The Hirshleifer Effect in a Dynamic Setting*

Thomas A. Maurer†   Ngoc-Khanh Tran‡

July 24, 2018

Abstract

We characterize a prominent tradeoff in the timing of public information releases in dynamic competitive economies populated by heterogeneous agents. While early information releases improve the consumption smoothing, they also reduce risk sharing incentives ex-ante. When either financial markets are under-developed to reap risk sharing potentials, or agents perceive large consumption smoothing benefits, early information releases are universally desirable to all agents, and vice versa. We analytically establish sufficient conditions to quantify these intuitions, and shed new light on the Hirshleifer effect of the information-based welfare reduction from a broader economic perspective incorporating intertemporal dynamics, market availability and heterogeneous agents.

JEL-Classification: D80, G14, D51.

Keywords: Hirshleifer effect, welfare, information, heterogeneous agents, incomplete markets.

*We are very grateful to Jonathan Berk, Antoine Bommier, Phil Dybvig, Mahmoud Elamin, Vincent Glode, Itay Goldstein, Piero Gottardi, Denis Gromb, Ohad Kadan, Naveen Khanna, Jeongmin "Mina" Lee, Hong Liu, Mark Loewenstein, Todd Milbourn, Giorgia Piacentino, Rudolfo Prieto, Rohit Rahi, Ernst-Ludwig von Thadden, Anjan Thakor, Liyan Yang and participants at ETH seminar, the North American Summer Meeting of the Econometric Society (2014), the European Summer Symposium in Financial Markets in Gerzensee (2014) and the Finance Theory Group 2015 Summer School Meeting for comments and discussions. We thank Xiaoxiao Tang for her excellent assistance in checking several results numerically.

†Olin Business School, Washington University in St. Louis, thomas.maurer@wustl.edu.

‡Olin Business School, Washington University in St. Louis, ntran@wustl.edu.

1 Introduction

Hirshleifer (1971) makes a surprising observation that releasing public information early may reduce risk sharing incentives, potentially leading to an adverse impact on agents in the economy from an ex-ante welfare perspective. This intriguing insight, known as the Hirshleifer effect, is originally formulated in a static setting in which the risk sharing across states is the predominant factor. We demonstrate the Hirshleifer effect and the lack thereof in a dynamic economic setting featuring intertemporal consumption and trade decisions, agents' disagreements and heterogeneous preferences, as well as various asset market configurations. Conceptually, our paper characterizes the welfare value of public information by contrasting two types of benefits: one from sharing risk across states (insuring against uncertainties), the other from smoothing consumption across dates. Technically, our paper identifies subtle conditions on agents' heterogeneities and asset markets at which one benefit unambiguously dominates the other for all agents, while tackling the key welfare-analysis challenge that equilibrium prices adjust endogenously and intricately with changes in the information structure.

Intuitions built upon the Hirshleifer effect have been employed in the insurance, information economic and policy literature to assess the timing, transparency and valuation of public information. We contribute an additional intertemporal perspective to these important questions by elaborating on Hirshleifer's original insight and its reappraisal in dynamic settings. We propose a simple and unified filtration framework incorporating alternative timings of the public information release and market availabilities, ranging from a perfect foresight to no signal on future states of the economy. Our framework allows for a systematic welfare comparison analysis across various scenarios of information and market timings, offering new perspectives beyond the original setting of the Hirshleifer's effect. Our main results are threefold.

First, when agents have access to complete asset markets before information arrives, an early release of public information is desired by all agents (information is Pareto improving) if (i) they have heterogeneous beliefs (i.e., disagree) about the future prospects of the economy, and (ii) aggregate risk dominates individual endowment risks. If beliefs are homogeneous, then agents are indifferent between early and late public information releases. These results are robust and hold for general additively separable preferences with any level of heterogeneity in agents' time discount rates, risk aversions, beliefs, and initial wealth distributions.

Second, when agents are unable to trade before information arrives, an early release of public information is desired by all agents if they have heterogeneous beliefs such that benefits from risk sharing across states are modest and dominated by benefits from intertemporal consumption smoothing. This result speaks directly to the Hirshleifer effect in a dynamic setting. It demonstrates that public information is valuable even when agents cannot trade financial assets before the information arrives.

Third, also when agents are unable to trade before information arrives, we further show that some agents always strictly prefer and others always strictly dislike early information releases if (i) agents' preferences and beliefs are similar, and (ii) aggregate risk dominates individual endowment risks. Interestingly, for certain levels of heterogeneity in beliefs we are also able to show that some agents always strictly prefer (and others always strictly dislike) an economy without trading before information arrives to one where trading in complete asset markets is possible before the information release. That is, going from an incomplete-market to a complete-market economy, early public information releases are not necessarily universally welfare-improving.

Our analysis is about the value of information and the timing relevance of information releases from an ex-ante perspective. It is not about (re-)trade induced by information releases. The question on (re-)trade has been answered by Jaffe (1975) and Hakansson et al. (1982) among others. These papers feature a general setting with two consumption dates, and they show that an unexpected information release induces (re-)trade if and only if agents have non-time-separable preferences, heterogeneous believes, or markets are incomplete. Our setting is different in three aspects: (a) agents expect the release of information, (b) we are interested in the ex-ante value of an early information release (compared to a late release), and (c) we assume more general endowment distributions for which a simple re-trade argument is not sufficient in our proofs.

The endowment stream of every agent is kept unchanged across different economies in our comparative analysis and we do not allow for ad-hoc wealth transfers designed by a benevolent social planner. As a result, the price effect – or how an economy's information and asset market structure affects consumption allocations via equilibrium prices – is highly nontrivial in our welfare analysis.[1] We demonstrate that controlling the price effect is the key to deliver unambiguous welfare results. We achieve this with sufficient conditions such as proportional endowments or particular

[1]If we let a benevolent social planner allocate ad-hoc state-contingent consumptions to agents (without being subject to individual budget constraints but only to the less restrictive resource constraint), then any consumption allocation in an economy without public information releases is also resource-feasible in an economy with early public information releases. Thus, a social planner is always able to find an allocation such that all agents are weakly better off when public information is released earlier.

preference structures.

We use disagreement as a realistic modeling device to illustrate the trade-off between benefits from trade to smooth consumption across states versus time. Alternatively, we could assume non-time separable utilities, which disentangle risk aversion from elasticity of intertemporal substitution. However, such preference specifications are more complex and equilibrium solutions are often not analytically tractable. For completeness, Appendix C offers a numerical perspective on this issue.

Our analysis employs the canonical definition of welfare in the presence of heterogeneous beliefs. We define an improvement in welfare (Pareto improvement) as an ex-ante increase in every agent's expected utility evaluated under the respective agent's belief. Welfare does not necessarily improve under other criteria, which consider speculative trading to be undesirable (Kim, 2012; Gilboa et al., 2014; Gayer et al., 2014; Brunnermeier et al., 2014). But even when speculative trading is not found to be socially beneficial under some welfare criterion, attempting to regulate speculation can be socially suboptimal because restrictions can leave unintended adverse effects on markets functioning as in Duffie (2014). We do not take a stance on which welfare criterion should be chosen by a benevolent social planner. We simply show under what conditions agents ex-ante prefer an early versus a late release of public information.[2]

Our analysis is robust to many extensions. The derivation is general enough to accommodate an imperfect quality of information and an arbitrary number of time periods. Instead of focusing on aggregate uncertainty, our models can be altered to address multiple endowment processes.

Related literature

The literature on the value of information arguably goes back to Blackwell (1953) who shows that, in a single-agent decision problem, early information releases have a positive effect on welfare. Many subsequent papers point out limitations to this result in an endowment economy. Most prominently, Hirshleifer (1971) argues that too early information releases reduce risk sharing and have negative implications on welfare. We generalize Hirshleifer (1971)'s static model to a dynamic setting with multiple consumption and trading dates and heterogeneous beliefs (i.e., disagreements). We show that the introduction of heterogeneous beliefs in an intertemporal consumption setting enables consumption smoothing benefits (given early public information) to dominate the Hirshleifer effect and information releases (before asset markets open) are Pareto improving. Our extensions are

[2] According to our definition, if all agents prefer an early (late) release of information, then we say an early release of information has positive (negative) social value and increases (decreases) welfare.

both natural and crucial to shed light on the robustness of the classic Hirshleifer effect. This is of particular interest since a sizable body of research in the literature builds on the Hirshleifer effect.

Schlee (2001) shows in a heterogeneous agent model that early information releases have adverse effects on risk sharing and every agent dislikes these releases, while the representative agent is indifferent. Therefore, the representative agent is normatively unrepresentative. Hakansson et al. (1982), Jaffe (1975), Marshall (1974), Ng (1975), Ng (1977) and Wilson (1975) show that information has no value if agents have time additive preferences, homogeneous beliefs, and markets are complete. Information may be valuable in case of heterogeneous beliefs (Hakansson et al., 1982; Marshall, 1974; Ng, 1977), incomplete markets or non-time-separable preferences (Hakansson et al., 1982; Jaffe, 1975). In general, the results showing that information is valuable hinge on the restrictive assumption that endowments are equilibrium consumption allocations in the economy without information releases. Thus, the value of information is proved by showing that information induces (re-)trade. Once this assumption on endowments is relaxed, numerical examples are provided to demonstrate the welfare value of information in specific cases (Hakansson et al., 1982; Marshall, 1974; Ng, 1977). Thus, their results are more interesting for the question of what conditions are necessary and sufficient for a public information release to induce (re-)trade. That is, they interpret the restrictive assumption on initial endowments as the outcome of previous trade given agents do not expect any release of information, and the subsequent (re-)trade result is with respect to an unexpected information release. We employ less restrictive wealth distributions and provide sufficient conditions when information is valuable in a multi-period model with heterogeneous beliefs, which imply unequal benefits from risk sharing across states and time.[3]

Berk and Uhlig (1993) show that with enough heterogeneity in wealth, there is at least one agent who benefits from generating public information in order to make markets incomplete, but they do not address the welfare aspects in the economy. Blume et al. (2006) and Gottardi and Rahi (2013) analyze necessary and sufficient conditions on the information structure under which an arrival of new information causes re-trade in incomplete markets.[4] Gottardi and Rahi (2014) discuss the trade-off between the Blackwell and the Hirshleifer effect in incomplete markets and show that in general a change in information does not lead to a socially optimal allocation. These papers work with an exchange economy with a single consumption date. In contrast, our paper addresses multiple consumption dates, which allows us the investigate the trade-off between risk

[3]We provide further details on the relation between our results and the literature in the main text.

[4]Incomplete markets in their settings is not that trading is impossible before the arrival of information as in our setting; they consider more general incompleteness.

sharing across states versus time.

Eckwert and Zilcha (2003), discuss the interesting trade-off between the Blackwell (early information arrival improves the capital allocation in a production economy) and the Hirshleifer effect (early information arrival destroys risk sharing). In a sense, they formalize the idea that information may improve real investment decisions, which is good for welfare (but see Goldstein and Sapra (2013) for an exception), but at the same time it also reduces risk sharing according to the classic Hirshleifer effect. Bond et al. (2012) provide a broader view of the effect of financial markets and information on real investments. While our paper does not feature real investments, we show that the latter statement is not necessarily true in a dynamic model. That is, an information release may improve risk sharing, if it improves risk sharing across time by a larger amount than it diminishes risk sharing across states. We further provide sufficient conditions, which are both natural and quite general, for benefits from risk sharing across time to dominate benefits from risk sharing across states.

Kurlat and Veldkamp (2015) show that information disclosure reduces risk and thus also expected returns in equilibrium, which leads to a reduction in welfare for investors. Dang et al. (2015) attribute the information insensitivity of debt, and thus their ability to enhance welfare (via the Hirshleifer effect), to the ubiquity of these financial instruments in intermediaries financing. Other papers explore the value of information releases in the context of coordination games (see Colombo et al. (2014) for a recent overview) and information asymmetries (see Goldstein and Yang (forthcoming) for a recent overview). Our focus is on the risk sharing implications of changes in the timing of public information releases. Our study does not feature information asymmetries, cross-agent and intertemporal transfers of endowments.

The paper is organized as follows. Section 2 introduces the setting of our analysis, then relates and contrasts it with the premise of the first welfare theorem. Section 3 derives a global result on the welfare value of information when asset markets are complete. Section 4 contrasts the welfare value of early versus too-early information releases. Section 5 demonstrates the reversal of the Hirshleifer effect where benefits from risk sharing across states are outweighed by benefits from risk sharing across time. Section 6 concludes. Appendix A provide technical derivations omitted in the main text. Appendix B generalizes our results in several directions. By construction, the tradeoff between the consumption smoothing and risk sharing can also arise from the decoupling of the elasticity of intertemporal substitution and risk aversion in agents' preferences. For completeness,

Appendix C presents a numerical analysis of such a tradeoff when agents have non-time separable utilities.

2 Setup

We consider a setting of a heterogeneous-agent endowment economy in discrete time. We consider a stylized three-period setting, $t \in \{0^-, 0, 1\}$, where $t = 0^-$ denotes the instant right before $t = 0$. The choice of a three-period setting is for convenience, and can be generalized to a multiple-period setting.[5]

Endowments: There is a single tree in the economy, which bears deterministic aggregate endowment e_0 in period $t = 0$, and state-contingent aggregate endowment $e_1(s)$ in period $t = 1$, with $s \in \Omega \equiv \{1, \ldots, N\}$ denoting the state at $t = 1$. Time-one endowment $e_1(s)$ is the sole source of uncertainty in the economy. The endowment good is perishable and must be consumed in the same period in which it arrives. There are neither endowments nor consumptions at $t = 0^-$. Asset markets open at time $t = 0^-$. Signals about future endowment $e_1(s)$ arrive either before $t = 0^-$ (too-early informed economy), after $t = 0^-$ (informed economy), or never (uninformed economy), depending on the informational setting we consider.

Agents: There are two agents, A and B, who differ in their risk aversions, time preferences, beliefs about the prospects of future endowments, and in their initial endowments. There is no information asymmetry, and agents' subjective beliefs may differ because agents agree to disagree. For tractability, we assume that agents maximize expected utilities with constant relative risk aversions (CRRA) and constant time discount factors. The utility of consumption at time t and state s, discounted to time zero, is,

$$U(c_{It}(s), t) = \beta_I^t u(c_{It}(s)) = \beta_I^t \frac{[c_{It}(s)]^{1-\gamma_I}}{1-\gamma_I}, \qquad \forall I \in \{A, B\}, t \in \{0, 1\}, s \in \Omega. \tag{1}$$

where $u(c)$ denotes period utility, and β_I and γ_I respectively agent I's time discount factor and relative risk aversion. We remark that all our results can be generalized to the setting of strictly increasing, strictly concave and continuously differentiable additively separable preferences, as explained in Appendix B.

[5] The three-period setting stylizes a period of possible early public information releases (being early with respect to trading in asset markets), a period of consumption with no aggregate uncertainty, and a future period of consumption with aggregate uncertainty. We can straightforwardly insert more periods in between these three distinct reference points in time.

Before state s is realized, agents' beliefs are characterized by two equivalent subjective probability measures $\mathbf{P}_I \equiv \{p_I(s)\}$, $I \in \{A, B\}$,

$$\sum_{s\in\Omega} p_A(s) = \sum_{s\in\Omega} p_B(s) = 1, \qquad p_A(s) \neq 0 \Longleftrightarrow p_B(s) \neq 0.$$

Agent $I \in \{A, B\}$ is endowed with e_{It} units of the consumption good at period $t \in \{0, 1\}$. In the aggregate, the consumption goods market clears at each time and state, as quantified in the following resource constraints,[6]

$$\sum_{I\in\{A,B\}} c_{I0} = \sum_{I\in\{A,B\}} e_{I0} = e_0, \qquad \sum_{I\in\{A,B\}} c_{I1}(s) = \sum_{I\in\{A,B\}} e_{I1}(s) = e_1(s), \quad \forall s \in \Omega. \tag{2}$$

The conditions for the existence and uniqueness of a competitive equilibrium in a heterogeneous-agent setting are discussed in Mas-Colell et al. (1995), and in the recent work by Hugonnier et al. (2012).[7]

2.1 Three Economies

To place the relevance of public information in perspectives of different asset market settings, we study the three economies specified next: uninformed, informed, and too-early informed economy. Our road map to assess the welfare value of information is to compare these three economies. For instance, when we move from the informed to the too-early informed economy, then we shut down markets which allow cross-state risk sharing; or when we move from the uninformed to the informed economy, then we open a new market which improves intertemporal consumption smoothing.

Uninformed economy

In this economy, no signals on future aggregate endowment e_1 are available before time $t = 1$. Asset markets are complete, and at $t = 0^-$, agents can trade a complete set of Arrow-Debreu (AD) contingent claims on future endowments[8] as indicated by Figure 1. Because there is no information on the endowment e_1 to be released before $t = 1$, agents are content with non-contingent

[6] Agents' time-zero endowments, e_{A0}, e_{B0}, like their aggregate counterpart e_0, are deterministic.

[7] Most relevant to our economies are Propositions 4.C.1, 4.C.2, 17.C.1, 17.D.1 and 17.F.2. in the book by Mas-Colell et al. (1995), and Proposition C.1 in the online supplement to Hugonnier et al. (2012).

[8] For the economies considered in this paper, whenever asset markets are complete, dynamic trading in multiple-period settings can be reduced to initial trades in AD assets.

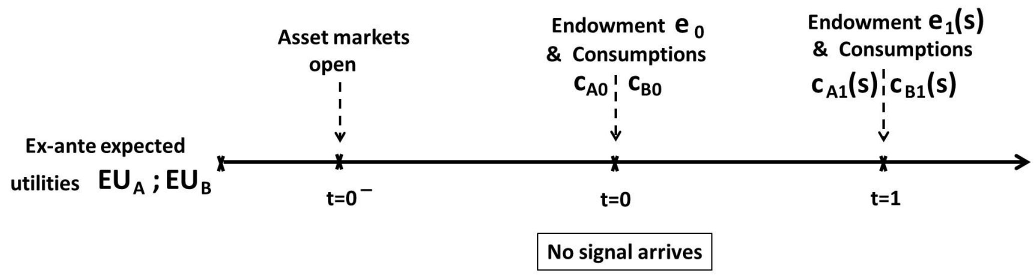

Figure 1: Time scheme of the uninformed economy.

consumptions at $t = 0$. Accordingly, agents trade and choose consumption plans to maximize their expected utilities[9] subject to their budget constraints,

$$EU_I \equiv \sup_{\{c_{I0}, c_{I1}(s)\}} E^I_{0^-} \left[u(c_{I0}) + \beta_I u(c_{I1}(s)) \right], \qquad \forall I \in \{A, B\}, \tag{3}$$

$$\text{s.t. } c_{I0} + \sum_{s\in\Omega} c_{I1}(s) q(s) \leq e_{I0} + \sum_{s\in\Omega} e_{I1}(s) q(s),$$

where $q(s)$ is the time-zero price of the s-th AD security paying one unit of the consumption good at $t = 1$ if state s realizes (and zero otherwise), and period utilities $u(c)$ are given in (1). In the aggregate, the resource constraints (2) must hold at each time and state. The optimality in this complete-market economy is achieved when agents perfectly share risks by equalizing their marginal utilities to the AD price in the corresponding state,

$$p_A(s)\beta_A \frac{u'_{A1}(s)}{u'_{A0}} = q(s) = p_B(s)\beta_B \frac{u'_{B1}(s)}{u'_{B0}}, \tag{4}$$

where "prime" denotes the first-order derivative with respect to consumption, $u'_{It}(s) \equiv \frac{\partial u(c_{It}(s))}{\partial c_{It}(s)}$. Substituting the AD prices into (3), we find that the following first order conditions (FOCs) and budget constraint formalize this equilibrium,

$$\text{at } t=0 \begin{cases} u'_{A0} = \lambda u'_{B0}, \\ c_{A0} + c_{B0} = e_0, \end{cases} \qquad \begin{matrix} \text{at } t=1 \\ \forall s \in \Omega \end{matrix} \begin{cases} p_A(s)\beta_A u'_{A1}(s) = \lambda p_B(s)\beta_B u'_{B1}(s), \\ c_{A1}(s) + c_{B1}(s) = e_1(s), \end{cases} \tag{5}$$

$$u'_{A0} c_{A0} + \sum_{s\in\Omega} p_A(s)\beta_A u'_{A1}(s) c_{A1}(s) = u'_{A0} e_{A0} + \sum_{s\in\Omega} p_A(s)\beta_A u'_{A1}(s) e_{A1}(s). \tag{6}$$

[9]Each agent computes expected utility under his subjective belief ex-ante at $t = 0^-$.

where λ is the Pareto weight. For the uninformed economy under consideration, asset markets are complete, thus λ is state-independent. The assumed property of strictly increasing preferences[10] implies that the budget constraint in equilibrium holds with equality as in (6). By Walras' law, the budget constraint for agent B is redundant.[11]

To analyze the equilibrium of the uninformed economy, we first take Pareto weight λ as given. The first (respectively, second) equation system in (5) then indicates how equilibrium consumptions at $t = 0$ (respectively, at $t = 1$) vary with λ and other exogenous factors, such as beliefs and endowments. Finally, substituting these consumptions into the budget constraint (6) then indicates how λ varies with the above exogenous factors.[12]

Informed economy

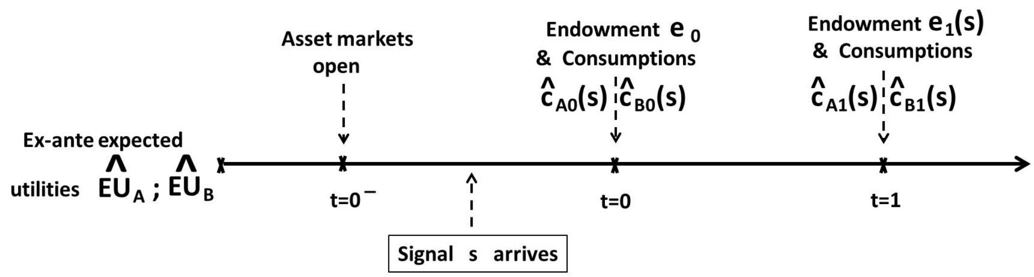

Figure 2: Time scheme of the informed economy.

