Nontraded Sector Growth Risks and Economic Sizes in International Asset Pricing *

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Abstract

Output fluctuations in nontraded sectors are an important country-specific risk factor because nontraded outputs are consumed almost entirely domestically. In interest rate markets, countries of higher nontraded output growth risks are associated with stronger precautionary savings motives and lower interest rates. In currency markets, strategies of higher exposures to nontraded output growth risks offer higher average excess returns. Economic sizes are a key factor to exacerbate the pricing impact of nontraded output growth risks because it requires a larger amount of international trade to mitigate larger economies' nontraded output risks. However, as economic sizes are functions of outputs in the endowment economy setting, their pricing impact is incorporated into that of output growth risks in equilibrium of such a setting.

JEL-Classification: F3, F31, F4, G0, G12, G15.

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1 Introduction

A fundamental risk-based asset pricing implication is that economic agents impacted by a non-diversifiable consumption risk require a return premium for holding assets that are exposed to such a risk. Uncertainties in the nontraded sector's growth present a non-diversifiable consumption risk because nontraded outputs are consumed entirely by domestic agents, regardless of whether financial markets are complete or not. Therefore, nontraded output growth risks should be a prominent pricing factor in developed and developing economies as well as international markets.

Motivated by this intuition, this paper focuses on the nontraded sector's growth risks and their asset pricing impacts in a cross section of countries. The paper's main insight is that output's nontradability exacerbates the impact of growth risks on the host country of any economic size. While the (imperfect) substitution of nontraded by traded consumptions and international trade (of traded consumption goods) help mitigate nontraded output growth risks, the mitigation is hindered by countries' sizes, implying that economic sizes further exacerbate the impacts of nontraded output growth risks. Nontraded outputs are goods and services that cannot be consumed outside of the host country. They contribute significantly to all countries' GDP and aggregate consumptions and are key to important sectors, such as, real estate, constructions, and domestic business activities and services.

Nontraded output growth risks underlie all results of the paper. First, countries of higher nontraded output variances feature higher precautionary savings motives and lower interest rates. Second, currency strategies sorted on their increasing exposures to nontraded output growth risks offer increasing average excess returns out of sample. Third, in the endowment economy framework, economic sizes are endogenously determined from fundamental and exogenous endowment streams in equilibrium. As a result, at a more fundamental level, the pricing impact of economic sizes can be decomposed into those associated with current and expected future endowment growths. Data from the Organisation for Economic Co-operation and Development (OECD) economies provides empirical evidences for these results, which we elaborate next.

Considering sovereign bond markets from a risk-based perspective, higher nontraded growth risks make risk-free bonds of the associated countries more valuable as hedge assets that deliver baskets of country-specific nontraded (and common traded) consumption goods. Higher bond prices translate into lower interest rates in those countries of higher nontraded output growth risks. Equivalently, from a consumption perspective, higher uncertainties in nontraded outputs induces higher precautionary savings motives and suppress interest rates. Nontraded outputs are primarily internalized (due to their nontraded nature), in contrast to country-specific traded outputs and risks, which are largely internationalized and diversified (via international trade). Empirically, we document a significant inverse relationship between nontraded output growth risks and interest rates in the cross section of OECD economies, after controlling for other country-specific risks and economic sizes (Section 2.3.2).

Considering FX markets from an asset pricing perspective, the currency risk premia in a currency H's denomination reflect the correlations between exchange rates (against currency H) and country H's marginal utility. In equilibrium, nontraded growth risks feature predominantly in both exchange rates and consumptions, driving the above correlations and hence the currency risk premia. Whereas, traded growth risks are largely diversified from the exchange rate dynamics. A FX strategy taking a short position in currency H and a long position in currency F offers H's investors a hedge against F's nontraded output risks.[1] Therefore, for a country F whose nontraded output growth risks are lower than H's, this FX strategy is of little value (inexpensive), and hence offers higher expected returns, to H's investors. Low nontraded output growth risks also reduce precautionary savings motives, hence boost the interest rate, in country F as elaborated earlier. Altogether, these observations show that nontraded output growth risks help to rationalize the high average excess returns earned in the well-known strategy of borrowing low interest-rate and lending high interest-rate currencies. Empirically, we document a monotone (inverse) relationship between excess returns (and Sharpe ratios) and the exposure to nontraded growth risks. Quantitatively, the currency excess returns based on nontraded growth risk exposures account for 40% and 80% of the excess currency returns based on interest rates for respectively the samples of OECD and developed countries (Sections 2.3.3-2.3.5).

