.87]	[8.46,11.22]
Vol (%)	18.36	14.06	20.95	13.08	9.31
	[17.32,19.51]	[13.25,14.42]	[19.29,22.97]	[11.89,14.21]	[8.71,9.76]
SR	1.10	0.85	0.43	0.64	1.05
	[0.96,1.31]	[0.76,1.06]	[0.34,0.57]	[0.59,0.79]	[0.92,1.29]
Skew	0.79	-0.48	-0.04	-0.62	0.10
	[0.40,1.19]	[-0.22,-0.70]	[0.03,-0.55]	[-0.37,-1.06]	[-0.00,0.50]
Kurt	7.23	3.88	8.00	4.91	5.62
	[5.74,9.51]	[3.15,4.94]	[5.95,11.17]	[4.10,6.12]	[4.35,8.45]
|MDD|/Mean	2.04	4.80	13.94	8.55	2.47
	[1.42,2.61]	[3.26,5.40]	[6.77,18.14]	[5.14,9.73]	[1.80,2.95]
Remove 2 currencies					

Continued on next page

Table 15 – continued from previous page

Mean (%)	19.86	11.89	9.10	8.52	9.64
	[15.21,23.60]	[8.87,14.37]	[4.39,12.22]	[6.32,10.57]	[7.58,11.57]
Vol (%)	18.47	14.11	21.22	13.24	9.30
	[15.69,21.19]	[12.42,14.91]	[18.78,31.52]	[11.34,15.17]	[8.07,10.40]
SR	1.08	0.84	0.43	0.64	1.04
	[0.89,1.31]	[0.66,1.08]	[0.14,0.57]	[0.51,0.82]	[0.85,1.30]
Skew	0.82	-0.47	-0.05	-0.57	0.10
	[0.02,1.38]	[-0.11,-0.84]	[0.01,-0.79]	[-0.13,-1.06]	[-0.00,-0.75]
Kurt	7.62	3.94	8.01	4.77	5.86
	[5.46,13.14]	[3.10,7.46]	[5.47,27.42]	[3.79,6.33]	[4.20,13.33]
|MDD|/Mean	2.12	4.79	13.68	8.26	2.54
	[1.37,3.30]	[3.19,6.99]	[5.73,50.52]	[5.19,11.36]	[1.67,3.59]
Remove 3 currencies					
Mean (%)	19.27	11.53	8.96	8.35	9.32
	[14.36,24.04]	[8.76,14.28]	[3.55,13.99]	[5.91,11.18]	[7.23,11.45]
Vol (%)	18.30	14.00	21.38	13.40	9.20
	[14.34,21.48]	[11.92,14.89]	[18.59,31.57]	[11.61,15.77]	[7.55,10.65]
SR	1.05	0.82	0.42	0.62	1.01
	[0.85,1.30]	[0.61,1.05]	[0.12,0.64]	[0.42,0.83]	[0.80,1.28]
Skew	0.83	-0.46	-0.13	-0.57	0.12
	[-0.06,1.75]	[-0.09,-1.03]	[0.00,-2.90]	[-0.15,-1.01]	[0.00,-0.95]
Kurt	7.68	3.95	8.40	4.92	5.86
	[5.66,11.07]	[2.95,6.60]	[5.19,32.51]	[3.81,7.34]	[3.95,10.35]
|MDD|/Mean	2.09	4.84	14.40	8.49	2.53
	[1.27,3.51]	[3.01,6.74]	[5.08,71.42]	[4.98,13.04]	[1.63,3.90]
Remove 4 currencies					
Mean (%)	18.59	11.43	9.31	8.53	9.05
	[12.17,24.35]	[8.36,14.81]	[4.06,15.80]	[5.54,12.56]	[5.90,11.75]
Vol (%)	18.16	13.99	21.62	13.68	9.15
	[13.74,22.47]	[12.19,15.09]	[18.65,31.45]	[11.34,17.22]	[7.13,10.76]
SR	1.02	0.82	0.43	0.62	0.99
	[0.82,1.30]	[0.62,1.09]	[0.15,0.65]	[0.44,0.88]	[0.80,1.29]
Skew	0.70	-0.48	-0.20	-0.61	0.03
	[-0.04,2.08]	[-0.11,-1.02]	[-0.00,-2.81]	[-0.01,-1.26]	[0.00,-1.28]
Kurt	8.42	4.23	7.93	5.03	6.73
	[5.87,17.85]	[3.02,8.71]	[5.27,27.69]	[3.79,8.37]	[4.21,17.62]

Continued on next page

Table 15 – continued from previous page

|MDD|/Mean	2.28	4.76	13.76	8.18	2.70
	[1.26,4.30]	[2.94,6.53]	[4.15,51.54]	[4.77,12.27]	[1.58,4.39]
Remove 5 currencies					
Mean (%)	17.84	11.14	8.89	8.33	8.76
	[11.56,24.19]	[7.73,14.28]	[0.07,16.90]	[4.12,12.52]	[5.83,11.74]
Vol (%)	17.78	13.85	23.28	14.03	8.99
	[13.23,21.48]	[12.39,15.51]	[18.92,68.93]	[11.24,17.45]	[6.79,10.83]
SR	1.00	0.81	0.41	0.59	0.98
	[0.76,1.26]	[0.57,1.08]	[0.00,0.70]	[0.31,0.89]	[0.72,1.25]
Skew	0.64	-0.45	-0.27	-0.64	0.03
	[-0.02,-2.67]	[0.04,-1.53]	[-0.00,-3.29]	[-0.01,-2.12]	[-0.01,-2.08]
Kurt	9.21	4.43	9.76	5.63	7.39
	[5.92,35.24]	[2.98,13.49]	[5.11,48.21]	[3.69,26.56]	[4.20,26.78]
|MDD|/Mean	2.44	4.84	53.71	8.93	2.82
	[1.28,6.08]	[2.90,7.96]	[4.00,4883.09]	[4.57,25.59]	[1.63,5.90]

Notes: The table reports the performance of MV, MV_{CV}, MV_{CY}, MV_{CN}, and MV_{FS} for subsets of currencies. Panel 1 considers the 15 subsets of 14 currencies. Panels 2-5 consider 100 random subsets of currencies after removing 2-5 currencies from the original set of 15 currencies. For each performance measure we report the average, minimum and maximum (in brackets on the next row) across all simulations.