We employ the sign "hat" to denote quantities associated with the informed economy. In this economy, agents expect that a signal about the future aggregate endowment $e_1(s)$ will be released before $t = 0$, but after $t = 0^-$ (when asset markets open) as indicated by Figure 2. Asset markets are complete and agents are able to contract their consumptions at both $t = 0$ and $t = 1$ on the public signal s revealed prior to $t = 0$. Compared to the uninformed economy, agents' optimal consumption plans here possess higher degree of state contingency due to the availability of public information. For simplicity we assume that the signal is perfect in the sense that its observation clears all uncertainties about future endowments. Appendix B addresses and generalizes our results to a setting of imperfect signals. Consumption smoothing is across both states and time. At pre-

[10]A weaker assumption of locally non-satiated preferences suffices to bind the budget constraints at the equilibrium.

[11]B's budget constraint is implied by A's budget constraint, the resource constraints, and the FOC.

[12]Take λ as given, the system at $t = 0$ in (5) has two equations and two unknowns (c_{I0}, for $I \in \{A, B\}$), the system at $t = 1$ in (5) has $2N$ equations and $2N$ unknowns ($c_{I1}(s)$, for $s \in \Omega \equiv \{1, \ldots, N\}$, $I \in \{A, B\}$). The budget constraint (6) is an additional equation to solve for λ. In total, the equilibrium system consists of $2N + 3$ unknowns (including λ) and $2N + 3$ equations.

signal time $t = 0^-$ (i.e., before the arrival of the signal), agents trade and choose consumption plans to maximize their expected utilities, subject to their budget constraints,

$$\widehat{EU}_I \equiv \sup_{\{\widehat{c}_{I0}(s), \widehat{c}_{I1}(s)\}} E^I_{0^-} \left[u(\widehat{c}_{I0}(s)) + \beta_I u(\widehat{c}_{I1}(s)) \right], \qquad \forall I \in \{A, B\}, \tag{7}$$

$$\text{s.t.} \sum_{s\in\Omega} \widehat{c}_{I0}(s)\widehat{q}_0(s) + \sum_{s\in\Omega} \widehat{c}_{I1}(s)\widehat{q}_1(s) \leq \sum_{s\in\Omega} e_{I0}\widehat{q}_0(s) + \sum_{s\in\Omega} e_{I1}(s)\widehat{q}(s),$$

where $\widehat{q}_0(s)$ and $\widehat{q}_1(s)$ are time-0^- prices of AD securities, paying one unit of consumption respectively at $t = 0$ and $t = 1$ when and only when signal s realizes. In the aggregate, the resource constraints (2) must hold at each time and state.

The optimality in this complete-market economy is achieved when agents perfectly share risks by equalizing their marginal utilities to the AD price in the corresponding state and time,

$$\frac{p_A(s)\widehat{u}'_{A0}(s)}{\sum_{s\in\Omega} p_A(s)\widehat{u}'_{A0}(s)} = \widehat{q}_0(s) = \frac{p_B(s)\widehat{u}'_{B0}(s)}{\sum_{s\in\Omega} p_B(s)\widehat{u}'_{B0}(s)}, \tag{8}$$

$$\frac{p_A(s)\beta_A\widehat{u}'_{A1}(s)}{\sum_{s\in\Omega} p_A(s)\widehat{u}'_{A0}(s)} = \widehat{q}_1(s) = \frac{p_B(s)\beta_B\widehat{u}'_{B1}(s)}{\sum_{s\in\Omega} p_B(s)\widehat{u}'_{B0}(s)}, \tag{9}$$

where $\widehat{u}'_{It}(s) \equiv \frac{\partial u(\widehat{c}_{It}(s))}{\partial \widehat{c}_{It}(s)}$. Substituting the AD prices into (7), we find that the following system of FOCs and budget constraint formalizes this equilibrium,

$$\begin{array}{ll} \text{at } t = 0 \\ \forall s \in \Omega \end{array} \begin{cases} p_A(s)\widehat{u}'_{A0}(s) = \widehat{\lambda} p_B(s)\widehat{u}'_{B0}(s), \\ \widehat{c}_{A0}(s) + \widehat{c}_{B0}(s) = e_0, \end{cases} \qquad \begin{array}{ll} \text{at } t = 1 \\ \forall s \in \Omega \end{array} \begin{cases} p_A(s)\beta_A\widehat{u}'_{A1}(s) = \widehat{\lambda} p_B(s)\beta_B\widehat{u}'_{B1}(s), \\ \widehat{c}_{A1}(s) + \widehat{c}_{B1}(s) = e_1(s), \end{cases} \tag{10}$$

$$\sum_{s\in\Omega} p_A(s)\widehat{u}'_{A0}(s)\widehat{c}_{A0}(s) + \sum_{s\in\Omega} p_A(s)\beta_A\widehat{u}'_{A1}(s)\widehat{c}_{A1}(s) = \sum_{s\in\Omega} p_A(s)\widehat{u}'_{A0}(s)e_{A0} + \sum_{s\in\Omega} p_A(s)\beta_A\widehat{u}'_{A1}(s)e_{A1}(s), \tag{11}$$

where $\widehat{\lambda}$ is the Pareto weight in the current informed economy. $\widehat{\lambda}$ is state-independent because asset markets are complete.

To analyze the equilibrium of this informed economy, we first take Pareto weight $\widehat{\lambda}$ as given. The first (respectively, second) equation system in (10) then indicates how equilibrium consumptions at $t = 0$ (respectively, at $t = 1$) vary with endogenous $\widehat{\lambda}$ and other exogenous factors, such as beliefs and endowments. Substituting these consumptions into the budget constraint (11) then indicates how $\widehat{\lambda}$ varies with the above exogenous factors.[13]

[13] In total, the equilibrium system (10)-(11) consists of $4N + 1$ unknowns ($c_{It}(s)$, for $s \in \Omega \equiv \{1, \ldots, N\}$, $t \in \{0, 1\}$,

Too-early informed economy

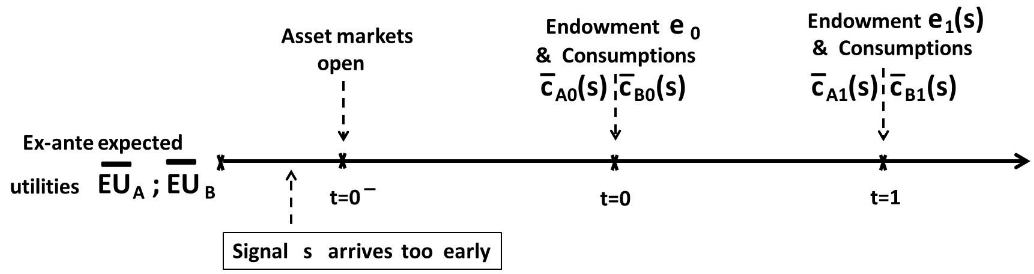

Figure 3: Time scheme of the uninformed economy.

We employ the sign "bar" to denote quantities associated with the too-early informed economy. In this economy, a perfect signal[14] about future aggregate endowment $e_1(s)$ is expected to be released before asset markets open at $t = 0^-$. In this sense, the information arrives "too early" and disrupts the risk sharing function of asset markets. After the signal arrives, at $t = 0^-$, agents can trade assets to intertemporally smooth their time-zero and time-one consumptions,[15] as indicated by Figure 3. Consequently, consumptions at $t = 0, 1$ reflect only the realizations of a single state s revealed by the signal arriving just before $t = 0^-$. Given the signal's informational content, agents trade and choose consumption plans to maximize their utilities separately for each state s, subject to their budget constraints,

$$\overline{EU}_{I,s} \equiv \sup_{\{\bar{c}_{I0}(s), \bar{c}_{I1}(s)\}} \left[u(\bar{c}_{I0}(s)) + \beta_I u(\bar{c}_{I1}(s)) \right], \qquad \forall I \in \{A, B\}, \tag{12}$$

$$\text{s.t. } \bar{c}_{I0}(s) + \bar{c}_{I1}(s)\bar{q}(s) \leq e_{I0} + e_{I1}(s)\bar{q}(s), \quad \text{for each } s \in \Omega,$$

where $\bar{q}(s)$ is time-0 price of the state-specific pure-discount security, which pays one unit of consumption at $t = 1$. Notice that $\overline{EU}_{I,s}$ is expected utility given state s. To compare utilities in the too-early informed economy to utilities in other settings, we compute ex-ante expected utility (before the information is released),

$$\overline{EU}_I = E^I_{0^-} \left[\overline{EU}_{I,s} \right] = \sum_s p_I(s) \left[u(\bar{c}_{I0}(s)) + \beta_I u(\bar{c}_{I1}(s)) \right].$$

$I \in \{A, B\}$; and λ) and $4N + 1$ equations.

[14] Appendix B addresses and generalizes our results to setting of imperfect signals.

[15] Because the signal already reveals the state, the consumption smoothing is purely intertemporally, but not across the states. Also, since there in no more uncertainty in the market after the signal arrives, a single pure-discount contract suffices to facilitate this consumption smoothing.

In the aggregate, the resource constraints (2) must hold at each time and state. For our comparative statics study, we always define agents' (ex-ante) welfare as their expected utilities at a pre-signal point in time.

The optimality in this too-early informed economy is achieved when agents perfectly smooth their consumptions intertemporally by equalizing their marginal utilities to the pure-discount security price in each corresponding state s,

$$\beta_A \frac{\overline{u}'_{A1}(s)}{\overline{u}'_{A0}(s)} = \overline{q}(s) = \beta_B \frac{\overline{u}'_{B1}(s)}{\overline{u}'_{B0}(s)}, \tag{13}$$

where $\overline{u}'_c \equiv \frac{\partial u(\overline{c})}{\partial \overline{c}}$. Substituting the AD prices into (12), we find that the following system of FOC and budget constraints formalizes this equilibrium, for each s separately,

$$\begin{matrix} \text{at } t=0 \\ \forall s \in \Omega \end{matrix} \begin{cases} \overline{u}'_{A0}(s) = \overline{\lambda}(s)\overline{u}'_{B0}(s), \\ \overline{c}_{A0}(s) + \overline{c}_{B0}(s) = e_0, \end{cases} \qquad \begin{matrix} \text{at } t=1 \\ \forall s \in \Omega \end{matrix} \begin{cases} \beta_A\overline{u}'_{A1}(s) = \overline{\lambda}(s)\beta_B\overline{u}'_{B1}(s), \\ \overline{c}_{A1}(s) + \overline{c}_{B1}(s) = e_1(s), \end{cases} \tag{14}$$

$$\overline{u}'_{A0}(s)\overline{c}_{A0}(s) + \beta_A\overline{u}'_{A1}(s)\overline{c}_{A1}(s) = \overline{u}'_{A0}(s)e_{A0} + \beta_A\overline{u}'_{A1}(s)e_{A1}(s), \tag{15}$$

where $\overline{\lambda}(s)$ is the state-specific (i.e., post-signal) Pareto weight in the too-early informed economy.

To analyze the equilibrium of this too-early informed economy, we can work with each state s separately. We first take Pareto weight $\overline{\lambda}(s)$ as given. The first (respectively, second) equation system in (14) then indicates how equilibrium consumptions at $t = 0$ (respectively, at $t = 1$) vary with endogenous $\overline{\lambda}(s)$ and exogenous endowments. Substituting these consumptions into the budget constraint (15) then indicates how $\overline{\lambda}(s)$ varies with the above exogenous factors.[16]

2.2 Relationship to the First Welfare Theorem

Before deriving comparative welfare results across the above economies in subsequent section, it is helpful to relate and contrast our setting and analysis approach with the implication and analysis of the first welfare theorem. Such a reflection helps position our new findings from a classic perspective.

The first welfare theorem assures that an economy's competitive equilibrium is (weakly) Pareto optimal when markets are complete, investors are price-takers and have non-satiated utilities. Therefore, from the ex-ante perspective (before the arrival of public signal), the first welfare theorem holds

[16]For each state s, the equilibrium system (14)-(15) consists of five unknowns ($c_{It}(s)$, for $t \in \{0, 1\}$, $I \in \{A, B\}$; and $\overline{\lambda}(s)$) and five equations.

in each of the informed and uninformed economies, but does not hold in the too-early-informed economy.[17] For the referencing with our findings later, we list below the implications (and the lack thereof) of the first welfare theorem on the comparative statics across all pairs of the three economies.

Uninformed versus informed economies: For additively separable preferences, equilibrium allocations $\{c_{I0}, c_{I1}(s)\}_{I\in\{A,B\}}$ of the uninformed economy can also be cast as legitimate consumption allocations of the informed economy in evaluating expected utilities.[18] The first welfare theorem, applied on the informed economy at uninformed economy's equilibrium allocations $(\{c_{I0}, c_{I1}(s)\}_{I\in\{A,B\}})$ and informed economy's equilibrium allocations $(\{\widehat{c}_{I0}(s), \widehat{c}_{I1}(s)\}_{I\in\{A,B\}})$, implies simply that the former allocation cannot Pareto dominate the latter. This theorem therefore *rules out* the following simultaneous improvements,

$$\begin{aligned} EU_A \equiv EU_A(\{c_{A0}, c_{A1}(s)\}) = \widehat{EU}_A(\{c_{A0}, c_{A1}(s)\}) > \widehat{EU}_A(\{\widehat{c}_{A0}(s), \widehat{c}_{A1}(s)\}) \equiv \widehat{EU}_A, \\ EU_B \equiv EU_B(\{c_{B0}, c_{B1}(s)\}) = \widehat{EU}_B(\{c_{B0}, c_{B1}(s)\}) > \widehat{EU}_B(\{\widehat{c}_{B0}(s), \widehat{c}_{B1}(s)\}) \equiv \widehat{EU}_B. \end{aligned} \tag{16}$$

Informed versus too-early-informed: Equilibrium allocations $\{\overline{c}_{I0}(s), \overline{c}_{I1}(s)\}_{I\in\{A,B\}}$ of the too-early-uninformed economy are also legitimate consumption allocations of the informed economy. Similar to (16), an application of the first welfare theorem on the informed economy at the too-early-informed economy's and the informed economy's equilibrium consumption allocations *rules out* the following simultaneous inequalities

$$\overline{EU}_A > \widehat{EU}_A, \qquad \overline{EU}_B > \widehat{EU}_B. \tag{17}$$

Too-early-informed versus uninformed economies: In the difference with the two comparative statics above, the first welfare theorem does not relate equilibrium consumption allocations of the too-early-informed with those of the uninformed economies. This is because while the first welfare theorem applies for the uninformed economy as mentioned earlier, the too-early-informed economy's equilibrium allocations (which are state-contingent in both dates $t \in \{0, 1\}$) in general

[17] This is because asset markets are incomplete (absent) in the too-early-informed economy before public signals arrive.

[18] Specifically,

$$\begin{aligned} EU_I(\{c_{I0}, c_{I1}(s)\}) = u_I(c_{I0}) + \beta_I \textstyle\sum_s p_I(s) u_I(c_{I1}(s)) \\ = \textstyle\sum_s p_I(s) \left[u_I(c_{I0}) + \beta u_I(c_{I1}(s))\right] = \widehat{EU}_I(\{c_{I0}, c_{I1}(s)\}), \qquad \forall I \in \{A, B\}. \end{aligned}$$

are not feasible allocations in the uninformed economy (which are necessarily state-independent in date $t = 0$). Such an incompatibility precludes the use of the first welfare theorem in the comparative analysis of the too-early-informed and uninformed economies. Our key findings on the Hirshleifer effect (Proposition 5) concern these two economies, and hence, they are not implied by the first welfare theorem.

3 Welfare Value of Information in Complete Asset Markets

To assess the welfare value of public information when asset markets are complete and frictionless, we compare agents' ex-ante (i.e., before $t = 0^-$) expected utilities in the uninformed and informed economies.[19] Under homogeneous beliefs, we show that agents are indifferent to the informed and uninformed economy (i.e., public information releases do not affect welfare; Proposition 1). In contrast, the informed economy (i.e. an early release of public information) is always preferred by all agents under heterogeneous beliefs (Proposition 2). Public information releases are valuable under heterogeneous beliefs in a complete market setting because they improve risk sharing in the sense that each agent is able to shift more consumption to states he believes are more likely.[20]

3.1 Informed versus Uninformed Economy: Homogeneous Beliefs

We assume that agents have identical beliefs, $p_A(s) = p_B(s)$, $\forall s \in \Omega$. Taking the ratio of the FOCs (10), (5) of the two economies at time $t = 0$ yields,

$$\frac{\widehat{u}'_{A0}(s)}{\widehat{u}'_{B0}(s)} = \frac{\widehat{\lambda}}{\lambda}\frac{u'_{A0}}{u'_{B0}},$$

that is, the ratio of agents' marginal utilities at $t = 0$ is state-independent, $\frac{\widehat{u}'_{A0}(s_1)}{\widehat{u}'_{B0}(s_1)} = \frac{\widehat{u}'_{A0}(s_2)}{\widehat{u}'_{B0}(s_2)}$, $\forall s_1, s_2 \in \Omega$. Because marginal utilities are monotone functions of consumptions, this relationship implies that agents A and B must exhibit the same ordering of time-zero contingent consumptions in the informed economy,

$$\widehat{c}_{A0}(s_1) \geq \widehat{c}_{A0}(s_2) \Longleftrightarrow \widehat{c}_{B0}(s_1) \geq \widehat{c}_{B0}(s_2) \quad \forall s_1, s_2 \in \Omega.$$

[19]The asset markets are indeed complete for both uninformed and informed economies.
[20]Detailed proofs are relegated to Appendix A.1.

Combining this observation with the state-independent resource constraints at $t = 0$ in (10), $\widehat{c}_{A0}(s_1) + \widehat{c}_{B0}(s_1) = e_0 = \widehat{c}_{A0}(s_2) + \widehat{c}_{B0}(s_2)$, it must be that time-zero consumptions of both agents are state-independent in the informed economy,

$$\widehat{c}_{A0}(s) = \widehat{c}_{A0}, \qquad \widehat{c}_{B0}(s) = \widehat{c}_{B0}, \quad \forall s \in \Omega.$$

The equilibrium systems (5) and (10) of respectively uninformed and informed economies are identical when agents have same beliefs. Under the technical assumption of the equilibrium's uniqueness,[21] we have,

Proposition 1 (Information irrelevance for homogeneous beliefs) *Assume (i) asset markets are complete and (ii) agents have identical beliefs about the prospects of future aggregate endowments. The equilibrium, and thus each agent's optimal consumption plan and expected utility, are identical in the uninformed and informed economies.*

$$\widehat{EU}_A = EU_A, \qquad \widehat{EU}_B = EU_B. \tag{18}$$

Therefore, early public information releases about future endowments have no ex-ante (i.e., before $t = 0^-$) value to welfare in homogeneous-belief complete-market environments.

Intuitively, because asset markets are complete, agents ex-ante optimally share risk by trading a full set of contracts contingent on any possible outcome of the economy. Since agents agree on the prospects of the economy, the arrival of a signal ex-post will not "surprise" agents beyond what is covered by their original optimal contingent consumption plans. Thus, ex-ante, the possibility of a signal's arrival will not add value to agents' utilities. Given this result we also note that agents have no desire to re-trade if they have reached the equilibrium under the uninformed (no-signal) premise and unexpectedly receive signal s. In Appendix B, we extend the public information irrelevance result to settings in which initial endowments are uncertain and signals are imperfect.

Proposition 1's result can be analyzed in light of the first welfare theorem as mentioned in Section 2.2. In particular, the first welfare theorem applied on the informed economy only rules out the simultaneous utility improvements (16). The simultaneous equalities (18) deliver a more concrete (yet still conforming with the first welfare theorem) welfare result that when agents hold

[21] Our Assumption 2 below on the proportional endowments ensures uniqueness of the equilibrium. For details, see Mas-Colell et al. (1995).

identical belief about the incoming public signal, they do neither gain nor lose anything from being able to trade contracts on this signal from an ex-ante perspective. Put differently, the informed economy's Pareto efficient allocations are fully implementable with the market structure of the uninformed economy when agents have homogeneous belief. This result hints at a joint relevance of heterogeneous expectations and early public information release, which we present in the next section.