Considering economic sizes from a reduced-form perspective, they are characterized by Pareto weights and intrinsically related to the pricing impact of nontraded growth risks. While nontraded output growth risks are mostly internalized, they can be (imperfectly) mitigated by international trade and (imperfect) substitution of traded and nontraded consumptions. A large country H finds it more difficult to secure sufficient traded consumptions (via international trade) to substitute

[1]Indeed, after a drop in F's nontraded output, currency F appreciates in real value (F's consumption basket is more valuable due to the scarcity of F's nontraded good). To make up for this drop, F increases the traded consumption, and via international market clearing of traded good, H decreases traded consumption and H's marginal utility increases. That is, this FX strategy makes a profit (currency F appreciates) to H's investors when their marginal utilities are high, i.e., a hedge asset.

for its losses in nontraded output and consumption. Therefore, the nontraded growth risk mitigation is hindered by H's size, i.e., economic sizes exacerbate the impact of nontraded growth risks. Importantly, in an endowment economy setting, economic sizes are determined in equilibrium as functions of traded and nontraded endowment streams, which are fundamentals to the setting. In this equilibrium perspective of economic sizes, their effects on asset pricing are subsumed into those of fundamental traded and nontraded endowments. In the setting, equilibrium sizes are non-linear functions of both current endowments and expected value of future endowments. Empirically, this feature requires a parametric estimation approach to evaluate the comprehensive and non-linear relationship between output growth risks and equilibrium asset prices. The method of moment yields significant estimates for both nontraded and traded consumption tastes of the underlying utility function, and quantifies a significant substitution channel of nontraded and traded consumptions for OECD economies (Section 3).

Related Literature: Our paper belongs to a broad research topic of macroeconomic uncertainties that are priced in asset markets. Adopting the consumption-based asset pricing framework of Lucas (1978) and Breeden (1979), we focus on how nontraded endowment risks affect equilibrium consumptions, marginal utilities, and asset prices in a cross section of countries. In international finance, various structural features and risk factors have been identified, among others, fundamental consumption risks in Lustig and Verdelhan (2007), recursive preferences in Colacito and Croce (2011) and Colacito et al. (2018), habit formations in Verdelhan (2010) and Stathopoulos (2017), incomplete markets and risk sharing in Sarkissian (2003), global imbalance risks in Della Corte et al. (2016), commodity versus final good productions in Ready et al. (2017), geopolitics and trade gravities in Lustig and Richmond (2019), and trade network centrality in Richmond (2019). Nontraded output growth risks fit in this set of factors because the trade network, trade gravities, and commodity versus final good productions are endogenous to historical, spatial, and political aspects of resources and economies, the same aspects that determine the nontradability structure of economies.

The pricing effects of output nontradability and trade costs have been studied in early work by Stulz (1987), Stockman and Dellas (1989), Backus and Smith (1993), Zapatero (1995), Dumas (1992) and Sercu et al. (1995). Our empirical analysis builds on this literature to work with various data series of OECD productions, exports, imports, price indices and revisions to classify and aggregate real traded and nontraded outputs at country level.[2] Hartzmark (2016) provides an em-

[2]For robustness, we also constructed the tradability index to account for the partial tradability at at industry

pirical analysis and evidences for an inverse relationship between uncertainties in macroeconomic (consumption, GDP, and industrial production) growths and interest rates, indicating a precautionary savings motive. Our empirical analysis considers uncertainties in (traded and nontraded) output growths, and provides evidences for the pricing of these risks in both exchange rates and interest rates. No-arbitrage factor pricing and strategies of currency markets are investigated in many papers, including Brunnermeier et al. (2009) and Burnside et al. (2011) on skewness risks, Menkhoff et al. (2012) on volatility risks, Mueller et al. (2017) on the cross-sectional dispersion of exchange rate correlations, Maurer et al. (2017) on principal component (PCA) factors. Our paper examines the performance of currency portfolios based on exposures to nontraded output growth risks, and estimates significant factor prices for both nontraded and traded consumption growth factors.

Our paper is closest to and builds upon Hassan (2013), who demonstrates the role of economic sizes in pricing interest rates and currencies. We focus on the role and pricing effects of nontraded growth risks, and extend the analysis to highlight that economic sizes are equilibrium quantities in the endowment economy setting. As a result, sizes and their asset pricing impacts are accounted for and incorporated into the pricing impact of output growth risks as fundamental primitives to the setting. Endogenizing economic sizes also shows that both current and expected future outputs have important asset pricing impacts.

The pricing of nontraded growth risks is also related to other well-known conceptual and empirical analysis of the international finance literature. Adopting the U.S. investors' perspective (the USD denomination), Lustig et al. (2011) construct two prominent pricing factors in currency markets, namely the Dollar factor and the HML factor. The output growth risks can be mapped into, and hence present a specific structural interpretation for, this reduced-form factor pricing framework. Specifically, in the USD denomination, the prevalence of the U.S. nontraded output risk implies that it is common to all components of Lustig et al. (2011)'s Dollar factor and drives this factor in our setting (Section (2.3.6)). Engel (1999) documents a robust and important result that movements in the real exchange rate growths of advanced currencies are mostly related to the movements in the relative traded good prices (between two countries). This empirical pattern can be rationalized in the current setting of nontraded output growth risks. In the setting, both real exchange rates and traded good prices are equilibrium quantities. Traded output growth risks are diversified from relative traded good prices and real exchange rates, leaving nontraded output

level.

growth risks the key driving factor of these two quantities, and hence, their significant co-movements as documented in Engel (1999) (Section (2.3.6)).

The paper is organized as follows. Section 2 addresses the setting in which economic sizes are taken as given; Sections 2.1 and 2.2 present the the model's setup and implications, Section 2.3 presents the empirical analysis (Sections 2.3.1-2.3.5) and its relevance to literature (Section 2.3.6). Section 3 addresses the setting in which economic sizes are determined in equilibrium from fundamental endowment inputs: Section 3.1 presents conceptual analysis and Section 3.2 presents the empirical estimates. Online Appendices present supplementary technical derivations.

2 Nontraded Risk and Asset Pricing

This section presents an international finance model to price the output risk of countries' nontraded and traded sectors (Section 2.1), in which countries' sizes are taken effectively as input parameters in the first setting (Section 2.2) and then obtained as equilibrium outputs in the second setting (Section 3). We discuss the asset pricing implications and tests concerning interest rates and currency risk premia.

2.1 Setup

We consider a parsimonious setup of an international real endowment economy that consists of home and foreign countries, indexed respectively by H and F.[3] Each country is endowed with a stream of country-specific nontraded consumption good (consumed only domestically) indexed by N, and a stream of common traded consumption good (consumed globally) indexed by T. All consumption goods are perishable and consumed in their entirety each period. Country-specific traded and nontraded endowments (also referred to as outputs in the current setup) $\{\Delta_T^I, \Delta_N^I\}$ are assumed to follow diffusion processes

$$d\log\Delta_{Tt+dt}^I = \mu_T^I dt + \sigma_T^I dZ_{Tt+dt}^I, \quad d\log\Delta_{Nt+dt}^I = \mu_N^I dt + \sigma_N^I dZ_{Nt+dt}^I, \quad I \in \{H, F\}, \tag{1}$$

where Z_{Tt}^I and Z_{Nt}^I denote the standard (possibly multi-dimensional) Brownian shocks to I's endowments, and μ_T^I, μ_N^I and σ_T^I, σ_N^I the (possibly time-varying) means and volatilities, of I's traded

[3] Adding more countries offers qualitatively similar results, and for simplicity of exposition is abstracted from our setup.