Proposition 1 generalizes the results in section 1.B in Marshall (1974). In his setting agents expect the release of information and have access to complete (insurance) markets before the signal arrives. The difference is that Marshall (1974) works in a model with a single consumption date, while Proposition 1 (see also Appendix B) features multiple consumption dates.[22]

3.2 Informed versus Uninformed Economy: Heterogeneous Beliefs

When agents have different expectations about future endowments, these different beliefs are not canceled in the equilibrium equations at $t = 0$ in (10), and affect equilibrium consumption plans in both periods $t \in \{0, 1\}$. We start with the observation that the Pareto weight, in (5) and (10), characterizes the relative importance of agent B in the economy,[23] and there is a monotone relationship between equilibrium consumptions and the Pareto weight,

Lemma 1 *Assume that agents have strictly increasing and concave utilities of consumption, and aggregate endowments and beliefs are fixed. Agent A's (agent B's) equilibrium consumption unambiguously decreases (increases) with the Pareto weight, at all times and states, in all economies under consideration,*

$$\frac{\partial c_{At}(s)}{\partial \lambda} < 0, \quad \frac{\partial \widehat{c}_{At}(s)}{\partial \widehat{\lambda}} < 0; \qquad \frac{\partial c_{Bt}(s)}{\partial \lambda} > 0, \quad \frac{\partial \widehat{c}_{Bt}(s)}{\partial \widehat{\lambda}} > 0; \qquad \forall t \in \{0, 1\}, \forall s \in \Omega.$$

Intuitively, a larger equilibrium Pareto weight implies that agent A has higher marginal utility in all states and dates (5), (10). Then, agent A's consumption drops in all states and dates when λ

[22] Hakansson et al. (1982), Jaffe (1975), Ng (1975), Ng (1977) and Wilson (1975) are not related to Proposition 1 because they address the Hirshleifer effect and compare an economy without information vs an economy with information but no trade before the information release (i.e., uninformed vs. too-early informed economies; see section 5). The proof of Proposition 1 is in the idea similar to the proofs of the no-trade results in the aforementioned papers.

[23] Recall that we can arrive at the complete-market FOCs (5) and (10) through the construction of a representative agent, whose utility is $u(c) \sim u_A(c_A) + \lambda u_B(c_B)$. That is, the Pareto weight characterize the agent B's contribution to the representative agent.

increases.[24] Observe however that the Pareto weight is an endogenous quantity in equilibrium and when aggregate endowments and beliefs are kept unchanged, the Pareto weight varies with agents' relative endowments via a wealth effect (see (25)).

We note that at $t = 1$, the equilibrium equations of the informed and uninformed economies are identical (equation systems at $t = 1$ are identical up to the Pareto weight in (5) and (10)). This is because asset markets are complete and agents perfectly share aggregate endowment risks by contracting their contingent consumptions at $t = 1$ in both economies. Following Lemma (1) we know that agent A's (B's) consumption at $t = 1$ is strictly decreasing (increasing) in the Pareto weight, and thus, agent A's (B's) utility of consumptions at $t = 1$ must be strictly decreasing (increasing) in the Pareto weight as well. Accordingly, Pareto weights are key to a welfare comparison across different economies, at least for utilities of consumption at $t = 1$.

Lemma 2 *Assume that agents have strictly increasing and concave utilities of consumption, and aggregate endowments and beliefs are fixed. If the Pareto weight is lower (higher) in the uninformed than in the informed economy, then agent A's expected utility of time-one consumption is higher (lower) in the uninformed than in the informed economy. The opposite holds for agent B's utility.*

$$\lambda \leq \widehat{\lambda} \Longrightarrow \begin{cases} EU_{A1} \geq \widehat{EU}_{A1}, \\ EU_{B1} \leq \widehat{EU}_{B1}, \end{cases} \qquad \lambda \geq \widehat{\lambda} \Longrightarrow \begin{cases} EU_{A1} \leq \widehat{EU}_{A1}, \\ EU_{B1} \geq \widehat{EU}_{B1}, \end{cases} \tag{19}$$

where EU_{I1} denotes agent I's expected utility at $t = 0$ of time-one consumption,

$$EU_{I1} \equiv \sum_{s \in \Omega} p_I(s) \beta_I u(c_{I1}(s)), \qquad \widehat{EU}_{I1} \equiv \sum_{s \in \Omega} p_I(s) \beta_I u(\widehat{c}_{I1}(s)), \quad I \in \{A, B\}. \tag{20}$$

In contrast, at $t = 0$, the equilibrium structures of the informed and uninformed economies are very different. In (10), the arrival of a signal in the informed economy fosters optimal state-contingent consumptions already at time $t = 0$, while this contingency is impossible in the uninformed economy (5). Consequently, the result of Lemma 1 can not be directly employed (as in Lemma 2) to compare utilities of time-zero consumptions between the uninformed and informed economies.

However, we are able to show that CRRA preferences with $\gamma_A \geq 1$ and $\gamma_B \geq 1$ are sufficient conditions for agents' utilities of time-zero consumptions $\widehat{c}_{I0}(s)$ (in the informed economy) to be convex

[24] We keep aggregate resource in all states and dates unchanged. The Pareto weight varies as we vary individual agents' endowments.

in the Pareto weight, and thus, by Jensen's inequality $\widehat{EU}_{I0} \equiv \sum_{s\in\Omega} p_A(s)\widehat{u}_{A0}(s) > u\left(c_{I0}\left(\widehat{\lambda}\right)\right)$. In other words, every agent's expected utility of time-zero consumption in the informed economy is larger than his utility in the uninformed economy if the Pareto weight in the uninformed economy was $\widehat{\lambda}$. The convexity argument can be interpreted as the ex-ante utility-gain from speculative trading due to the heterogeneity in beliefs. This insight is most apparent in the special case of log-utility (see Appendix A.1). This argument is key to derive unambiguous comparative statics, which leads us to the following Assumption:

Assumption 1 (Risk aversions) *Assume that agents have CRRA utilities with relative risk aversions satisfying $\gamma_A \geq 1$ and $\gamma_B \geq 1$.*

We, then, combine the convexity result with Lemma 1,

Lemma 3 *Suppose that (i) asset markets are complete (ii) Assumption 1 holds and (iii) agents have heterogeneous beliefs about the prospect of future aggregate endowments. If the Pareto weight is lower (higher) in the uninformed than in the informed economy, then agent B's (agent A's) expected utility in period $t = 0$ is higher in the informed than in the uninformed economy,*

$$\lambda \leq \widehat{\lambda} \implies \widehat{EU}_{B0} > u_{B0}; \qquad \lambda \geq \widehat{\lambda} \implies \widehat{EU}_{A0} > u_{A0}.$$

where

$$u_{I0} \equiv u(c_{I0}), \qquad \widehat{EU}_{I0} \equiv \sum_{s\in\Omega} p_I(s) u(\widehat{c}_{I0}(s)). \quad I \in \{A, B\}. \tag{21}$$

We first note that as a sufficient condition, Assumption 1 can be weakened substantially (see Appendix B). It can also be reformulated to accommodate non-CRRA preferences (equation (60)).

Together Lemma 2 and 3 provide use with a clear preference of agent A for an early information release if $\lambda > \widehat{\lambda}$ but we cannot tell agent B's preference. Similar, if $\lambda < \widehat{\lambda}$, then we know that agent B prefers an early release of information but we do not know agent A's preference. Only in the special case when the Pareto weights are the same in the two economies (which does not hold in general), both A's and B's expected utilities are higher in the informed than in the uninformed economy,

$$\lambda = \widehat{\lambda} \implies \quad \widehat{EU}_{A0} > u_{A0}; \quad \text{and} \quad \widehat{EU}_{B0} > u_{B0}.$$

In order to get an unambiguous preference of both agents for an early release of information if $\lambda \neq \widehat{\lambda}$, we assume that initial endowments are proportional,

Assumption 2 (Proportional endowments) *Let agents A and B be endowed respectively with fraction k and $1-k$ of the endowment tree of the economy,*

$$e_{At}(s) = \frac{k}{k+1} e_t(s), \qquad e_{Bt}(s) = \frac{1}{k+1} e_t(s), \quad \forall t \in \{0,1\}, \forall s \in \Omega. \tag{22}$$

for some constant $k > 0$.

Proportional endowments can be understood as a modeling device to deliver the dominance of aggregate endowment risks (over individual endowment risks). This is a reasonable assumption in many contexts.

This assumption on initial endowments ensures that the wealth ratio between agents is unaffected by a change in the information structure of the economy and a subsequent (endogenous) change in prices. To understand this in more detail we start pointing out that for CRRA preferences, there is a close relationship between wealth and expected utilities. This relationship stems from a property of the power function (specifically marginal utilities), $u'_c = \frac{(1-\gamma_I)u}{c_I}$, and holds for both the uninformed and the informed economy. Intuitively, because wealth finances consumption streams, agent I's initial wealth w_I is related to his expected utility EU_I. For the uninformed economy, wealth is given by

$$w_I = c_{I0} + \sum_{s\in\Omega} q(s) c_{I1}(s) = \frac{(1-\gamma_I)\, EU_I}{\sum_{s\in\Omega} p_I(s) u'_{I0}(s)}, \qquad I \in \{A, B\},$$

where we have used expressions (4) for AD prices $q(s)$, and EU_I is agent I's ex-ante expected utility (3), which also quantifies his welfare. Note that $EU_I = u_{I0} + EU_{I1}$; u_{I0} is the expected utility of time-zero consumption addressed by Lemma 3, and EU_{I1} is the expected utility of time-one consumption addressed by Lemma 2, for the uninformed economy. Combining the wealth ratio of the two agents with the FOC (5) yields the following relationship,

$$\frac{w_A}{w_B} = \frac{1-\gamma_A}{1-\gamma_B} \frac{1}{\lambda} \frac{EU_A}{EU_B}. \tag{23}$$

The intuition is simple. An agent's higher relative wealth translates, in expectation, into a higher relative utility. However, the translation is not necessarily linear, due to the presence of price effects captured by the Pareto weight λ. That is, prices vary endogenously with states; equilibrium consumption goods tend to be cheaper in states of high equilibrium consumptions.

For the informed economy, the corresponding relationship between agent I's initial wealth $\widehat{w}_I$, expected utility $\widehat{EU}_I$ (7), AD prices $\widehat{q}_0$ (8), $\widehat{q}_1$ (9) reads,

$$\widehat{w}_I = \sum_{s\in\Omega} [\widehat{q}_0(s)\widehat{c}_{I0}(s) + \widehat{q}_1(s)\widehat{c}_{I1}(s)] = \frac{(1-\gamma_I)\,\widehat{EU}_I}{\sum_{s\in\Omega} p_I(s)\widehat{u}'_{I0}(s)}, \qquad I \in \{A, B\},$$

which implies,

$$\frac{\widehat{w}_A}{\widehat{w}_B} = \frac{1-\gamma_A}{1-\gamma_B}\frac{1}{\widehat{\lambda}}\frac{\widehat{EU}_A}{\widehat{EU}_B}. \tag{24}$$

Again, agent I's ex-ante (pre-signal) expected utility $\widehat{EU}_I = \widehat{EU}_{I0} + \widehat{EU}_{I1}$; $\widehat{EU}_{I0}$ is the expected utility of time-zero consumption addressed by Lemma 3, and $\widehat{EU}_{I1}$ is the expected utility of time-one consumption addressed by Lemma 2, for the informed economy.

Combining (23) with (24) gives a simple cross-economy relationship,

$$\frac{\widehat{EU}_A}{EU_A} = \left(\frac{\widehat{w}_A/w_A}{\widehat{w}_B/w_B}\right)\frac{\widehat{\lambda}}{\lambda}\frac{\widehat{EU}_B}{EU_B} \tag{25}$$

This equation provides us with a deeper insight into the limitation of Lemmas 2 and 3. When $\widehat{\lambda} \geq \lambda$, Lemmas 2 and 3 imply that $\widehat{EU}_B > EU_B$, but this is not enough to assure $\widehat{EU}_A > EU_A$, because of the endogenous price effect which is captured by the wealth ratio $\frac{\widehat{w}_A/w_A}{\widehat{w}_B/w_B}$. It could be that, when B is relatively more influential in the informed than in the uninformed economy, (that is, $\widehat{\lambda} \geq \lambda$, and thus $\widehat{EU}_B \geq EU_B$), his relative wealth is even more dominant, $\left(\frac{\widehat{w}_A/w_A}{\widehat{w}_B/w_B} \ll 1\right)$, such that $\widehat{EU}_A \leq EU_A$ by (25). Similar, if $\widehat{\lambda} \leq \lambda$, Lemmas 2 and 3 imply $\widehat{EU}_A > EU_A$, but this is not enough to assure $\widehat{EU}_B > EU_B$, again because the wealth ratio $\frac{\widehat{w}_A/w_A}{\widehat{w}_B/w_B}$ may reverse the latter inequality. This insight suggests that one way to obtain an unambiguous welfare result is to control for the variation in agents' relative wealth induced by endogenous equilibrium prices. Assuming proportional endowments (Assumption 2) is a sufficient condition to limit this price effect.

Note that proportional endowments limit agents' risk sharing capacities and benefits, in the sense that all shocks to individual agents' endowments are aggregate shocks. Proportional endowments are also part of a sufficient condition for the uniqueness of the competitive equilibrium in an endowment economy with multiple agents (Mas-Colell et al. (1995), Chapter 17). In Appendix B we discuss the merits and alternatives to this assumption.

An agent's wealth is the value of his endowments priced at equilibrium prices, as in budget constraints (3), (7). Thus, (22) implies that the wealth ratio is independent of equilibrium prices,

$$\frac{w_A}{w_B} = \frac{\widehat{w}_A}{\widehat{w}_B} = k.$$

Hence, Assumption 2 eliminates the endogenous price effect on the equilibrium wealth ratio across different economies, rendering $\frac{\widehat{w}_A/w_A}{\widehat{w}_B/w_B} = 1$ in (25), or

$$\frac{\widehat{EU}_A}{EU_A} = \frac{\widehat{\lambda}}{\lambda} \frac{\widehat{EU}_B}{EU_B} \tag{26}$$

holds under Assumption 2. The combination of Lemma 2, Lemma 3, and equation (26) yields a key unambiguous comparative statics for the uninformed and informed economies,

Proposition 2 (Information relevance for heterogeneous beliefs) *Suppose that (i) asset markets are complete (ii) Assumption 1 holds ($\gamma_A \geq 1$, $\gamma_B \geq 1$), and (iii) Assumption 2 holds ($e_{At}(s)/e_{Bt}(s) = k$, $\forall t, s$). When agents differ in their beliefs about the prospect of future endowments, $p_A \neq p_B$, both agents have higher ex-ante expected utilities in the informed than in the uninformed economy,*

$$\widehat{EU}_A > EU_A, \qquad \widehat{EU}_B > EU_B. \tag{27}$$

Therefore, early public information releases about future endowments have an unambiguously positive ex-ante (i.e., before $t = 0^-$) value to welfare in a heterogeneous-beliefs complete-market environment.

It is instructive to understand the distinct impacts of homogeneous (Proposition 1) and heterogeneous priors (Proposition 2) on the welfare value of public information releases for complete markets. When agents have homogeneous priors, after the information arrives, both agents benefit equally from the arrival of information (both agents were equally pessimistic/optimistic before the signal arrives). Access to complete asset markets before the information arrives ensures agents adhere to their contingent consumption plan agreed upon ex-ante. Whereas, when agents have heterogeneous priors, the arrival of information induces agents to adjust their beliefs differentially and contract their consumptions (in both periods) on the information. As a result, if the price effect does not put either agent too much at a disadvantage (e.g. under Assumption 2), then *in expectation*, both agents gain from trading contingent claims on the information to be released.

The strength of Proposition 2's result can also be elucidated in light of the first welfare theorem

(Section 2.2). Simultaneous inequalities (27) deliver much more incisive welfare results than what can be broadly implied from the first welfare theorem (which simply rules out (16)). Under the sufficient conditions specified in Proposition 2, results (27) show that both agents are strictly better off in the informed economy with early and contractible public information release. Early public information then clearly is welfare-improving in such premise, a result that is consistent with, but not implied by, the first welfare theorem.

Our analysis takes into account equilibrium price effects due to changes in the information structure (i.e., a change from the uninformed to the informed economy).[25] This evidently makes the analysis complex and requires additional assumptions on agents' preferences and endowments. In particular, the assumption on proportional endowments, or some weaker version of it (Appendix B), is crucial to deliver our unambiguous welfare result.[26] Proportional endowments have a simple economic interpretation: aggregate risk dominates individual endowment risks.

Suppose now agents' endowments, $\{e_{A1}(s)\}$, $\{e_{B1}(s)\}$, were not tightly related, so that aggregate risk does not dominate individual endowment risks (a violation of Assumption 2). Even when we maintain asset market completeness, this implies ambiguous welfare results due to the price effect. We could design appropriate wealth transfers and combine them with the early release of information (i.e., a benevolent social planner imposes these transfers and releases information at the same time) to offset the price effect in (25) and ensure that an early release of information is always Pareto improving with arbitrary endowments (see footnote 1). However, the implementation of ideal transfers is close to impossible in practice. Thus, we focus on the welfare implications of an early information release in a setting without transfers.

Our result is related to Marshall (1974), who provides a single numerical example with heterogeneous beliefs where all agents prefer an early information release if they can trade in complete markets before the information arrives. He concludes that this proves the existence of settings where information is socially valuable. We go further and provide an analytical proof that an early release of information is always (ex-ante) preferred by all agents assuming aggregate risk dominate individual endowment risks. In Appendix B we further show that the assumption of proportional endowments can be weaken. Our analytical solutions provide us with a deep understanding of all

[25]This is in contrast, for instance, to Ng (1977) (Proposition 2), who assumes prices do not change.

[26]The proportional endowment assumption is, however, much weaker than the strong assumption that endowments are equilibrium allocations in the uninformed economy, as it is for instance assumed in the re-trade results of Hakansson et al. (1982), Jaffe (1975) or Ng (1977). Notice that these papers compare the uninformed with the too-early informed economy and are in that sense not directly related to Proposition 2.

economic forces at work. This allows us to understand the conditions and develop a clear intuition when early information releases are valuable. Purely numerical examples cannot provide the same insights.

Figure 4 numerically illustrates Proposition 2 and its underlying intuitions. The left panel shows the setting with proportional endowment and Assumption 2 holds. Agents' expected utilities in the informed economy are indeed higher than their counterpart values in the uninformed economy (left panel, first and second rows), and the relative wealth ratio equals unity, $\frac{\widehat{w}_A/w_A}{\widehat{w}_B/w_B} = 1$ (left panel, third row), for *all* heterogeneous belief configurations. The right panel illustrates the setting when proportional endowment Assumption 2 does not hold. Agent A's expected utility in the informed economy can be either lower or higher than its counterpart value in the uninformed economy (right panel, first row), and the wealth ratio generally differs from unity, $\frac{\widehat{w}_A/w_A}{\widehat{w}_B/w_B} \neq 1$ (right panel, third row). A reassuring exception is the homogeneous belief configuration ($p_B(H) = 0.5$, right panel), in which every agent has identical expected utility in the informed and uninformed economy, as implied by Proposition 1. These numerical results vividly demonstrate the potency of the price effect arising from non-proportional endowments. Even in complete asset markets, not all agents benefit ex-ante from the expected early resolution of uncertainties featured in the informed economy.

In Appendix B we discuss and provide weaker versions of the assumptions underlying Proposition 2 while maintaining unambiguous welfare implications. For now we note that, by contrasting the results of Propositions 1 and 2, given the dominance of aggregate risks (Assumption 2), heterogeneous beliefs are the key factor delivering the unambiguous value of public information in complete asset markets.

4 Welfare Value of 'Too-Early' Released Infromation

To assess the welfare value of public information under various asset market conditions, we compare agents' ex-ante (i.e., pre-signal) expected utilities in the informed and too-early informed economies. Information is expected to be released in both economies. For the too-early informed economy, information arrives before asset markets open, whereas for the informed economy, information arrives after asset markets open. To produce comparative statics, without loss of generality, we fix the belief of agent A and vary that of agent B. We first identify a particular belief of B, under which the equilibria of the two economies are identical (see (28); Lemma 4). When agent B (slightly) deviates from this belief (28), moving across the two economies, we show a surprising result: in

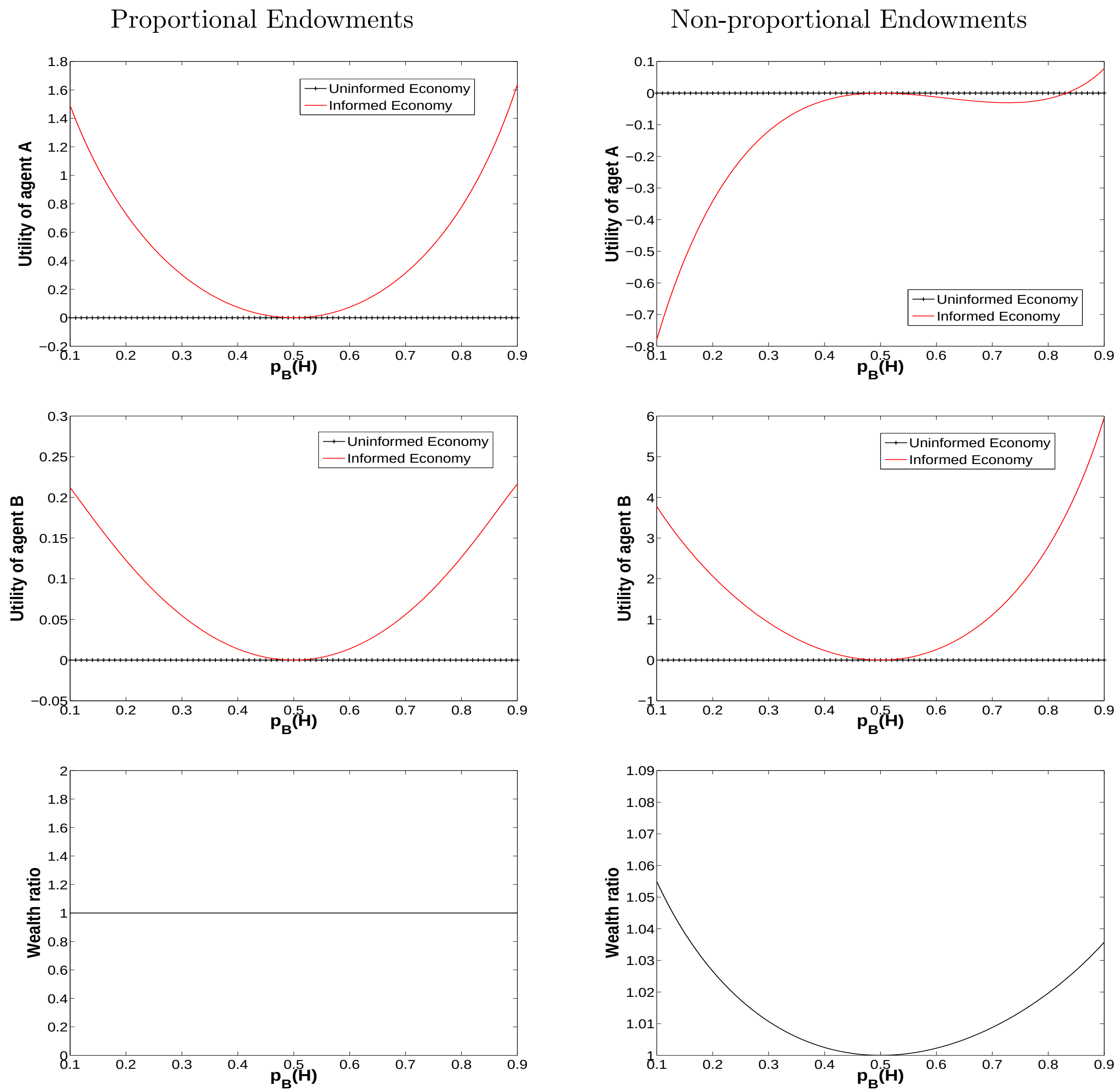

Figure 4: First row: agent A's expected utility $\widehat{EU}_A$ in the informed economy (normalized by its value EU_A in the uninformed economy). Second row: agent B's expected utility $\widehat{EU}_B$ in the informed economy (normalized by its value EU_B in the uninformed economy). Third row: agents' wealth ratio $\frac{\widehat{w_A/w_B}}{w_A/w_B}$ in the informed economy (normalized by its value in the uninformed economy). Left panel: proportional endowment configuration (Assumption 2). Right panel: non-proportional endowment configuration. All quantities are plotted against agent B's belief $p_B(H)$ of future state H. Parameters are as follows (common to all panels): state space $\Omega = \{L, H\}$; CRRA preferences $\gamma_A = 10$, $\gamma_B = 2$; time discount factors $\beta_A = \beta_B = 0.9996$, agent A's subjective belief $p_A(L) = p_A(H) = 0.5$; aggregate endowments $e_0 = 100$, $e_1(L) = 100 \times e^{-0.02}$, $e_1(H) = 100 \times e^{0.08}$. Case of proportional endowments (left panel): agent A's endowments $e_{A0} = 0.5 \times e_0$, $e_{A1}(L) = 0.5 \times e_1(L)$, $e_{A1}(H) = 0.5 \times e_1(H)$. Case of non-proportional endowments (right panel): agent A's endowments $e_{A0} = 100$, $e_{A1}(L) = 0.1 \times e_1(L)$, $e_{A1}(H) = e_1(H)$.

general, one agent is strictly better off and the other strictly worse off as measured by their expected utilities (Proposition 3).[27] That is, allowing agents to trade before the arrival of the information is in general not Pareto improving if beliefs are in the neighborhood of the configuration in (28).

Informed versus Too-Early Informed Economy: Convergence

Given agent A's belief $\{p_A(s)\}$, we consider first a particular belief of agent B,[28]

$$p_B^*(s) = \frac{p_A(s)\overline{\lambda}(s)}{\widehat{\lambda}^*}, \qquad \forall s \in \Omega, \tag{28}$$

$$\text{where:} \qquad \widehat{\lambda}^* \equiv \sum_{s\in\Omega} p_A(s)\overline{\lambda}(s). \tag{29}$$

Note that $\{p_B^*(s)\}$ (i) are constructed solely from the state-specific Pareto weights $\{\overline{\lambda}(s)\}$ of the too-early informed economy (14) and agent A's belief, (ii) are all positive and (iii) sum to one.[29] For these heterogeneous beliefs $\{p_A, p_B^*\}$, the informed economy equilibrium system (10) becomes,

$$\widehat{u}'_{A0}(s) = \frac{\widehat{\lambda}}{\widehat{\lambda}^*}\overline{\lambda}(s)\widehat{u}'_{B0}(s), \qquad \widehat{u}'_{A1}(s) = \frac{\widehat{\lambda}}{\widehat{\lambda}^*}\overline{\lambda}(s)\widehat{u}'_{B1}(s). \tag{30}$$

Under belief $\{p_B^*(s)\}$ in (28), $\widehat{\lambda}$ equals $\widehat{\lambda}^*$ and the above system is identical to the FOC system (14) of the too-early informed economy. Therefore, the too-early informed economy's equilibrium consumptions $\{\overline{c}_{It}(s)\}$, $t \in \{0, 1\}$, $s \in \Omega$, together with $\widehat{\lambda}^*$ of (29), solve the informed economy's FOCs when agent B has the particular belief $\{p_B^*(s)\}$ (28). The informed economy's resource constraints evidently also holds at all times and states because aggregate endowments are identical in the two economies.[30] Moreover, taking the weighted average under A's belief of the too-early informed economy's budget constraint (15) yields,

$$\sum_{s\in\Omega} p_A(s)\frac{\partial u\left(\overline{c}_{A0}(s)\right)}{\partial \overline{c}_{A0}(s)}\overline{c}_{A0}(s) + \sum_{s\in\Omega} p_A(s)\beta_A\frac{\partial u\left(\overline{c}_{A1}(s)\right)}{\partial \overline{c}_{A1}(s)}\overline{c}_{A1}(s)$$

[27] In Appendix A.2 we address a special case in which agents are locally indifferent around the beliefs in (28) between the two market settings.

[28] An intuition for the specific beliefs of agent B is provided in the paragraph following Lemma 4.

[29] The Radon-Nikodym derivative characterizing these heterogeneous beliefs is the ratio of Pareto weights in the two economies, $\xi^*(s) \equiv \frac{p_B^*(s)}{p_A(s)} = \frac{\overline{\lambda}(s)}{\widehat{\lambda}^*}$, $\forall s \in \Omega$.

[30] That is, when $\overline{c}_{It}(s)$ is identified with the informed economy's equilibrium solution $\widehat{c}_{It}(s)|_{p_B=p_B^*}$, $I \in \{A, B\}$ for particular belief configuration $\{p_A, p_B^*\}$, we have $\overline{c}_{At}(s) + \overline{c}_{Bt}(s) = e_t(s)$.

$$= \sum_{s \in \Omega} p_A(s) \frac{\partial u\left(\bar{c}_{A0}(s)\right)}{\partial \bar{c}_{A0}(s)} e_{A0} + \sum_{s \in \Omega} p_A(s) \beta_A \frac{\partial u\left(\bar{c}_{A1}(s)\right)}{\partial \bar{c}_{A1}(s)} e_{A1}(s).$$

This is the informed economy's budget constraint (11) evaluated at the too-early informed economy's equilibrium consumptions $\{\bar{c}_{It}(s)\}$, $I \in \{A, B\}, t \in \{0, 1\}$. It is worthwhile to observe that the too-early informed economy's equilibrium consumptions $\{\bar{c}_{It}(s)\}$ do not depend on either agents' beliefs. The particular belief p_B^* offsets and eliminates the effects of all subjective beliefs on the informed economy's equilibrium. The above analysis shows that the equilibria of the two economies converge in this particular belief configuration.

Lemma 4 *Given the heterogeneous beliefs $\{p_A, p_B^*\}$, where p_B^* is specified in (28), the informed and too-early informed economies have identical equilibrium consumption allocations,*

$$\widehat{c}_{It}(s)|_{\{p_B = p_B^*\}} = \bar{c}_{It}(s), \qquad \forall I \in \{A, B\}, t \in \{0, 1\}, s \in \Omega.$$

The Pareto weights in the two economies are related by (29). Consequently, the availability of asset markets before the arrival of information has no impact on agents' welfare, as measured by their ex-ante (pre-signal) expected utilities,

$$\widehat{EU}_A\Big|_{\{p_B = p_B^*\}} = \overline{EU}_A\big|_{\{p_B = p_B^*\}}, \qquad \widehat{EU}_B\Big|_{\{p_B = p_B^*\}} = \overline{EU}_B\big|_{\{p_B = p_B^*\}}.$$

The intuition behind this asset-market irrelevance is as follows. Generally, agents benefit from trading contingent assets on incoming signals (or other contractible realizations) because they can share the associated risk ex-ante by way of smoothing consumptions across states of the world. When asset markets are complete, the risk is perfectly shared and agents' marginal utilities are equalized in every state. However, when agent B's expectation is related to that of A by (28), B believes that he is able to achieve the perfect risk sharing with A (i.e., equalizing their marginal utilities) without trading endowments across states. As a result, in terms of utility, there are no benefits to either agent between (i) trading contingent assets before the arrival of the signal and (ii) waiting for the realization of the signal before executing signal-specific trades. In other words, under belief configuration (28) the benefits to share risk across states is zero. Finally, observe that the endowment is not equal to the optimal consumption allocation (i.e., not a no-trade setting); there is trade towards the optimal sharing rule along the time dimension.

By symmetry, given agent B's belief $\{p_B(s)\}$, when A has a particular belief defined by,

$$p_A^*(s) = \frac{p_B(s)\frac{1}{\overline{\lambda}(s)}}{\sum_{s\in\Omega} p_B(s)\frac{1}{\overline{\lambda}(s)}}, \qquad \forall s \in \Omega, \tag{31}$$

the informed and too-early informed economies have identical equilibria. Consequently, asset market frictions have no impact on agents' welfare when their beliefs are $\{p_A^*, p_B\}$.

Obviously, either belief configuration $\{p_A, p_B^*\}$ and $\{p_A^*, p_B\}$ is a special case. We are interested in more generic settings in which B's belief differs from the particular distribution $\{p_B^*(s)\}$ (or A's belief differs from $\{p_A^*(s)\}$).

Informed versus Too-Early Informed Economy: Divergence

To assess the impact of public information on agents' welfare in different asset market settings, we perform a differential analysis around the belief configurations for which the equilibria of informed and too-early informed economies converge. Without loss of generality, we take agent A's belief as given, and vary agent B's around (28),

$$p_B^*(s) \longrightarrow p_B(s) = p_B^*(s) + dp_B(s), \quad \text{s.t.} \quad \sum_{s\in\Omega} dp_B(s) = 0. \tag{32}$$

The last equation assures that the new belief configuration $\{p_B(s)\}$ is properly normalized. Without loss of generality, we only vary agent B's belief in our analysis. The variations $\{dp_B(s)\}$ induce changes in the agents' consumptions and welfare along the equilibrium path. Therefore, the equilibrium equation systems (10) and (14) hold at all times. The differential analysis is fit to generate comparative statics for different economies under consideration.[31]

We first note that for each state s, within an economy type, the system of FOCs and resource constraint at $t = 0$ and $t = 1$ are similar.[32] We focus on the differential analysis in each period $t \in \{1, 2\}$ separately. For the informed economy, under the variation of agent B's belief (32), the

[31] A drawback of the differential analysis is that its applicability is limited to local exogenous variations. For the current paper, the differential analysis serves the purpose of demonstrating the relevance or irrelevance of public information unambiguously, albeit locally.

[32] For the informed economy (10), this system involves both $p_A(s)$ and $p_B(s)$ in both periods. For the too-early informed economy (14), this system involves neither $p_A(s)$ nor $p_B(s)$ at both periods.

change in agent B's expected utility in period t reads,

$$d\widehat{EU}_{Bt} = (\beta_B)^t \sum_{s'\in\Omega} \left.\frac{d\left[\sum_{s\in\Omega} p_B(s) u\left(\widehat{c}_{Bt}(s)\right)\right]}{dp_B(s')}\right|_{p_B=p_B^*} dp_B(s')$$

$$= (\beta_B)^t \sum_{s'\in\Omega} \left.\left[u\left(\widehat{c}_{Bt}(s')\right) + \sum_{s\in\Omega} p_B(s) \frac{du\left(\widehat{c}_{Bt}(s)\right)}{dp_B(s')}\right]\right|_{p_B=p_B^*} dp_B(s'),$$

and for the too-early informed economy,

$$d\overline{EU}_{Bt} = (\beta_B)^t \sum_{s'\in\Omega} \left.\frac{d\left[\sum_{s\in\Omega} p_B(s) u\left(\overline{c}_{Bt}(s)\right)\right]}{dp_B(s')}\right|_{p_B=p_B^*} dp_B(s') = (\beta_B)^t \sum_{s'\in\Omega} u\left(\overline{c}_{Bt}(s')\right)\big|_{p_B=p_B^*} dp_B(s'),$$

where the last equality arises from the property that the equilibrium consumptions in the too-early informed economy do not depend on agents' beliefs. Using the result from Lemma 4 that at p_B^*, $\widehat{c}_{Bt}(s) = \overline{c}_{Bt}(s)$, $\forall t, s$, the cross-economy change in B's expected utility in period t is,

$$d\widehat{EU}_{Bt} - d\overline{EU}_{Bt} = (\beta_B)^t \sum_{s', s\in\Omega} p_B(s) \left.\frac{du\left(\widehat{c}_{Bt}(s)\right)}{dp_B(s')}\right|_{p_B=p_B^*} dp_B(s'), \qquad \forall t \in \{0, 1\}. \tag{33}$$

Similarly, for agent A, the corresponding relationship for the informed economy reads,

$$d\widehat{EU}_{At} = (\beta_A)^t \sum_{s'\in\Omega} \left.\frac{d\left[\sum_{s\in\Omega} p_A(s) u\left(\widehat{c}_{At}(s)\right)\right]}{dp_B(s')}\right|_{p_B=p_B^*} dp_B(s') = (\beta_A)^t \sum_{s', s\in\Omega} p_A(s) \left.\frac{du\left(\widehat{c}_{At}(s)\right)}{dp_B(s')}\right|_{p_B=p_B^*} dp_B(s')$$

$$= (\beta_A)^t \sum_{s', s\in\Omega} \left.\left[p_A(s) \widehat{u}'_{At}(s) \frac{d\widehat{c}_{At}(s)}{dp_B(s')}\right]\right|_{p_B=p_B^*} dp_B(s') = -\left(\beta_B\right)^t \sum_{s', s\in\Omega} \left.\left[p_B(s) \widehat{\lambda} \widehat{u}'_{Bt}(s) \frac{d\widehat{c}_{Bt}(s)}{dp_B(s')}\right]\right|_{p_B=p_B^*} dp_B(s'), \tag{34}$$

where in the last equality we have used the FOCs (10), and the fact that aggregate endowments do not vary with variations in agent B's belief, i.e., $d\widehat{c}_{At}(s) = -d\widehat{c}_{Bt}(s)$, $\forall t, s$. For the too-early informed economy, agent A's equilibrium consumptions do not depend on B's belief at any time and state,

$$d\overline{EU}_{At} = 0, \qquad \forall t \in \{0, 1\}, \tag{35}$$

where $\overline{EU}_{At}$ denotes A's expected utility of time-t consumption.

We analyze each agent's welfare, as measured by his pre-signal expected utility, across the informed and too-early informed economies. Using (35), we compare (33) with (34), and note that these equations hold for any t. We obtain the following comparative statics when agent B's belief deviate from (28).

Proposition 3 (Pareto efficiency of too-early informed economy: Version A) *Consider the deviation of agents' beliefs locally from the heterogeneous belief configuration* $\{p_A, p_B^*\}$ (28)*. If agent* A *is strictly better off in one (either informed, or too-early informed) economy, then agent* B *is strictly worse off in that economy,*

$$\left[d\widehat{EU}_B - d\overline{EU}_B \right]\Big|_{\{p_B = p_B^*\}} = -\left[\widehat{\lambda} \left(d\widehat{EU}_A - d\overline{EU}_A \right) \right]\Big|_{\{p_B = p_B^*\}}. \tag{36}$$

Symmetrically, similar results hold when agents' beliefs deviate locally from the heterogeneous belief configuration $\{p_A^*, p_B\}$ (31)*,*

$$\left[d\widehat{EU}_A - d\overline{EU}_A \right]\Big|_{\{p_A = p_A^*\}} = -\left[\frac{1}{\widehat{\lambda}} \left(d\widehat{EU}_B - d\overline{EU}_B \right) \right]\Big|_{\{p_A = p_A^*\}}.$$

The same key intuition underlies both Proposition 3 and equality (34). By virtue of the resource constraint, if a deviation in B's belief increases one agent's equilibrium consumption at time t in state s, it must decrease the other agent's consumption by the same amount at the same (t, s). Because asset markets are complete in the informed economy, agents equalize their marginal utilities along the equilibrium path, up to the Pareto weight $\widehat{\lambda}$. As a result, the increment in one agent's utility of consumption at any (t, s) (and his expected utility), is exactly opposite and proportional to the increment in the other agent's utility of consumption at the same (t, s) (and his expected utility). This argument applies to the equilibrium path within a single (either informed or too-early informed) economy. But Lemma 4 shows that the two economies' equilibria converge at the starting point $\{p_A, p_B^*\}$ (or $\{p_A^*, p_B\}$) of our differential analysis. As a result, Proposition 3 offers a cross-economy comparative statics, albeit locally.

The results of Proposition 3 not only are consistent with the implication of the first welfare theorem (Section 2.2), they further *rule out* stronger simultaneous inequalities (locally at belief configuration $\{p_A, p_B^*\}$),

$$\overline{EU}_A < \widehat{EU}_A, \qquad \overline{EU}_B < \widehat{EU}_B.$$

This impossibility result is interesting because intuitively we would expect that there is more risk sharing in the informed economy (due to trade in complete markets before the arrival of information) than in the more restrictive, too-early informed economy. We would, therefore, expect that all agents prefer the informed economy, and allowing agents to trade in complete markets before the information arrival is Pareto improving. The reason why one agent is strictly worse off (without loss of generality, be this agent A) is due to a change in equilibrium prices as we switch from the too-early informed to the informed economy. In particular, price changes place agent A at a disadvantage such that the consumption allocation in the too-early informed economy is no longer in the budget set of agent A in the informed economy.

Further remarks are in order. First, while the derivation leading to Proposition 3 explicitly involves the variation of only one agent's belief,[33] Proposition 3 holds for any local (possibly simultaneous) deviation of both agents' beliefs $\{p_A, p_B\}$ around $\{p_A, p_B^*\}$ (or $\{p_A^*, p_B\}$). This is because when p_B^* is specified in (28) starting from p_A, the relationship is bi-directional. Given p_B^* in (28), we can invert it to obtain uniquely, $p_A(s) = \frac{p_B^*(s)/\overline{\lambda}(s)}{\sum_s p_B^*(s)/\overline{\lambda}(s)}$. Here $\overline{\lambda}(s)$ is the state-specific Pareto weight for the too-early informed economy and does not depend on either agent's belief. Therefore, we can equivalently derive Proposition 3's comparative statics by formally fixing p_B^* while varying p_A. The combination of these two derivation procedures immediately imply yet another derivation procedure in which we vary both agents' beliefs around the configurations $\{p_A, p_B^*\}$ (or $\{p_A^*, p_B\}$).

Second, when either agent's utility changes by a strictly non-zero amount as agents' beliefs deviate from where the two economies converge, Proposition 3 rules out, at least locally, any welfare dominance, i.e., both agents would see higher expected utilities in one economy than in the other. Proposition 3 applies virtually for any endowment configuration. Therefore, endowment restrictions similar to Assumption 2 are unlikely to change the result.