and nontraded endowment growths. We assume that financial markets are complete. As a result, each country is represented by an agent who derives an utility each period from consuming both traded and nontraded goods,

$$U^I(C^I,t) = e^{-\rho t}\frac{(C_t^I)^{1-\gamma}}{1-\gamma} = e^{-\rho t}\frac{1}{1-\gamma}\left[\omega_T (C_{Tt}^I)^{1-\epsilon} + \omega_N (C_{Nt}^I)^{1-\epsilon}\right]^{\frac{1-\gamma}{1-\epsilon}}, \qquad I \in \{H,F\}, \tag{2}$$

where C_{Tt}^I and C_{Nt}^I denote I's per-capita traded and nontraded consumptions, C_t^I the consumption aggregator, ρ the time discount factor, ω_T and ω_N the tastes respectively for traded and non traded consumption goods (normalized by $\omega_T + \omega_N = 1$), $\frac{1}{\gamma}$ the intertemporal elasticity of consumption, and $\frac{1}{\epsilon}$ the elasticity of substitution between traded and nontraded consumption goods. We assume that the elasticity of the consumption substitution across traded and nontraded goods is larger than that of the intertemporal consumption substitution in all countries,

$$\frac{1}{\epsilon} > \frac{1}{\gamma}. \tag{3}$$

This assumption posits that countries are more concerned about the variation of consumptions across time than across traded and nontraded goods. As a result, to improve their expected utilities, countries primarily focus on smoothing their consumption aggregator C_t^I (2) intertemporally, and do so by substituting nontraded and traded consumption components $\{C_{Nt}^I, C_{Tt}^I\}$. This important assumption provides an interpretation for the impact of countries' traded and nontraded endowments on their traded consumptions in equilibrium (Equation (6) below).

While the market for the nontraded consumption good clears at the country level, the market for the traded consumption good clears at the global level,

$$C_{Nt}^I = \Delta_{Nt}^I, \quad \forall I \in \{H,F\}, \qquad \sum_{I\in\{H,F\}} C_{Tt}^I = \sum_{I\in\{H,F\}} \Delta_{Tt}^I \equiv \Delta_{Tt}, \tag{4}$$

where Δ_{Tt} is the aggregate traded endowment. For simplicity, we assume that nontraded and traded endowments are mutually uncorrelated. The aggregate traded endowment dynamics, $d\log\Delta_{Tt+dt} \equiv \mu_T dt + \sigma_T dZ_{Tt+dt}$, therefore reflect the diversification of country-specific traded endowment risks via international trade and are determined in equilibrium.

2.2 Exogenous Economic Sizes

In equilibrium, complete financial markets imply that countries are able to perfectly share traded endowment risks, resulting in equal marginal utilities of the traded consumption good across countries

$$\Lambda^H \frac{\partial U^H}{\partial C^H_{Tt}} = \Lambda^F \frac{\partial U^F}{\partial C^F_{Tt}} \equiv M_{Tt}, \tag{5}$$

where $\{\Lambda^I\}$, $I \in \{H, F\}$ are Pareto weights, and M_{Tt} the pricing kernel that prices assets in units of the traded consumption good. We first follow the customary (reduced-form) approach in the literature to employ countries' economic sizes (or GDPs) as proxies for these weights. The subsequent section explicitly relates $\{\Lambda^I\}$ to the fundamental (traded and nontraded) endowments of home and foreign economies. We log-linearize the nonlinear first-order conditions (FOCs) (5) to obtain an approximate equilibrium solution for an intuitive analysis.

Equilibrium traded consumptions: Let the lower-case letters denote the respective log quantities, $c \equiv \log C$, $\delta_T \equiv \log \Delta_T$, $\delta_N \equiv \log \Delta_N$. The equilibrium log (per-capita) traded consumption of the home country is (Online Appendix A.1),[4]

$$c^H_{Tt} = \delta_{Tt} + \alpha \left\{ -\rho t - (\gamma - \epsilon)\omega_N \left(\left[1 - \frac{\Lambda^H}{\Lambda} \right] \delta^H_{Nt} - \frac{\Lambda^F}{\Lambda} \delta^F_{Nt} \right) \right\}, \tag{6}$$