Finally, the technical and special case that both sides of (36) in Proposition 3 equal zero would lead to the possibility that both agents are locally indifferent between the informed and too-early informed economies when agents' beliefs deviate from the transiting configurations ($\{p_A, p_B^*\}$ or $\{p_A^*, p_B\}$). However, this technical case can be ruled out in general (Appendix A.2).

[33] We either vary p_B around p_B^* while fixing p_A, or symmetrically we vary p_A around p_A^* while fixing p_B.

5 The Hirshleifer Effect Revisited

To assess the welfare value of public information if it arrives before asset markets open, we compare agents' ex-ante expected utilities in the uninformed and too-early informed economies.[34] This comparison speaks to the Hirshleifer effect. In the uninformed economy, without public information but with complete asset markets, agents can trade to smooth their consumption across states and time. Consumption smoothing across time is limited, which is reflected in the rigid (non-state-contingent) time-zero consumptions c_{I0} for all agents $I \in \{A, B\}$. In the too-early informed economy, agents smooth consumptions only across time given the hindsight conveyed by the signal, but no trade is allowed to share risk across states. This is reflected in the independence between equilibrium consumptions and agents' beliefs. In general, it is not clear whether the benefit of risk sharing across states outweighs that of intertemporal consumption smoothing given signals on the future state.[35] In section 5.1 we demonstrate the reversal of the Hirshleifer effect (the too-early informed economy Pareto dominates the uninformed economy) under two conditions: (i) aggregate risk dominates individual endowment risks and (ii) beliefs are heterogeneous. In section 5.2 we show that if (i) aggregate risk dominates individual endowment risks but (ii) beliefs are homogeneous and agents have similar preferences, then none of the two economies (uninformed vs too-early informed) Pareto dominates the other.

5.1 Reversal of the Hirshleifer Effect under Heterogeneous Beliefs

Our analysis on the Hirshleifer effect when aggregate risk dominates individual endowment risks and agents disagree about the future prospect of the economy relies on the global result in Proposition 2. When agent B's belief p_B is exactly at p_B^* (28), then Lemma 4 implies that equilibria of the informed and too-early informed economies converge. An application of Proposition 2 then indicates that both agents have higher expected utilities in the too-early informed economy than in the uninformed economy.

Proposition 4 (Reversal of the Hirshleifer Effect) *Suppose that (i) Assumption 1 holds ($\gamma_A \geq 1$, $\gamma_B \geq 1$), (ii) Assumption 2 holds ($e_{At}(s)/e_{Bt}(s) = k$, $\forall t, s$), and (iii) agents have either hetero-*

[34] In the uninformed economy agents do not have any information before they trade and consume. In the too-early informed economy agents have information before they are able to trade and consume.

[35] We note that risk sharing benefits depend on agents' risk aversions. Intertemporal consumption smoothing benefit depends on the elasticities of intertemporal substitution. For additively separable preferences, these two characteristics are confounded. However, differences in beliefs generate similar first order conditions as if agents differ in risk aversion or intertemporal substitution.

geneous risk aversions ($\gamma_A \neq \gamma_B$) or heterogeneous time discount factors ($\beta_A \neq \beta_B$). When agent B's belief p_B is given by p_B^* (28), both agents unambiguously have strictly higher expected utilities in the too-early informed economy than in the uninformed economy.

$$\overline{EU}_A\big|_{p_B=p_B^*} > EU_A\big|_{p_B=p_B^*}\,, \qquad \overline{EU}_B\big|_{p_B=p_B^*} > EU_B\big|_{p_B=p_B^*}\,.$$

Therefore, in this setting, an early release of public information is unambiguously Pareto-improving even if no trade is possible before the arrival of the information.

Under the listed assumptions, Proposition 4 shows analytically that, when agents disagree ($p_B = p_B^* \neq p_A$), the Hirshleifer effect is reversed and all agents unambiguously prefer an early release of information even though they cannot trade before the information's arrival. All inequalities in Proposition 4 are strict. Then, by analytic continuation on regular economies, Proposition 4 holds not only at exactly $p_B = p_B^*$, but also for all B's beliefs p_B sufficiently close to p_B^* in (28).

As we discuss in Section 2.2, the first welfare theorem does not relate equilibrium consumption allocations of the uniformed economy with those of the too-early-informed economy. Therefore, the reversal of the Hirshleifer effect reported in Proposition 4 is not an implication of the first welfare theorem.

Proposition 4 holds for any heterogeneity in agents' relative risk aversions (as long as $\gamma_I \geq 1$) and time discount factors. The result is general and robust with respect to preferences. Since the result is a direct consequence of Lemma 4 and Proposition 2, it neither requires a specific relationship between agents' risk (and time) preferences, nor does it rely on a local differential analysis. But with the assumption of proportional endowments (as required by Proposition 2), we need either $\gamma_A \neq \gamma_B$ or $\beta_A \neq \beta_B$, otherwise $p_B^* = p_A$ (homogeneous beliefs). When agents have homogeneous beliefs, $p_A = p_B$, then Proposition 1 applies instead of Proposition 2, and the uninformed and the too-early informed economy coincide.

Our result is related to the re-trade results by Hakansson et al. (1982), Ng (1975), Ng (1977) and Jaffe (1975). Making the restrictive assumption that endowments are equilibrium consumptions in the informed economy, these authors show that the Hirshleifer effect reverses under heterogeneous beliefs, non-time-separable preferences or incomplete markets. This strong assumption on endowments simplifies the analysis substantially and their results are a straightforward implication of re-trade and revealed preference arguments. Thus, their results are more interesting for the question of what conditions are necessary and sufficient for a public information release to induce (re-)trade. That is, they interpret the restrictive assumption on initial endowments as the outcome of previous trade given agents do not expect any release of information, and the subsequent (re-)trade result is with respect to an unexpected information release. We relax their assumption on endowments and show that a weaker and intuitive assumption that aggregate risk dominates individual endowment risks suffices for the Hirshleifer effect to reverse if beliefs are heterogeneous according to equation (28). In contrast to the re-trade results in the previous literature, in our setting, there is non-trivial trade in both the uninformed and the too-early informed economy. Intuitively, as explained in section 3.2, the assumption that aggregate risk dominates individual endowment risks is important to assure that endogenous price effects are moderate. That is, changes in prices due to changes in the information structure do not lead to large relative wealth transfers putting either agent at a strong disadvantage. The particular beliefs p_B^* in (28) are such that benefits from risk sharing across states are modest, so that the loss of these benefits (due to too-early releases of public information) is smaller than the welfare value of public information due to an improvement in the intertemporal sharing rule.

Our result is also related to Theorem 2 in Hakansson et al. (1982). They show that if markets are complete, beliefs are homogeneous and preferences are time-separable, then for arbitrary endowments the too-early informed economy never Pareto dominates the uninformed economy. They conclude that if either of these three conditions is violated, then the too-early informed economy might Pareto dominate the uninformed economy (but they do not provide any sufficient conditions). This result is rather weak and uninformative. They also provide a numerical example with a single consumption date and heterogeneous beliefs where the Hirshleifer effect reverses. Preferences are constructed in a very special way to deliver the desired result; they are not in the HARA-class of preferences. Ng (1977) also provides a numerical example with a single consumption date where the Hirshleifer effect reverses if both agents have identical endowments, log-utility, and disagree about the prospects of the economy. His example further assumes that utilities in both uninformed and too-early informed economies are computed ex-post (after the signal has been released), which is different from our setting (we work with ex-ante beliefs). Besides small differences in the setup, our result is more general, because we provide both analytical sufficient conditions and clear underlying intuitions for the Hirshleifer effect to reverse.

Figure 5 numerically illustrates Proposition 4. Given agent A's belief $p_A(H) = p_A(L) = 0.5$, when agent B has a belief quantified by $p_B(H) \in (0.22, 0.45)$, *every agent* has higher expected

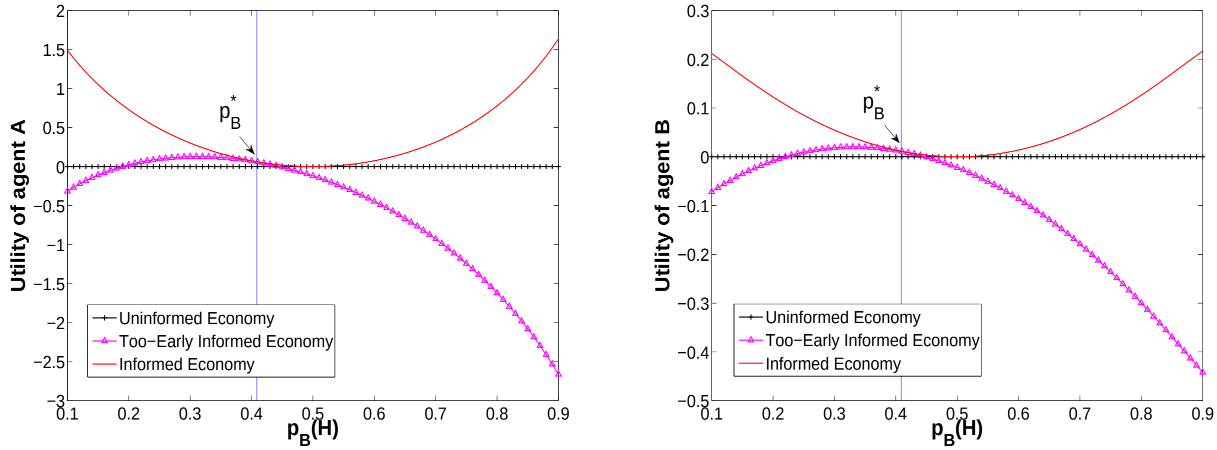

Figure 5: Left panel: agent A's expected utilities $\widehat{EU}_A$ in the informed economy, and $\overline{EU}_A$ in the too-early informed economy (normalized by its value EU_A in the uninformed economy). Right panel: agent B's expected utilities $\widehat{EU}_B$ in the informed economy, and $\overline{EU}_B$ in the too-early informed economy (normalized by its value EU_B in the uninformed economy). All quantities are plotted against agent B's belief $p_B(H)$ of future state H. The blue vertical line indicates $p^*_B(L) = 0.59, p^*_B(H) = 0.41$. Parameters are identical to those associated with Figure 4: state space $\Omega = \{L, H\}$; CRRA preferences $\gamma_A = 10$, $\gamma_B = 2$; time discount factors $\beta_A = \beta_B = 0.9996$, agent A's subjective belief $p_A(L) = p_A(H) = 0.5$; aggregate endowments $e_0 = 100$, $e_1(L) = 100 \times e^{-0.02}$, $e_1(H) = 100 \times e^{0.08}$; agent A's (proportional) endowments $e_{A0} = 0.5 \times e_0$, $e_{A1}(L) = 0.5 \times e_1(L)$, $e_{A1}(H) = 0.5 \times e_1(H)$.

utility in the too-early informed than in the uninformed economy. This range of B's belief is quite similar to that of agent A.

5.2 No Pareto Dominance under Homogeneous Beliefs

We make a series of simple arguments based on previous results of Sections 3 and 4 to infer a local comparison between the uninformed and the too-early informed economy. We start with the observation that in the setting of homogeneous risk and time preferences, $\gamma_A = \gamma_B \equiv \gamma$ and $\beta_A = \beta_B \equiv \beta$, agents have proportional consumptions (56) in equilibrium. Consequently, Assumption 2 implies a no-trade equilibrium in the too-early informed economy (equation (22), for $\gamma_A = \gamma_B$ and $\beta_A = \beta_B$),

$$\bar{c}_{It}(s) = e_{It}(s), \qquad \forall I \in \{A, B\}, t \in \{0, 1\}, s \in \Omega.$$

As a result, Pareto weights $\overline{\lambda}(s)$ are constant by virtue of FOC (14), and (22),

$$\overline{\lambda}(s) = \left(\frac{\overline{c}_{B1}(s)}{\overline{c}_{A1}(s)}\right)^{\gamma} = k^{-\gamma}, \qquad \forall s \in \Omega.$$

From the specification (28), then, follows that the particular belief p_B^* is also the homogeneous belief configuration when agents have same time and risk preferences,

$$\left.\begin{array}{rl} \text{Proportional endowments:} & \frac{e_{At}(s)}{e_{Bt}(s)} = k \\ \text{Homogeneous risk preferences:} & \gamma_A = \gamma_B \\ \text{Homogeneous time preferences:} & \beta_A = \beta_B \end{array}\right\} \quad \Longrightarrow \quad p_B^*(s) = p_A(s), \;\; \forall s \in \Omega. \tag{37}$$

By analytic continuation, this result implies that a small heterogeneity in agents' risk aversions induces a small deviation of the belief configuration $\{p_A, p_B^*\}$ from the homogeneous belief $\{p_A, p_B = p_A\}$. Hence, our differential analysis in Proposition 3 in the proximity of $\{p_A, p_B^*\}$ (where the equilibria of the informed and the too-early informed economy converge, Lemma 4) can be applied to the homogeneous belief configuration $\{p_A, p_B = p_A\}$ (where the equilibria of informed and uninformed economies converge, Proposition 1). We carry out a detailed analysis along this line of reasoning.

We take agents' risk and time preferences $\{\gamma_A,\, \gamma_B,\, \beta_A,\, \beta_B\}$ and agent A's belief p_A as exogenously given. We assume that agents have identical time discount factors and strictly different risk aversions, but the difference is sufficiently small. Equivalently, we may also assume that agents have identical risk aversions and strictly different time discount factors.[36] Then, relationship (37) implies that belief configuration p_B^* defined in (28) is also strictly different from agent A's belief, but their difference is sufficiently small, because our economies are regular by construction.

$$\left.\begin{array}{l} \gamma_A \approx \gamma_B \\ \beta_A = \beta_B \end{array}\right\} \Longrightarrow p_B^* \approx p_A. \tag{38}$$

We consider a variation of agent B's belief p_B from p_B^* to p_A along the equilibrium path. Three arguments are in order.

First, because γ_A strictly differs from γ_B, condition (54) does not hold in general, by a similar argument underlying Lemma 6. As a result, our choice of strictly heterogeneous risk aversions,

[36] All we need for our differential analysis is that agents exhibit strict but sufficiently small heterogeneous preferences toward either risk or time dimension.

$\gamma_A \neq \gamma_B$, implies an ambiguous implication of information on welfare.[37]

Second, as $p_B^* \approx p_A$ (38), the entire variation path of p_B from p_B^* (the start of the variation) to p_A (the end of the variation) can be considered local around initial configuration p_B^* and Proposition 3 applies. As a result, one agent is strictly better off, the other agent is strictly worse off, in the informed than in the too-early informed economy. Therefore, we have two (and only two) mutually exclusive possibilities at $p_B = p_A$ (homogeneous belief configuration), listed below,

$$\text{(I)}: \begin{cases} \widehat{EU}_A\Big|_{p_B=p_A} > \overline{EU}_A\big|_{p_B=p_A}, \\ \\ \widehat{EU}_B\Big|_{p_B=p_A} < \overline{EU}_B\big|_{p_B=p_A}, \end{cases} \qquad \text{(II)}: \begin{cases} \widehat{EU}_A\Big|_{p_B=p_A} < \overline{EU}_A\big|_{p_B=p_A}, \\ \\ \widehat{EU}_B\Big|_{p_B=p_A} > \overline{EU}_B\big|_{p_B=p_A}. \end{cases} \tag{39}$$

Note that all inequalities above are strict inequalities, because γ_A strictly differs from γ_B.

Third, at the homogeneous belief configuration $p_B = p_A$, Proposition 1 holds. As a result, we have both,

$$\widehat{EU}_A\Big|_{p_B=p_A} = EU_A\big|_{p_B=p_A}, \qquad \text{and} \qquad \widehat{EU}_B\Big|_{p_B=p_A} = EU_B\big|_{p_B=p_A} \tag{40}$$

But at $p_B = p_A$, either possibility (I) or (II) in (39) must arise. Combining (40) with possibility (I), or combining (40) with possibility (II), yields two (and only two) possibilities (I') and (II') reported in the following Proposition,

Proposition 5 (Pareto efficiency of too-early informed economy: Version B) *Suppose that (i) Assumption 1 holds ($\gamma_A \geq 1$, $\gamma_B \geq 1$), (ii) Assumption 2 holds ($e_{At}(s)/e_{Bt}(s) = k$, $\forall t, s$), (iii) agents' relative risk aversions are close to each other but not identical ($|\gamma_A - \gamma_B| = \epsilon > 0$), and (iv) agents' time discount factors are identical.*[38] *When agent B's belief p_B is identical to agent A's belief p_A, there are two (and only two) mutually exclusive possibilities*

$$\textit{(I')}: \begin{cases} EU_A\big|_{p_B=p_A} > \overline{EU}_A\big|_{p_B=p_A}, \\ \\ EU_B\big|_{p_B=p_A} < \overline{EU}_B\big|_{p_B=p_A}, \end{cases} \qquad \textit{(II')}: \begin{cases} EU_A\big|_{p_B=p_A} < \overline{EU}_A\big|_{p_B=p_A}, \\ \\ EU_B\big|_{p_B=p_A} > \overline{EU}_B\big|_{p_B=p_A}. \end{cases}$$

[37] We assume throughout section 5 that agents have proportional endowments. But as noted in the discussion below Lemma 6, proportional endowments are not sufficient to assure an unambiguous Pareto-improving value of information. Therefore, an ambiguous impact of public information on agents' welfare can arise even under the assumption of proportional endowments.

[38] As we noted in Footnote 36, this proposition equally holds when agents' time discount factors differ by a small amount, and in that case agents' risk aversions can be either identical or different by small amount.

Therefore, one (and only one) agent is strictly better off, the other agent is strictly worse off in the too-early informed economy than in the uninformed economy.

The Hirshleifer effect categorically captures the notion that early releases of public information impair risk sharing and reduce welfare. In our setting, this means that both agents prefer the uninformed economy over the too-early informed economy. Under the assumptions listed above, Proposition 5 shows analytically that the Hirshleifer effect does not arise. When agents have identical beliefs, too early public information releases decrease ex-ante expected utility of exactly one agent, while utility of the other agent increases.[39] Similar to the observation we made below Proposition 4 and Section 2.2, the absence of the Hirshleifer effect found in Proposition 5 is not implied by the first welfare theorem.

Intuitively, key assumptions underlying Proposition 5 point to moderate risk sharing demands and benefits. When risk aversions are similar, the risk sharing incentive is mild. When endowments are proportional, the risk sharing capacity is limited. The belief configurations, to which Proposition 5 applies, are where agents have same beliefs, $p_B = p_A$, or quite similar beliefs, $p_B = p_B^* \approx p_A$ (38). Thus, risk sharing is reduced by a small amount. At the same time, the early release of information changes prices and the wealth ratio, leaving one agent relatively better off while putting the other at a disadvantage.

Proposition 5 is related to Theorem 2 by Hakansson et al. (1982) (see discussion following Proposition 4 for details). Their result cannot tell whether the too-early informed economy is Pareto inferior to the uninformed economy. Our Proposition 5 provides sufficient conditions (agents have similar preferences, homogeneous beliefs and proportional endowments) so that the too-early release of information does not unambiguously decrease welfare.

6 Conclusion

We analyze the social value of public information in a competitive endowment economy with a special focus on how the timing of public information releases affects benefits from trade in the two dimensions: (i) benefits of sharing risk across states of the economy (insuring against uncertainty) and (ii) benefits of smoothing consumption intertemporally. Our analysis addresses various dimensions of heterogeneity across agents and settings with versus without the possibility to trade before

[39] Note that analytical results of Proposition 5 can only be demonstrated locally within the specified premises.

the arrival of information. While our results are consistent with the first welfare theorem, they are not implied by this theorem.

First, we provide a global result that an early release of information is desired by all agents (information is Pareto improving) if agents have heterogeneous beliefs, aggregate risk dominates individual endowment risks and asset markets are complete (Proposition 2).[40] If beliefs are homogeneous, agents are indifferent between early and late information releases (Proposition 1). These results hold for time additive preferences with any heterogeneity in agent's subjective time discount rates, risk aversions, beliefs, and for any initial wealth distribution.

Second, we prove that for certain levels of disagreement, all agents prefer an early release of information (information is Pareto improving) even if agents are not able to trade in asset markets before the information arrives (Proposition 4). This result is related to Hirshleifer (1971). He argues that if information is released too early (before agents are able to trade), then it has an adverse impact on risk sharing and reduces welfare. Interestingly, when we introduce heterogeneous beliefs and extend his model to multiple consumption dates, then the Hirshleifer effect can reverse, i.e., an early release of information implies an increase in welfare. Key is the understanding that there are two distinct dimensions of benefit from trade: (i) sharing risk across states of the economy and (ii) trading across time (intertemporal consumption smoothing). Early information arrivals negatively affect risk sharing across states (as pointed out by Hirshleifer (1971)) but have a positive effect on the sharing rule across the time dimension. A single-period model like in Hirshleifer (1971) naturally focuses on the first dimension and omits the second one. We, then, explore diverse settings and are able to provide sufficient conditions under which unambiguously the time (respectively cross-state) dimension of risk sharing dominates and early information releases increase (respectively decrease) welfare.

Third, we show that there always exist some agents who strictly prefer and at the same time others who strictly dislike an early information release if trading is impossible before the arrival of the information and agents have similar preferences, homogeneous beliefs and aggregate risk dominates individual risks (Proposition 5). Surprisingly, for certain levels of heterogeneity in beliefs, there are always some agents who strictly prefer and others who strictly dislike an economy where trading is impossible compared to an economy where complete asset markets are available before information is released (Proposition 3).

[40]Markets are complete in the sense that agents can also contract on the information before it is released.

References

Berk, Jonathan, and Harald Uhlig, 1993, The Timing of Information in a General Equilibrium Framework, *Journal of Finance* 59, 275–287.

Blackwell, David, 1953, Equivalent comparison of experiments, *Annals of Mathematical Statistics* 24, 265–272.

Blume, Lawrence, Tarek Coury, and David Easley, 2006, Information, trade and incomplete markets, *Economic Theory* 29, 379–394.

Bond, Philip, Alex Edmans, and Itay Goldstein, 2012, The Real Effects of Financial Markets, *Annual Review of Financial Economics* 4, 339–360.

Brunnermeier, Markus K., Alp Simsek, and Wei Xiong, 2014, A Welfare Criterion For Models With Distorted Beliefs, *The Quarterly Journal of Economics* 129, 1753–1797.

Colombo, Luca, Gianluca Femminis, and Alessandro Pavan, 2014, Information Acquisition and Welfare, *The Review of Economic Studies* 81, 1438–1483.

Dang, Tri Vi, Gary Gorton, and Bengt Holmström, 2015, Ignorance, Debt and Financial Crises, Working paper, Columbia University, MIT and Yale University.

Duffie, Darrell, 2014, Challenges to A Policy Treatment of Speculative Trading Motivated by Differences in Beliefs, *Journal of Legal Studies* 43, 173–182.

Eckwert, Bernhard, and Itzhak Zilcha, 2003, Incomplete risk sharing arrangements and the value of information, *Economic Theory* 21, 43–58.

Epstein, Larry, and Stanley Zin, 1989, Substitution, Risk Aversion, and the Temporal Behavior of Consumption and Asset Returns: A Theoretical Framework, *Econometrica* 57, 937–969.

Gayer, Gabrielle, Itzhak Gilboa, Larry Samuelson, and David Schmeidler, 2014, Pareto Efficiency with Different Beliefs, *Journal of Legal Studies* 43, 151–171.

Gilboa, Itzhak, Larry Samuelson, and David Schmeidler, 2014, No-Betting-Pareto Dominance, *Econometrica* 82, 1405–1442.

Goldstein, Itay, and Haresh Sapra, 2013, Should Banks' Stress Test Results Be Disclosed? An Analysis of the Costs and Benefits, *Foundationas and Trends in Finance* 8, 1–54.

Goldstein, Itay, and Liyan Yang, forthcoming, Good Disclosure, Bad Disclosure, *Journal of Financial Economics* .

Gottardi, Piero, and Rohit Rahi, 2013, Risk-Sahring and Retrading in Incomplete Markets, *Economic Theory* 54, 287–304.

Gottardi, Piero, and Rohit Rahi, 2014, Value of Information in Competitive Economies with Incomplete Markets, *International Economic Review* 55, 57–81.

Hakansson, Nils, Gregory Kunkel, and James A. Ohlson, 1982, Sufficient and Necessary Conditions for Information to Have Social value in Pure Exchange, *Journal of Finance* 37, 1169–1181.

Hirshleifer, Jack, 1971, The Private and Social Value of Information and the Reward to Inventive Activity, *American Economic Review* 61, 561–574.

Hugonnier, Julien, Semyon Malamud, and Eugene Trubowitz, 2012, Endogenous Completeness of Diffusion Driven Equilibrium Markets, *Econometrica* 80, 1249–1270.

Jaffe, Jeffrey F., 1975, On the Use of Public Information in Financial Markets, *Journal of Finance* 30, 831–839.

Kim, Jeong-Ho, 2012, Assessing Welfare in Models with Heterogeneous Beliefs, Working paper, Princeton University.

Kreps, David, and Evan Porteus, 1978, Temporal Resolution of Uncertainty and Dynamic Choice Theory, *Econometrica* 46, 185–200.

Kurlat, Pablo, and Laura Veldkamp, 2015, Should We Regulate Financial Information?, *Journal of Economic Theory* 158, 697–720.

Marshall, John M., 1974, Private Inventives and Public Information, *American Economic Review* 64, 373–390.

Mas-Colell, Andreu, Michael Whinston, and Jerry Green, 1995, Microeconomic Theory, *Oxford University Press,* 1008 pp.

Ng, David S., 1975, Information Accuracy and Social Welfare under Homogeneous Beliefs, *Journal of Finance Economics* 2, 53–70.

Ng, David S., 1977, Pareto-Optimality of Authentic Information, *Journal of Finance* 32, 1717–1728.

Schlee, Edward E., 2001, The value of information in efficient risk sharing arrangements, *American Economic Review* 91, 509–524.

Wilson, Robert, 1975, Informational Economies of Scale, *Bell Journal of Economics* 6, 184–195.

Appendices

A Derivations and Proofs

A.1 Informed versus Uninformed Economy: Heterogeneous Beliefs

When agents have different expectations about future endowments, these different beliefs are not canceled in the equilibrium equations at $t = 0$ in (10), and affect equilibrium consumption plans in both periods $t \in \{0, 1\}$. First, we prove Lemma 1. Second, Lemma 2 follows straightforward from Lemma 1 (see main text). We, then, move to utilities derived from consumption at time $t = 0$ and prove Lemma 3. Lemma 2 and 3 together lead us to Proposition 2 as discussed in the main text. Finally, we solve the special case of Log-utility explicitly to illustrate the insight that an early information release increases welfare due to gains from speculative trading.

Proof of Lemma 1

We take the partial derivative with respect to the Pareto weight of the FOCs. For the uninformed economy (5), this operation yields, for all states $s \in \Omega$,

$$\frac{\partial c_{A0}}{\partial \lambda} = \frac{1}{\lambda}\left[\frac{u''_{A0}}{u'_{A0}} + \frac{u''_{B0}}{u'_{B0}}\right]^{-1} < 0, \qquad \frac{\partial c_{A1}(s)}{\partial \lambda} = \frac{1}{\lambda}\left[\frac{u''_{A1}(s)}{u'_{A1}(s)} + \frac{u''_{B1}(s)}{u'_{B1}(s)}\right]^{-1} < 0$$

$$\frac{\partial c_{B0}}{\partial \lambda} = \frac{-1}{\lambda}\left[\frac{u''_{A0}}{u'_{A0}} + \frac{u''_{B0}}{u'_{B0}}\right]^{-1} > 0, \qquad \frac{\partial c_{B1}(s)}{\partial \lambda} = \frac{-1}{\lambda}\left[\frac{u''_{A1}(s)}{u'_{A1}(s)} + \frac{u''_{B1}(s)}{u'_{B1}(s)}\right]^{-1} > 0.$$

Clearly, equilibrium consumptions are decreasing for A, and increasing for B, in λ because agents' utilities are increasing and concave. Similarly, for the informed economy (10), for all states $s \in \Omega$,

$$\frac{\partial \widehat{c}_{A0}(s)}{\partial \widehat{\lambda}} = \frac{1}{\widehat{\lambda}}\left[\frac{\widehat{u}''_{A0}(s)}{\widehat{u}'_{A0}(s)} + \frac{\widehat{u}''_{B0}(s)}{\widehat{u}'_{B0}(s)}\right]^{-1} < 0, \qquad \frac{\partial \widehat{c}_{A1}(s)}{\partial \widehat{\lambda}} = \frac{1}{\widehat{\lambda}}\left[\frac{\widehat{u}''_{A1}(s)}{\widehat{u}'_{A1}(s)} + \frac{\widehat{u}''_{B1}(s)}{\widehat{u}'_{B1}(s)}\right]^{-1} < 0, \tag{41}$$

$$\frac{\partial \widehat{c}_{B0}(s)}{\partial \widehat{\lambda}} = \frac{-1}{\widehat{\lambda}}\left[\frac{\widehat{u}''_{A0}(s)}{\widehat{u}'_{A0}(s)} + \frac{\widehat{u}''_{B0}(s)}{\widehat{u}'_{B0}(s)}\right]^{-1} > 0, \qquad \frac{\partial \widehat{c}_{B1}(s)}{\partial \widehat{\lambda}} = \frac{-1}{\widehat{\lambda}}\left[\frac{\widehat{u}''_{A1}(s)}{\widehat{u}'_{A1}(s)} + \frac{\widehat{u}''_{B1}(s)}{\widehat{u}'_{B1}(s)}\right]^{-1} > 0.$$

Proof of Lemma 3: Expected utilities of consumption at period $t = 0$

We concentrate on the utilities that agents expect to derive from their consumptions at time $t = 0$. We observe that at $t = 0$, the equilibrium structure of the informed and uninformed economies are very different. In (10), the arrival of signals in the informed economy fosters optimal state-contingent consumptions already at time $t = 0$, while this contingency is impossible in the uninformed economy (5). Consequently, the result of Lemma 1 can not be directly employed to derive comparative statics for utilities of time-zero consumptions in the uninformed and informed economies.

The key to an unambiguous welfare analysis at $t = 0$ lies with the convexity of agents' utilities of time-zero consumptions. We compute, for each state s, the first and second-order partial derivatives of the informed economy's equilibrium consumption at $t = 0$ with respect to the Pareto weight,

$$\frac{\partial \widehat{c}_{A0}(s)}{\partial \widehat{\lambda}(s)} = \frac{-1}{\widehat{\lambda}(s)} \left[\frac{\gamma_A}{\widehat{c}_{A0}(s)} + \frac{\gamma_B}{\widehat{c}_{B0}(s)} \right]^{-1}, \qquad \text{where} \quad \widehat{\lambda}(s) \equiv \frac{p_B(s)}{p_A(s)} \widehat{\lambda},$$

$$\frac{\partial^2 \widehat{c}_{A0}(s)}{\partial \left(\widehat{\lambda}(s)\right)^2} = \frac{1}{\left(\widehat{\lambda}(s)\right)^2} \left[\frac{\gamma_A}{\widehat{c}_{A0}(s)} + \frac{\gamma_B}{\widehat{c}_{B0}(s)} \right]^{-3} \left[\frac{\gamma_A}{\widehat{c}_{A0}(s)} \left(\frac{\gamma_A + 1}{\widehat{c}_{A0}(s)} + \frac{2\gamma_B}{\widehat{c}_{B0}(s)} \right) + \frac{\gamma_B(\gamma_B - 1)}{\widehat{c}^2_{B0}(s)} \right]. \tag{42}$$

Throughout the above derivative analysis, we keep aggregate endowments and beliefs unchanged. The introduction of the state-dependent parameter $\widehat{\lambda}(s)$ simplifies our exposition and is for pure convenience. For each state s, $\widehat{\lambda}(s)$ differs from the Pareto weight $\widehat{\lambda}$ by a non-material multiplicative factor.[41] It is clear from (42) that when $\gamma_B \geq 1$, A's time-zero equilibrium state-contingent consumption $\widehat{c}_{A0}(s)$ is a strictly convex function of the Pareto weight for all states s. Equivalently, consumption goods markets clear at $t = 0$ (2), so that $\frac{\partial \widehat{c}_{B0}(s)}{\partial \widehat{\lambda}(s)} = -\frac{\partial \widehat{c}_{A0}(s)}{\partial \widehat{\lambda}(s)}$ and $\frac{\partial^2 \widehat{c}_{B0}(s)}{\partial \left(\widehat{\lambda}(s)\right)^2} = -\frac{\partial^2 \widehat{c}_{A0}(s)}{\partial \left(\widehat{\lambda}(s)\right)^2}$. Therefore,

$$\frac{\partial \widehat{c}_{A0}(s)}{\partial \widehat{\lambda}(s)} > 0; \qquad \text{and} \qquad \frac{\partial \widehat{c}_{B0}(s)}{\partial \widehat{\lambda}(s)} < 0;$$

and $\gamma_B \geq 1$ is a sufficient condition for $\widehat{c}_{B0}(s)$ to be a strictly concave function of the Pareto weight for all state s,

$$\gamma_B \geq 1 \quad \Longrightarrow \qquad \frac{\partial^2 \widehat{c}_{A0}(s)}{\partial \left(\widehat{\lambda}(s)\right)^2} > 0; \quad \text{and} \quad \frac{\partial^2 \widehat{c}_{B0}(s)}{\partial \left(\widehat{\lambda}(s)\right)^2} < 0, \quad \forall s \in \Omega.$$

[41] That is, for each state s, the state-contingent equilibrium consumption $\widehat{c}_{A0}(s)$, (or $\widehat{c}_{B0}(s)$) depends on $\widehat{\lambda}(s)$ and $\widehat{\lambda}$ in identical way. This is because the factor $\frac{p_B(s)}{p_A(s)}$ that sets $\widehat{\lambda}(s)$ and $\widehat{\lambda}$ apart is constant within each state s.

The mechanism underlying this convexity is as follows. All else being equal, a drop in agent B's time-zero consumption means an increase in B's marginal utility, a decrease in A's marginal utility, and hence, a surge in the Pareto weight as asserted by Lemma 1. The condition $\gamma_B \geq 1$ suffices for the Pareto weight to increase (weakly) faster than linearly with agent B's time-zero consumption, i.e., the convexity in $\widehat{c}_{B0}(s)$.[42] To put it the other way around, when $\gamma_B \geq 1$, B's time-zero consumption in the informed economy is a strictly concave function of the Pareto weight for all states s. Because the consumption good market clears in each state, A's time-zero consumption is a strictly convex function of the Pareto weight.

Similar to the above argument, when $\gamma_B \geq 1$, for each state s, agent A's indirect utility at $t = 0$ is also unambiguously strictly convex in the Pareto weight. Indeed, taking the second derivative of $u(\widehat{c}_{A0}(s)$ with respect to $\widehat{\lambda}(s)$, using (41) and (42), yields for all s,

$$\frac{\partial^2 u(\widehat{c}_{A0}(s))}{\partial\left(\widehat{\lambda}(s)\right)^2} = \frac{[\widehat{c}_{A0}(s)]^{-\gamma_A}}{\left(\widehat{\lambda}(s)\right)^2}\left[\frac{\gamma_A}{\widehat{c}_{A0}(s)} + \frac{\gamma_B}{\widehat{c}_{B0}(s)}\right]^{-3}\left[\frac{\gamma_A}{\widehat{c}_{A0}(s)}\left(\frac{1}{\widehat{c}_{A0}(s)} + \frac{\gamma_B}{\widehat{c}_{B0}(s)}\right) + \frac{\gamma_B(\gamma_B - 1)}{\widehat{c}^2_{B0}(s)}\right]. \tag{43}$$

By a symmetric argument, the dependence of agent B's time-zero consumption $\widehat{c}_{B0}(s)$ and utility $u(\widehat{c}_{B0}(s))$ on $\frac{1}{\widehat{\lambda}(s)} = \frac{p_A(s)}{p_B(s)}\frac{1}{\widehat{\lambda}}$ exactly mirrors the dependence of $\widehat{c}_{A0}(s)$ and $u(\widehat{c}_{A0}(s))$ on $\widehat{\lambda}(s)$.[43] Consequently, we see that $\gamma_A \geq 1$ is a sufficient condition for agent B's time-zero utility to be unambiguously strictly convex in $\frac{1}{\widehat{\lambda}(s)}$ in all states s of the informed economy. We see this convexity explicitly in the following expression, which mirrors (43),

$$\frac{\partial^2 u(\widehat{c}_{B0}(s))}{\partial\left(\frac{1}{\widehat{\lambda}(s)}\right)^2} = \frac{[\widehat{c}_{B0}(s)]^{-\gamma_A}}{\left(\widehat{\lambda}(s)\right)^{-2}}\left[\frac{\gamma_A}{\widehat{c}_{A0}(s)} + \frac{\gamma_B}{\widehat{c}_{B0}(s)}\right]^{-3}\left[\frac{\gamma_B}{\widehat{c}_{B0}(s)}\left(\frac{1}{\widehat{c}_{B0}(s)} + \frac{\gamma_A}{\widehat{c}_{A0}(s)}\right) + \frac{\gamma_A(\gamma_A - 1)}{\widehat{c}^2_{A0}(s)}\right]. \tag{44}$$

A direct application of Jensen's inequality on agents A's and B's strictly convex utility functions results in a comparative statics between the informed and uninformed economies in period $t = 0$. To facilitate this application, we note that, for each state s, time-zero state-contingent utility $\widehat{u}_{A0}(s)$ is an (indirect) function of $\widehat{\lambda}(s)$.[44] Accordingly, in what follows, we employ notations

[42]Quantitatively, for each state s, the FOC (10) and resource constraint $\widehat{c}_{A0}(s) + \widehat{c}_{B0}(s) = \widehat{e}_0$ at $t = 0$ imply the relationship, $\widehat{\lambda}(s) = (\widehat{c}_{B0}(s))^{\gamma_B}(e_0 - \widehat{c}_{B0}(s))^{-\gamma_A}$. When $\gamma_B \geq 1$, $\widehat{\lambda}(s)$ is convex in $\widehat{c}_{B0}(s)$.

[43]To see this symmetry, we note that the FOC at time $t = 0$ (10) can be written in two equivalent ways, for all s,

$$\widehat{u}_{A0}(s) = \frac{p_B(s)}{p_A(s)}\widehat{\lambda}\widehat{u}_{B0}(s) \equiv \widehat{\lambda}(s)\widehat{u}_{B0}(s) \quad \Longleftrightarrow \quad \widehat{u}_{B0}(s) = \frac{p_A(s)}{p_B(s)}\frac{1}{\widehat{\lambda}}\widehat{u}_{A0}(s) \equiv \frac{1}{\widehat{\lambda}(s)}\widehat{u}_{A0}(s).$$

[44]This is because in equilibrium this time-zero utility's only argument $c_{A0}(s)$ is also a function of $\widehat{\lambda}(s)$. A similar

$\widehat{u}_{A0}(s) = \widehat{u}_{A0}\left(\widehat{\lambda}(s)\right)$ and $\widehat{u}_{B0}(s) = \widehat{u}_{B0}\left(\widehat{\lambda}(s)^{-1}\right)$.

Assuming $\gamma_B \geq 1$ and taking the average of (43) (under agent A's belief) yield Jensen's bound concerning A's expected utilities in period $t = 0$, which is the counterpart of (20),

$$\begin{aligned} \widehat{EU}_{A0} &\equiv \textstyle\sum_{s\in\Omega} p_A(s)\widehat{u}_{A0}(s) = \sum_{s\in\Omega} p_A(s)\widehat{u}_{A0}\left(\widehat{\lambda}(s)\right) \\ &> \widehat{u}\left(\textstyle\sum_{s\in\Omega} p_A(s)\widehat{\lambda}(s)\right) = \widehat{u}\left(\sum_{s\in\Omega} p_A(s)\frac{p_B(s)}{p_A(s)}\widehat{\lambda}\right) = \widehat{u}_{A0}\left(\widehat{\lambda}\right) = u\left(c_{A0}\left(\widehat{\lambda}\right)\right). \end{aligned} \tag{45}$$

Similarly, assuming $\gamma_A \geq 1$ and taking taking the average of (44) (under agent B's belief) yields,

$$\begin{aligned} \widehat{EU}_{B0} &\equiv \textstyle\sum_{s\in\Omega} p_B(s)\widehat{u}_{B0}(s) = \sum_{s\in\Omega} p_B(s)\widehat{u}_{B0}\left(\frac{1}{\widehat{\lambda}(s)}\right) \\ &> \widehat{u}\left(\textstyle\sum_{s\in\Omega} p_B(s)\frac{1}{\widehat{\lambda}(s)}\right) = \widehat{u}\left(\sum_{s\in\Omega} p_B(s)\frac{p_A(s)}{p_B(s)}\frac{1}{\widehat{\lambda}}\right) = \widehat{u}_{B0}\left(\frac{1}{\widehat{\lambda}}\right) = u\left(c_{B0}\left(\widehat{\lambda}\right)\right). \end{aligned} \tag{46}$$

As an implication of Lemma 1, when the uninformed economy's Pareto weight is larger, $\lambda \geq \widehat{\lambda}$, agent A's time-zero consumption and hence utility in the informed economy are higher than their counterparts in the uninformed economy,

$$\lambda \geq \widehat{\lambda} \Longrightarrow c_{A0}\left(\widehat{\lambda}\right) \geq c_{A0}\left(\lambda\right) \Longrightarrow u\left(c_{A0}(\widehat{\lambda})\right) \geq u\left(c_{A0}(\lambda)\right). \tag{47}$$

Symmetrically, also as a result of Lemma 1, when the informed economy's Pareto weight is larger, $\widehat{\lambda} \geq \lambda$, agent B's time-zero consumption and hence utility in the informed economy are higher than their counterparts in the uninformed economy,

$$\widehat{\lambda} \geq \lambda \Longrightarrow c_{B0}\left(\widehat{\lambda}\right) \geq c_{B0}\left(\lambda\right) \Longrightarrow u\left(c_{B0}(\widehat{\lambda})\right) \geq u\left(c_{B0}(\lambda)\right). \tag{48}$$

Combining (47) with Jensen's inequality (45), and similarly (48) with Jensen's inequality (46) motivate Assumption 1 and imply Lemma 3.

Lemma 3 aims to extend the comparative statics at $t = 1$ of Lemma 2 to period $t = 0$. Though, the difference in time-zero equilibrium structures of the informed and uninformed economies is profound and goes beyond the difference of λ versus. $\widehat{\lambda}$. As a result, the comparative statics at $t = 0$ arises only under additional assumptions. The gist behind the result reported in Lemma 3 is

observation applies for $\widehat{u}_{B0}(s)$ as an (indirect) function of $\frac{1}{\widehat{\lambda}(s)}$.

Jensen's inequality, which applies when agents' preferences are sufficiently non-linear, $\gamma_A, \gamma_B > 1$, our additional assumptions mentioned earlier.[45] Even with these additional assumptions in place, the time-zero comparative statics can be obtained only for a single agent at a time when the two Pareto weights differ, $\lambda \neq \widehat{\lambda}$. The difficulty is not in achieving both (45) and (46), but in having *simultaneously* $c_{A0}\left(\widehat{\lambda}\right) \geq c_{A0}(\lambda)$ *and* $c_{B0}\left(\widehat{\lambda}\right) \geq c_{B0}(\lambda)$ in (47) and (48). Lemma 3 renders this simultaneity unlikely in a heterogeneous-belief setting. In fact, only when $\lambda = \widehat{\lambda}$ such unambiguous comparative statics exists. But when agents differ in their beliefs, the equality of Pareto weights in the uninformed and informed economies does not generically hold.

This is an indication of the challenge in establishing the welfare value of public information, unambiguously and comprehensively for both periods $t \in \{0, 1\}$, when agents disagree about the future prospects of the economy. We take up this task next.

Special case of Log-Utility

In the special case of log-utility we are able to derive explicit closed-form solutions for the equilibrium consumption allocation and agents' utilities. The explicit solutions make it is easy to understand how and why the release of public information improves risk sharing and expected utilities of all agents.

We first derive explicit solutions in the uninformed economy. Plugging the first order condition (4) into the budget constraint (6) and assuming proportional endowments yields,

$$\frac{c_{A0}(1+\beta_A)}{c_{B0}(1+\beta_B)} = k.$$

Moreover, the first order condition can now be rewritten as

$$\frac{c_{A1}(s)}{c_{B1}(s)} = k\frac{p_A(s)}{p_B(s)}\frac{\beta_A}{\beta_B}\frac{1+\beta_B}{1+\beta_A}.$$

Imposing market clearing we get $\forall t \in \{0, 1\}$,

$$e_t(s) \quad = \quad c_{At}(s) + c_{Bt}(s) = c_{At}(s)\left(1 + \frac{c_{Bt}(s)}{c_{At}(s)}\right),$$

[45] Note that the assumptions $\gamma_A, \gamma_B \geq 1$ are sufficient, but not necessary, conditions.

or

$$
\begin{aligned}
c_{At}(s) &= e_t(s)\frac{k(p_A(s)\beta_A)^t(1+\beta_B)}{(p_B(s)\beta_B)^t(1+\beta_A)+k(p_A(s)\beta_A)^t(1+\beta_B)},\\
c_{Bt}(s) &= e_t(s)\frac{(p_B(s)\beta_B)^t(1+\beta_A)}{(p_B(s)\beta_B)^t(1+\beta_A)+k(p_A(s)\beta_A)^t(1+\beta_B)}.
\end{aligned}
$$

Ex-ante expected utilities in the uninformed economy are therefore,

$$
\begin{aligned}
EU_A &= \ln(e_0)+\beta_A E^A_{0^-}\left[\ln(e_1(s))\right]-\ln\left(1+\frac{1}{k}\frac{1+\beta_A}{1+\beta_B}\right)\\
&\quad -\beta_A E^A_{0^-}\left[\ln\left(1+\frac{1}{k}\frac{p_B(s)\beta_B(1+\beta_A)}{p_A(s)\beta_A(1+\beta_B)}\right)\right],\\
EU_B &= \ln(e_0)+\beta_B E^B_{0^-}\left[\ln(e_1(s))\right]-\ln\left(1+k\frac{1+\beta_B}{1+\beta_A}\right)\\
&\quad -\beta_B E^B_{0^-}\left[\ln\left(1+k\frac{p_A(s)\beta_A(1+\beta_B)}{p_B(s)\beta_B(1+\beta_A)}\right)\right].
\end{aligned}
$$

We now turn to the informed economy. Plugging (8) and (9) into (11) and assuming proportional endowments yields,

$$
k\widehat{\lambda}=\frac{1+\beta_A}{1+\beta_B},
$$

Plugging this into (10) gives $\forall t\in\{0,1\}$,

$$
\frac{\widehat{c}_{At}(s)}{\widehat{c}_{Bt}(s)}=k\frac{p_A(s)}{p_B(s)}\frac{\beta_A^t}{\beta_B^t}\frac{1+\beta_B}{1+\beta_A}.
$$

Imposing market clearing, we get,

$$
\begin{aligned}
\widehat{c}_{At}(s) &= e_t(s)\frac{kp_A(s)\beta_A^t(1+\beta_B)}{p_B(s)\beta_B^t(1+\beta_A)+kp_A(s)\beta_A^t(1+\beta_B)},\\
\widehat{c}_{Bt}(s) &= e_t(s)\frac{p_B(s)\beta_B^t(1+\beta_A)}{p_B(s)\beta_B^t(1+\beta_A)+kp_A(s)\beta_A^t(1+\beta_B)}.
\end{aligned}
$$

Ex-ante expected utilities in the informed economy are therefore,

$$\begin{aligned}
\widehat{EU}_A &= \ln(e_0) + \beta_A E^A_{0^-}\left[\ln(e_1(s))\right] - E^A_{0^-}\left[\ln\left(1 + \frac{1}{k}\frac{p_B(s)(1+\beta_A)}{p_A(s)(1+\beta_B)}\right)\right] \\
&\quad -\beta_A E^A_{0^-}\left[\ln\left(1 + \frac{1}{k}\frac{p_B(s)\beta_B(1+\beta_A)}{p_A(s)\beta_A(1+\beta_B)}\right)\right], \\
\widehat{EU}_B &= \ln(e_0) + \beta_B E^B_{0^-}\left[\ln(e_1(s))\right] - E^B_{0^-}\left[\ln\left(1 + k\frac{p_A(s)(1+\beta_B)}{p_B(s)(1+\beta_A)}\right)\right] \\
&\quad -\beta_B E^B_{0^-}\left[\ln\left(1 + k\frac{p_A(s)\beta_A(1+\beta_B)}{p_B(s)\beta_B(1+\beta_A)}\right)\right].
\end{aligned}$$

Expected utilities in the two economies EU_I and $\widehat{EU}_I$ only differ with respect to the third terms. Jensen's inequality tells us that

$$\begin{aligned}
E^A_{0^-}\left[\ln\left(1 + \frac{1}{k}\frac{p_B(s)(1+\beta_A)}{p_A(s)(1+\beta_B)}\right)\right] &\leq \ln\left(E^A_{0^-}\left[1 + \frac{1}{k}\frac{p_B(s)(1+\beta_A)}{p_A(s)(1+\beta_B)}\right]\right) \\
&= \ln\left(1 + \frac{1}{k}\frac{1+\beta_A}{1+\beta_B}\right),
\end{aligned}$$

and therefore, $\widehat{EU}_A \geq EU_A$. The proof for $\widehat{EU}_B \geq EU_B$ is equivalent.

The gain in utility through to the Jensen term is clearly due to the heterogeneity in beliefs, $p_A(s) \neq p_B(s)$. In a similar spirit, in EU_I the fourth term depends on the disagreement, $\frac{p_A(s)}{p_B(s)}$. Jensen's inequality implies

$$E^A_{0^-}\left[\ln\left(1 + \frac{1}{k}\frac{p_B(s)\beta_B(1+\beta_A)}{p_A(s)\beta_A(1+\beta_B)}\right)\right] \leq \ln\left(1 + \frac{1}{k}\frac{\beta_B(1+\beta_A)}{\beta_A(1+\beta_B)}\right),$$

where the latter term is understood as a case if there was no disagreement, i.e. homogeneous beliefs.

Intuitively, every agent's utility increases due to speculation if beliefs are heterogeneous. Disagreement about the probability distribution over future endowments implies disagreement about the "fundamental value" of contingent claims. Agents speculate on their beliefs and ex-ante all agents feel wealthier and are better off in terms of expected utility. Each agent believes that his trading counter-party has wrong beliefs and given the respective probability distribution function each agent expects to make a profit off the other agents. If the state of the world is revealed at time 1 (uninformed economy), expected profits from speculation are paying off only at time 1 and agents cannot borrow against the expected wealth from speculation because of the disagreement. In this case of late uncertainty resolution agents can only increase their expected consumption at time 1 but not at time 0. In contrast, if a public signal is known to reveal the state of the world already at time 0 (informed economy), then agents expect to realize their profits from speculation at time 0 and are able to increase both their expected consumption at time 0 and time 1. An early release of information improves expected utilities of all agents (Pareto improvement).

Jensen's inequality further suggests that an early release of public information is relatively more valuable if the disagreement is relatively large. Large disagreement means that the ratio $\frac{p_A(s)}{p_B(s)}$ varies a lot from 1 and across the state space. Intuitively, it makes sense that agents expect larger speculative profits if they believe their speculative counter-party is very wrong rather than if they were in a situation where agents only slightly disagree.

A.2 Informed versus Too-Early Informed Economy: Unlikely Convergence

In this Appendix, we rule out the technical case that both sides of equation (36) in Proposition 3 vanish. Evidently, in such case, both agents' are locally indifferent between the informed and too-early informed economies, when agents' beliefs deviate from the transiting configurations ($\{p_A, p_B^*\}$ or $\{p_A^*, p_B\}$). Therefore, the following two hypothetical scenarios,

$$\begin{aligned} \text{scenario (i):} \quad & \left[d\widehat{EU}_B - d\overline{EU}_B\right]\Big|_{\{p_B=p_B^*\}} = \left[d\widehat{EU}_A - d\overline{EU}_A\right]\Big|_{\{p_B=p_B^*\}} = 0, \\ \text{scenario (ii):} \quad & \left[d\widehat{EU}_B - d\overline{EU}_B\right]\Big|_{\{p_A=p_A^*\}} = \left[d\widehat{EU}_A - d\overline{EU}_A\right]\Big|_{\{p_A=p_A^*\}} = 0. \end{aligned} \tag{49}$$

are compatible with Proposition 3.

If either of the two scenarios holds, then public information has an unambiguous welfare value (in conjunction with asset market availability). *Both* agents are better off or worse off, at least locally, in the informed economy (compared to the too-early informed economy) as measured by their pre-signal expected utilities. We next derive a necessary and sufficient condition for the scenarios in (49) to hold. The condition is very restrictive and is satisfied only for a particular and narrow set of agents' heterogeneous preferences. Therefore, the possibility of an unambiguous welfare value of public information in the presence of asset market frictions exists, but is highly unlikely.

Without loss of generality, we work with scenario (i) in (49). Our analysis and results can be immediately established for scenario (ii) by symmetric arguments. We begin with an explicit expression for the variation of agent A's expected utility, obtained from totally differentiating the expected utility $\widehat{EU}_A$ and employing FOC (10),

$$d\widehat{EU}_A = \sum_{s\in\Omega} dp_B(s)\frac{p_A(s)}{p_B(s)}\left(N(s) - \frac{E_A[N]}{E_A[M]}M(s)\right), \tag{50}$$

where $E_A[X] \equiv \sum_{s\in\Omega} p_A(s)X(s)$ denotes the average value of a generic state-contingent quantity X under A's belief. The state-contingent equilibrium quantities $N(s)$ and $M(s)$ respectively are,

$$N(s) \equiv \sum_{t=0}^{1} \frac{-(\beta_A)^t (\widehat{c}_{At}(s))^{-\gamma_A}}{\left(\frac{\gamma_A}{\widehat{c}_{At}(s)} + \frac{\gamma_B}{\widehat{c}_{Bt}(s)}\right)}, \tag{51}$$

$$M(s) \equiv \sum_{t=0}^{1} \frac{-(\beta_A)^t \left[(\widehat{c}_{At}(s))^{-\gamma_A} - \gamma_A (\widehat{c}_{At}(s))^{-\gamma_A - 1}(\widehat{c}_{At}(s) - e_{At}(s))\right]}{\left(\frac{\gamma_A}{\widehat{c}_{At}(s)} + \frac{\gamma_B}{\widehat{c}_{Bt}(s)}\right)}. \tag{52}$$

Because equilibrium in the too-early informed economy does not depend on agents' beliefs (35), the hypothesis $d\widehat{EU}_A - d\widehat{EU}_A = 0$ in (49) is equivalent to $d\widehat{EU}_A = 0$, which we now investigate.

Given the normalization constraint $\sum_{s\in\Omega} p_B(s) = 0$ (32) in the variations of B's belief, $d\widehat{EU}_A$ vanishes if and only if the expression associated with each perturbation $dp_B(s)$ in (50) is state-independent,[46]

$$\left.\left[\frac{p_A(s)}{p_B(s)}\left(N(s) - \frac{E_A[N]}{E_A[M]}M(s)\right)\right]\right|_{p_B=p_B^*} = K, \qquad \forall s \in \Omega, \tag{53}$$

where K is constant across states $s \in \Omega$. Taking the average (under distribution $\{p_B^*(s)\}$) of both sides of the above equation determines K unambiguously,

$$\sum_{s\in\Omega} p_B^*(s) \left.\left[\frac{p_A(s)}{p_B(s)}\left(N(s) - \frac{E_A[N]}{E_A[M]}M(s)\right)\right]\right|_{p_B=p_B^*} = \sum_{s\in\Omega} p_B^*(s)K \implies 0 = K.$$

Substituting this solution of K into (53) implies that $d\widehat{EU}_A$ (50) vanishes if and only if,

$$\left.\left[N(s) - \frac{E_A[N]}{E_A[M]}M(s)\right]\right|_{p_B=p_B^*} = 0, \qquad \forall s \in \Omega.$$

Since $N(s)$ in (51) is non-zero for all $s \in \Omega$, the above condition is equivalent to that the ratio of $M(s)$ and $N(s)$ (at p_B^*) be equal to the ratio of their means (under A's belief), $\left.\frac{M(s)}{N(s)}\right|_{p_B^*} = \frac{E_A[M]}{E_A[N]}$, for all s. We thus have the following necessary and sufficient condition for the special case in which

[46] This is a result of a standard Lagrangian optimization subject to an equality constraint.

the equation in Proposition 3 is equal to zero.

Lemma 5 *Both agents' expected utility increments* (36) *stall locally at the belief configuration* $\{p_A, p_B^*\}$ *if and only if the ratio of* $M(s)$ (51) *and* $N(s)$ (52) *is state-independent,*

$$\begin{cases} \left[d\widehat{EU}_A - d\overline{EU}_A\right]\Big|_{\{p_B=p_B^*\}} = 0, \\ \left[d\widehat{EU}_B - d\overline{EU}_B\right]\Big|_{\{p_B=p_B^*\}} = 0 \end{cases} \iff \quad \frac{M(s)}{N(s)}\bigg|_{p_B=p_B^*} = \frac{M(s')}{N(s')}\bigg|_{p_B=p_B^*}, \quad \forall s, s' \in \Omega. \quad (54)$$

We recall from Lemma 4 that exactly at the belief configuration $\{p_A, p_B^*\}$ (or $\{p_A^*, p_B\}$), the equilibria of the informed and the too-early informed economies converge. When agents' beliefs are not exactly at these particular configurations, and across the two economies, most likely only one agent is better off (the other worse off), again by Lemma 4. However, Lemma 5 goes a step further by quantifying the likelihood of the above ambiguity in agents' cross-economy welfare. Both agents have higher expected utilities in the informed than in the too-early informed economy if and only if condition (54) holds.[47] However, this unambiguous comparative statics holds only locally around a particular belief configuration $\{p_A, p_B^*\}$.

We now take a closer look at the necessary and sufficient condition (54). Note that the condition is always evaluated locally at the belief configurations where informed and too-early informed economies' equilibria converge. We can substitute too-early informed equilibrium consumptions $\bar{c}_{It}$ for all consumption quantities in that condition. For CRRA preferences, using expressions for $N(s)$ (51) and $M(s)$ (52), expression (54) is equivalent to the following explicit condition,

$$\frac{1}{\left(\frac{\gamma_A}{\bar{c}_{A0}(s)} + \frac{\gamma_B}{\bar{c}_{B0}(s)}\right)} + \bar{q}(s)\frac{1}{\left(\frac{\gamma_A}{\bar{c}_{A1}(s)} + \frac{\gamma_B}{\bar{c}_{B1}(s)}\right)} \quad (55)$$

$$= L\left[\frac{e_{A0}}{\bar{c}_{A0}(s)}\frac{1}{\left(\frac{\gamma_A}{\bar{c}_{A0}(s)} + \frac{\gamma_B}{\bar{c}_{B0}(s)}\right)} + \bar{q}(s)\frac{e_{A1}(s)}{\bar{c}_{A1}(s)}\frac{1}{\left(\frac{\gamma_A}{\bar{c}_{A1}(s)} + \frac{\gamma_B}{\bar{c}_{B1}(s)}\right)}\right], \quad \forall s \in \Omega,$$

where L is some state-independent coefficient, and $\bar{q}(s) = \beta_A\left(\frac{\bar{c}_{A0}(s)}{\bar{c}_{A1}(s)}\right)^{-\gamma_A}$ are AD prices (13).

To see how restrictive condition (54) is, two observations on its equivalent version (55) are in order. First, condition (54) calls for the equality (55) to hold for each state s, but with a common

[47] The Hessian matrix with respect to the variations $\{dp_B(s)\}$ of the expected utility increment $\widehat{EU}_I - \overline{EU}_I$. $\forall I \in \{A, B\}$ is negative semidefinite at p_B^*. For for both agents, the utility increment thus has a local maximum at $\{p_A, p_B^*\}$.

state-independent L. Second, the too-early informed equilibrium consumptions $\{\bar{c}_{At}(s), \bar{c}_{Bt}(s)\}$ are entirely determined by the FOCs–resource constraint system (14) and the budget constraint (15) for each state s independently. Therefore, when we substitute this equilibrium solution $\{\bar{c}_{At}(s), \bar{c}_{Bt}(s)\}$ into (55) to back out L, we generally obtain a state-dependent coefficient $L(s)$. Hence, condition (54), necessary and sufficient for an unambiguously positive impact of early information releases together with complete asset markets on welfare (Lemma 5) generally does not hold.

Condition (55), though, holds for a restrictive set of preferences. In particular, when agents have identical CRRA, $\gamma_A = \gamma_B$, the FOC (14) for the too-early informed economy reduces to the equality of any agent's intertemporal consumption ratio and that of aggregate endowments,

$$\frac{\bar{c}_{A1}(s)}{\bar{c}_{A0}(s)} = \frac{\bar{c}_{B1}(s)}{\bar{c}_{B0}(s)} = \frac{e_{A1}(s)}{\bar{c}_{A0}(s)}, \qquad \forall s \in \Omega. \tag{56}$$

As a result, agent A's budget constraint, $\bar{c}_{A0}(s) - e_{A0}(s) + \bar{q}(s)\left(\bar{c}_{A1}(s) - e_{A1}(s)\right) = 0$ immediately implies (55), which in turn indicates that the equation in Proposition 3 is equal to zero. Combining this result with the discussion following Lemma 5, we have

Lemma 6 *Assume that agents have identical relative risk aversions $\gamma_A = \gamma_B$. Then $N(s) = M(s)$, $\forall s \in \Omega$, and condition (54) holds. Therefore, both agents have higher expected utilities in the informed than in the too-early informed economy, locally around the belief configurations at which the two economies' equilibria converge.*

$$\gamma_A = \gamma_B \quad \Longrightarrow \quad \left[\widehat{EU}_A \geq \overline{EU}_A\right]\Big|_{\{p_B = p_B^* + \epsilon\}} = 0, \quad \text{and} \quad \left[\widehat{EU}_B \geq \overline{EU}_B\right]\Big|_{\{p_B = p_B^* + \epsilon\}} = 0.$$

While this result and its derivation appear to adhere explicitly to CRRA preferences (and at the belief configuration $\{p_A, p_B^*\}$), Lemma 6 applies to all additively separable utilities (as well as to the other belief configuration $\{p_A^*, p_B\}$). Neither Assumption 2 of proportional endowments, nor the assumption of homogeneous beliefs alone is sufficient to resurrect condition (54). But assumptions on agents' risk preferences are sufficient to deliver (54), as Lemma 6 demonstrates.[48] Lemma 6 offers an unambiguous welfare value of public information (in conjunction with early asset market availability) only locally around particular belief configuration $\{p_A, p_B^*\}$ (and $\{p_A^*, p_B\}$).

[48]It is interesting to note that, beyond the CRRA class, we can also show that condition (54) holds when agents have constant absolute risk aversion (CARA), for any level of heterogeneity in beliefs.

B Extensions

Our analysis and results on the welfare value of public information can be generalized to broader economic settings and under weaker assumptions. The generalization is important not only for a wider applicability of the results, but also for the better understanding of the economic factors driving these results. This is because different economic factors and their contributions may be confounded when the settings at hand are specific. In this section, we discuss aspects of imperfect information, general additively separable utilities, and possible weakened versions of assumptions previously needed to deliver the main results of the paper.

Imperfect signals

Up to this point, our analysis has focused on the perfect foresight of future endowments, whenever public signals are available. When imperfect signals are observed agents can reduce, but not completely eliminate, the uncertainty about future endowments. We now briefly examine the relevance of imperfect signals on the improvement of risk sharing and welfare of agents.

We consider a parsimonious setting of imperfect signals. In the economies in which such signals are available, at some time before $t = 0$, agents observe perfect signals only on a portion $e_1(s_0)$ of the future aggregate endowment,

$$e_1(s_0, s_1) = e_1(s_0) + \epsilon(s_1),$$

to be realized at $t = 1$. The noise $\epsilon(s_1)$ (or state s_1) intends to model the imperfection of signals s_0, and can only be observed at $t = 1$. Endowments $e_0 = e_{A0} + e_{B0}$ at time $t = 0$ remain constants and known to all agents as before.

Evidently, the imperfect signals do not affect the uninformed economy, in which no signal arrives ever. For the informed economy, before the arrival of signals, complete asset markets allow agents to trade two sets of AD securities $\{\widehat{q}_0(s_0)\}$, $\{\widehat{q}_1(s_0, s_1)\}$ paying off at $t = 0$ and $t = 1$ respectively.[49]

[49]In place of (8), (9), here the AD prices read,

$$\frac{p_A(s_0)\widehat{u}'_{A0}(s_0)}{\sum_{s_0} p_A(s_0)\widehat{u}'_{A0}(s_0)} = \widehat{q}_0(s_0) = \frac{p_B(s_0)\widehat{u}'_{B0}(s_0)}{\sum_{s_0} p_B(s_0)\widehat{u}'_{B0}(s_0)},$$

$$\frac{p_A(s_0, s_1)\beta_A\widehat{u}'_{A1}(s_0, s_1)}{\sum_{s_0,s_1} p_A(s_0, s_1)\widehat{u}'_{A0}(s_0, s_1)} = \widehat{q}_1(s_0, s_1) = \frac{p_B(s_0, s_1)\beta_B\widehat{u}'_{B1}(s_0, s_1)}{\sum_{s_0,s_1} p_B(s_0, s_1)\widehat{u}'_{B0}(s_0, s_1)}.$$

As a result, agents can devise (pre-signal) optimal consumptions $\{\widehat{c}_{I0}(s_0), \widehat{c}_{I0}(s_0, s_1)\}$ contingent on signal s_0 at $t = 0$, and on the full state of the economy (s_0, s_1) to be realized at $t = 1$. In place of (10)–(11) for perfect signals, the FOC and resource constraints for the current case of imperfect signals read,

$$\begin{array}{l} \text{at } t = 0 \\ \forall s_0 \end{array} \quad \begin{cases} p_A(s_0)\widehat{u}'_{A0}(s_0) = \widehat{\lambda} p_B(s_0)\widehat{u}'_{B0}(s_0), \\ \widehat{c}_{A0}(s_0) + \widehat{c}_{B0}(s_0) = e_0, \end{cases}$$

$$\begin{array}{l} \text{at } t = 1 \\ \forall s_0, s_1 \end{array} \quad \begin{cases} p_A(s_0, s_1)\beta_A \widehat{u}'_{A1}(s_0, s_1) = \widehat{\lambda} p_B(s_0, s_1)\beta_B \widehat{u}'_{B1}(s_0, s_1), \\ \widehat{c}_{A1}(s_0, s_1) + \widehat{c}_{B1}(s_0, s_1) = e_1(s_0, s_1), \end{cases}$$

and agent A's budget constraint,

$$\sum_{s_0} \widehat{q}_0(s_0)\widehat{c}_{A0}(s_0) + \sum_{s_0, s_1} \widehat{q}_1(s_0, s_1)\widehat{c}_{A1}(s_0, s_1) = \sum_{s_0} \widehat{q}_0(s_0)\widehat{e}_{A0}(s_0) + \sum_{s_0, s_1} \widehat{q}_1(s_0, s_1)\widehat{e}_{A1}(s_0, s_1).$$

where p_I denotes agent I's expectation, $I \in \{A, B\}$.

When agents have homogeneous beliefs, this common belief is canceled out in the FOC at both periods. Therefore, at $t = 0$, since there is no aggregate uncertainty, the solution of FOC and resource constraint must be state-independent, $\widehat{c}_{I0}(s_0) = \widehat{c}_{I0}$, $\forall s, I$. By identical arguments leading to Proposition 1, imperfect signals are irrelevant to welfare when agents have homogeneous beliefs and asset markets are complete. Proposition 1 remains valid when public information is imperfect.

When agents have different beliefs, by virtue of the Assumption 2 of proportional endowments, again only aggregate endowments e_0 and $e_1 + \epsilon$ enter the equilibrium equation system in both periods. By identical arguments leading to Proposition 2, imperfect signals make both agents' better off, in expectation, in the informed economy under the assumption of proportional endowments (and Assumption 1). Proposition 2 remains valid when public information is imperfect.

General additively separable preferences

Most of the results presented earlier, including Assumption 1 ($\gamma_A, \gamma_B \geq 1$), have been derived explicitly for CRRA utilities for ease of exposition. Our results hold for general additively separable preferences. For illustration, let us derive a sufficient condition for the convexity of agents' equilibrium consumptions in the Pareto weight (42) for general preferences. This condition replaces Assumption 1, and is a key to generalize Proposition 2 to general preferences beyond the CRRA class.

For each time and state, we keep beliefs and aggregate endowments unchanged, and vary the Pareto weight to model the changes in agents' relative endowments in our comparative statics investigation. The analysis is identical for $t = 0$ and $t = 1$. Below we work with period $t = 0$. We begin with totally differentiating the FOC (10) for state s, which yields

$$\Big(p_A(s)\widehat{u}''_{A0}(s) + \widehat{\lambda} p_B(s)\widehat{u}''_{B0}(s)\Big) \frac{d\widehat{c}_A(s)}{d\widehat{\lambda}} = p_B(s)\widehat{u}'_{B0}(s). \tag{57}$$

Totally differentiating one more time, we obtain the second-order derivative of consumption in the Pareto weight,

$$\begin{aligned} &\Big[p_A(s)\widehat{u}''_{A0}(s) + \widehat{\lambda} p_B(s)\widehat{u}''_{B0}(s)\Big] \frac{d^2\widehat{c}_A(s)}{d\widehat{\lambda}^2} \\ = &\frac{-d\widehat{c}_A(s)}{d\widehat{\lambda}} \left[2p_B(s)\widehat{u}''_{B0}(s) + \Big\{p_A(s)\widehat{u}'''_{A0}(s) - \widehat{\lambda} p_B(s)\widehat{u}'''_{B0}(s)\Big\} \frac{d\widehat{c}_A(s)}{d\widehat{\lambda}}\right]. \end{aligned} \tag{58}$$

Following (57), when utilities are increasing and concave in consumption, A's consumption decreases in the Pareto weight, $\frac{d\widehat{c}_A(s)}{d\widehat{\lambda}} < 0$. Therefore, A's consumption is convex in the Pareto weight when the expression inside square brackets in (58) is negative. Using FOC, and (58), we can express this sufficient condition for $\frac{d^2\widehat{c}_A(s)}{d\widehat{\lambda}^2} > 0$ as,

$$2\frac{\widehat{u}''_{A0}(s)}{\widehat{u}''_{B0}(s)} + \frac{\widehat{u}'_{B0}(s)}{\big(\widehat{u}''_{B0}(s)\big)^2}\widehat{u}'''_{A0}(s) + \widehat{\lambda}\left[2 - \frac{\widehat{u}'_{B0}(s)\widehat{u}'''_{B0}(s)}{\big(\widehat{u}''_{B0}(s)\big)^2}\right] \geq 0. \tag{59}$$

Therefore a sufficient condition for A's consumption to be convex in the Pareto weight is

$$\widehat{u}'''_{A0}(s) \geq 0, \qquad \text{and} \qquad \frac{\widehat{u}'_{B0}(s)\widehat{u}'''_{B0}(s)}{\big(\widehat{u}''_{B0}(s)\big)^2} \leq 2. \tag{60}$$

For CRRA utilities, the above sufficient condition reduces to $\gamma_B \geq 1$ in Assumption 1. We remark that, for general additively separable preferences, the above condition involves the third-order derivative of agents' utility functions (or prudence), which is quite obscured in Assumption 1 for the class of CRRA utilities (Appendix C offers a numerical perspective on non-time separable preferences).

Weakening assumptions

Sufficient assumptions, which are key to our earlier results, can also be weakened. Assumption 1, or its version for general preferences (60), can be substantially weakened. This is because the original requirement of the convexity of equilibrium consumptions and indirect utilities in the Pareto weight ((43), (44) or (59)) for the application of Jensen's inequality can be met by contributions of other terms in these expressions.

Assumption 2 of proportional endowments arises when agents are born with shares of the endowment trees, which is plausible in our stylized three-period setting. This assumption's primary role is to make aggregate risks dominant by placing some discipline on the idiosyncratic movements of agent-specific components of endowments. If those components are not highly correlated, then either (i) the risk sharing benefits are considerable that public information would have an adverse effect on agents' welfare by means of impairing these benefits (Hirshleifer effect), or (ii) a new dimension of uncertainties (associated with agents' relative wealth) enters the equilibrium dynamics in quite an arbitrary way that no unambiguous welfare implication of public information exists. We view the assumption of proportional endowments in this light, and logically any other endowment configurations that do not render large risk sharing benefits would also fulfill the role of Assumption 2.

C Non-Time Separable Preferences

The tradeoff between the consumption smoothing across time and and the risk sharing across states is also self-evident when the elasticity of intertemporal substitution (EIS) is not confounded with the relative risk aversion (RRA) as in non-time separable preferences (Kreps and Porteus (1978)). We now briefly discuss such a tradeoff for Epstein and Zin (1989)'s specification of non-time separable preferences,

$$U_I\left(c_I\right) = \left(E_{0^-}^I\left[\left(c_{I0}^{\rho_I} + \beta_I\left(E_0^I\left[c_{I1}^{1-\gamma_I}\right]\right)^{\frac{\rho_I}{1-\gamma_I}}\right)^{\frac{1-\gamma_I}{\rho_I}}\right]\right)^{\frac{1}{1-\gamma_I}}, \quad I \in \{A, B\},$$

where agent I's EIS is $\frac{1}{1-\rho_I}$, and RRA is γ_I. When $\gamma_I = 1-\rho_I$, I's preference becomes the standard (time separable) power utility. By construction, agent $I \in \{A, B\}$ has a preference for early (late) uncertainty resolution if $\gamma_I > (<)\, 1 - \rho_I$. In the uninformed economy (Figure 1), preferences simplify to,

$$U_I\left(c_I\right) = \left(c_{I0}^{\rho_I} + \beta_I\left(E_0^I\left[\left(c_{I1}\left(s\right)\right)^{1-\gamma_I}\right]\right)^{\frac{\rho_I}{1-\gamma_I}}\right)^{\frac{1}{\rho_I}}, \quad I \in \{A, B\}.$$

Whereas, in the informed economy (Figure 2), preferences simplify to,

$$U_I(\widehat{c}_I) = \left(E_{0^-}^I \left[\left((\widehat{c}_{I0}(s))^{\rho_I} + \beta_I (\widehat{c}_{I1}(s))^{\rho_I} \right)^{\frac{1-\gamma_I}{\rho_I}} \right] \right)^{\frac{1}{1-\gamma_I}}, \quad I \in \{A, B\}.$$

In the special case of homogeneous beliefs and power utilities, the two economies possess identical equilibria. In contrast, in the general case of $\gamma_I \neq 1 - \rho_I$ for some $I \in \{A, B\}$, the two economies possess different equilibria. While this general case is complex, we opt for a numerical analysis of the two economies to illustrate the tradeoff between consumption smoothing across dates and risk sharing across states.

For simplicity, our numerical analysis features two agents $\{A, B\}$ having same RRAs, same time discount factors, same beliefs and proportional endowments (however, agents differ in their EIS). As a result of these symmetries, the uninformed economy (no information is ever released, Figure 1) and the too-early-informed economy (state at $t = 1$ is known before markets open, Figure 3) have identical equilibrium allocations. Hence, our reference to the uninformed economy can be equally taken as a reference to the too-early-informed economy in the current specific numerical analysis (this appendix). The numerical specification details are as follows. State space has two possibilities, $s \in \Omega = \{low, high\}$. Aggregate endowments are $e_0 = 1$ (at $t = 0$), and $e_1(high) = e^{\frac{0.03}{12} + 0.08\sqrt{\frac{1}{12}}}$ and $e_1(low) = e^{\frac{0.03}{12} - 0.08\sqrt{\frac{1}{12}}}$ (at $t = 1$). For simplicity, we assume agents have different (but proportional) endowments ($\frac{e_B}{e_A + e_B} = 0.8$) and different preferences for consumption smoothing (A has non-time separable utility with $EIS_A = \frac{1}{1-\rho_A} \in (0, 2]$ and $\gamma_A = 10$. B has power utility with $\frac{1}{EIS_B} = 1 - \rho_B = \gamma_B = 10$). Otherwise, agents have identical time preferences ($\beta_A = \beta_B = 0.9996$), RRAs ($\gamma_A = \gamma_B = 10$), and beliefs ($p_A(high) = p_B(high) = 0.5$).

To visualize a welfare comparison, let x denote the percentage of additional lifetime consumption agent $I \in \{A, B\}$ requires in the uninformed economy in order to be indifferent to the equilibrium consumption allocation he would receive in the informed economy (recall that above utilities are homogeneous functions of degree one),

$$U_I(\widehat{c}_I) = U_I((1 + x) c_I), \qquad \text{or equivalently} \qquad x = \frac{U_I(\widehat{c}_I)}{U_I(c_I)} - 1. \tag{61}$$

That is, agent $I \in \{A, B\}$ is indifferent between the informed economy (while consuming the stream $\widehat{c}_I$) and the uninformed economy (while consuming the stream $(1 + x) c_I$).

In Figure 6, the red line with circle markers is always above the green solid line, implying that

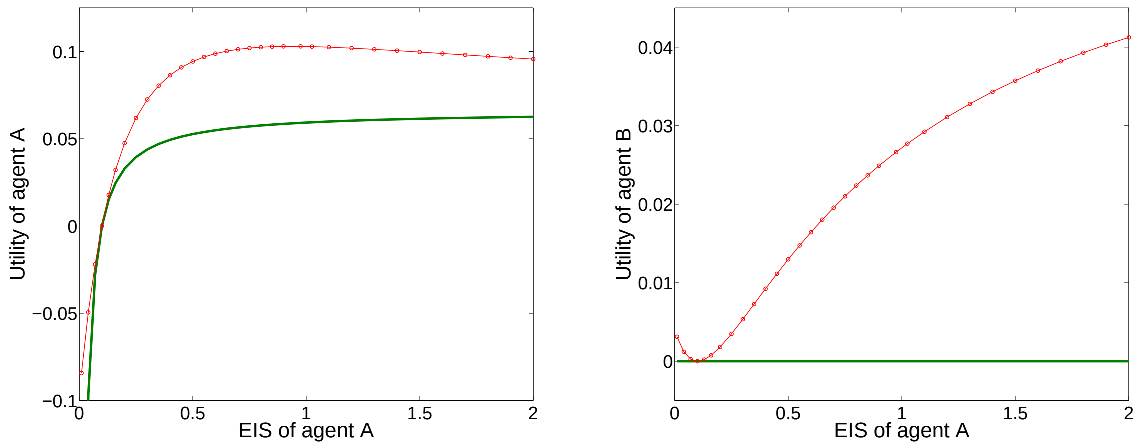

Figure 6: The horizontal axis represents agent A's EIS. The vertical axis represents values in percentage points. The red line with circle markers plots the consumption percentage (61). The green solid line plots a similar consumption percentage (but agent $I \in \{A, B\}$ are stuck with the uninformed economy's equilibrium consumption allocations c_I). The left panel is for the case of $I = A$, and right panel $I = B$.

early information releases improve risk-sharing.[50] Intuitively, in the case of $EIS_A < (>) \, EIS_B = 0.1$, A is relatively more (less) concerned about consumption smoothing over time than B. Agent B (A) will help agent A (B) to smooth A's (B's) consumption over time and requires a premium for the service. This Pareto improving trade is particularly beneficial in the informed economy (in which the uncertainty is resolved early at $t = 0$). In the benchmark uninformed economy, consumptions at $t = 1$ are state-contingent at $t = 0$ and agents benefit relatively less from consumption smoothing.

Figure 6 also illustrates that when $EIS_A < \frac{1}{\gamma_A}$, A's inherent preference for late uncertainty resolution dominates the benefits of risk-sharing (and A prefers that no information is released at time 0). In contrast, B of power utility always prefers early information releases because of the risk-sharing benefits. In frictionless markets Coase Theorem implies that a Pareto optimal allocation will be achieved, but it is unclear whether an early release of information is Pareto optimal in general. In relation to the discussion on time additive preferences and heterogeneity in beliefs in the main text, early information releases are not only Pareto improving (if $EIS_A > \frac{1}{\gamma_A}$) but also welfare improving according to the stricter criterion of Kim (1012), Brunnermeier et al. (2013) and Gilboa et al. (2013).

[50] Agent A prefers late resolution of uncertainty when $EIS_A < \frac{1}{\gamma_A} = 0.1$ and the green solid line in the left panel of Figure 6 is below zero. A prefers early uncertainty resolution when $EIS_A > \frac{1}{\gamma_A} = 0.1$ and the green solid line is above zero. At $EIS_A = \frac{1}{\gamma_A} = 0.1$ the green solid line is exactly zero because power utility implies indifference about the timing of uncertainty resolution. Agent B has power utility and does not care about the timing of uncertainty resolution (green solid line is zero for any value of EIS_A in the right panel of Figure 6).

We also observe that an early release of public information has substantial implications for asset pricing when agents have non-time separable preferences due to the associated highly non-linear pricing dynamics. (In comparison, the pricing impact of public information releases is considerably smaller for time separable preferences, and hence, is omitted in our paper). Figure 7 illustrates that for $EIS_A < (>) \, EIS_B$, the ex-ante equity premium of a dividend strip, which pays aggregate endowment at $t = 1$, is substantially higher (lower) in the informed economy than in the benchmark uninformed economy. The black solid line at 6.3% is the equity premium in the uninformed economy. The red line with circle markers is the premium in the informed economy. The

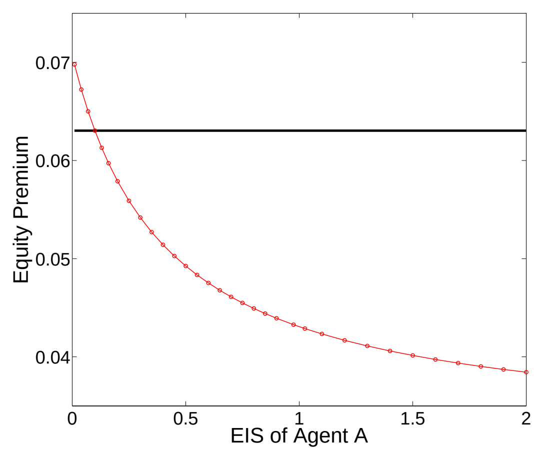

Figure 7: The horizontal axis represents agent A's EIS. The vertical axis represents the annualized equity premium of a dividend strip which pays aggregate endowment at $t = 1$. The black solid line plots the equity premium in the uninformed economy, the red line with circle markers the premium in the informed economy.

intuition underlying this pricing result is clear by examining B's pricing kernel (of power utility), $\frac{q_{1s}}{p_B(s)} = \frac{\beta_B (c_{B1}(s))^{-\gamma_B}}{E^B\left[(c_{B0}(s))^{-\gamma_B}\right]}$, which only depends on the consumption allocation c_B (but not on the timing of uncertainty resolution). As discussed above, for $EIS_A < (>) \, EIS_B$, agent B (A) helps agent A (B) to smooth the latter's consumption over time, implying that B's equilibrium consumption plan $\widehat{c}_{B1}(s)$ in the informed economy is more (less) volatile across states than his consumption plan $c_{B1}(s)$ in the uninformed economy. Accordingly, for $EIS_A < (>) \, EIS_B$, the pricing kernel $\frac{q_{1,s}}{p_B(s)}$ covaries strongly (weakly) negatively with endowment $e_1(s)$. As a result, the equity premium is higher (lower) in the informed economy than that in the uninformed economy.