where $\alpha \equiv (\gamma\omega_T + \epsilon\omega_N)^{-1}$ is a weighted elasticity of consumption substitution (recall that $\omega_T + \omega_N = 1$), and $\Lambda \equiv \sum_{I \in \{H,F\}} \Lambda^I$ is a measure of the global GDP. Hence $\frac{\Lambda^I}{\Lambda}$ characterizes country I's relative size. A country H's traded consumption c^H_{Tt} (6) increases when (i) aggregate traded endowment δ_{Tt} surges, (ii) H's nontraded endowment δ^H_{Nt} drops (H's traded consumption demand increases as a substitution), and (iii) F's nontraded endowment δ^F_{Nt} surges (F's traded consumption demand decreases). Aggregate traded endowment δ_{Tt} affects countries' log traded consumption uniformly (effect (i) above) due to frictionless trading and perfect risk sharing in traded endowment outputs. Whereas, nontraded endowments' impacts on c^H_{Tt} are enforced by countries' sizes (effect (ii) subdued by the home size in $1 - \frac{\Lambda^H}{\Lambda}$, and effect (iii) amplified by the foreign size in $\frac{\Lambda^F}{\Lambda}$) because economic sizes influence countries' traded consumption demands and supplies in equilibrium (5). Equation (6) (and the definition of total size $\Lambda \equiv \sum_{I \in \{H,F\}} \Lambda^I$) further shows how traded and nontraded

[4] The foreign log traded consumption c^F_{Tt} is obtained from (6) by interchanging H and F indices.

output growth risks affect traded consumption variance in equilibrium,

$$Var_t\left[c^H_{Tt+dt}\right] = \sigma_T^2 + \left(\alpha(\gamma-\epsilon)\omega_N\frac{\Lambda^F}{\Lambda}\right)^2\left([\sigma_N^H]^2+[\sigma_N^F]^2\right). \tag{7}$$

First, a larger foreign country amplifies the impact from the foreign nontraded growth shock on the home traded consumption adjustment. Second, via the size normalization $\Lambda \equiv \sum_{I\in\{H,F\}}\Lambda^I$, a larger foreign country also implies that the home country is relatively smaller and more flexible to adjust the home traded consumption. Both effects indicate a more volatile home traded consumption adjustment (to either home or foreign nontraded shocks) when the foreign country is larger. Section 2.3.5 below examines this testable relationship in the data.

Stochastic discount factors (SDFs): In consumption-based settings, the country-specific consumption basket is the lowest-cost consumption basket delivering one unit of the country's utility, and the country-specific SDF is the marginal utility of consuming the country-specific basket, $M_t^I = \frac{\partial U^I}{\partial C^I}$. The country-specific log SDF, $m_t^I \equiv \log M_t^I$, is (Online Appendix A.1)

$$m_t^I = \underbrace{-\rho t - \gamma\omega_T\delta_{Tt} - \alpha\gamma(\gamma-\epsilon)\omega_N\omega_T\sum_{I\in\{H,F\}}\frac{\Lambda^I}{\Lambda}\delta^I_{Nt}}_{\text{common component}} \quad + \quad \underbrace{-\alpha\gamma\epsilon\omega_N\delta^I_{Nt}}_{\text{country-specific component}}. \tag{8}$$

Note that M_t^I prices assets from the country I's perspective and in the numeraire of I's consumption basket. Following the decomposition of the country-specific log SDF (8) into a common and a country-specific component, several observations are in order. First, the country-specific component clearly shows that the variation across countries' SDFs, which is crucial to international asset pricing, is driven by the variation in countries' nontraded endowments $\{\delta^I_{Nt}\}$ emphasized by the current paper. Second, the common component shows that the priced nontraded output risks enter countries' SDFs in the form of a size-weighted average $\sum_{I\in\{H,F\}}\frac{\Lambda^I}{\Lambda}\delta^I_{Nt}$. Larger economies contribute more to, and also face a stronger effect from, this internationally non-diversifiable nontraded risk component even in the presence of complete financial markets as emphasized by Hassan (2013). The impact of nontraded endowment risks in international settings can also be delineated by examining the SDF growth volatility differential

$$Var_t\left(dm^F_{t+dt}\right) - Var_t\left(dm^H_{t+dt}\right) = \alpha^2\gamma^2\epsilon\omega_N^2\left\{\left[\epsilon + 2(\gamma-\epsilon)\omega_T\frac{\Lambda^F}{\Lambda}\right]\left(\sigma_N^F\right)^2 - \left[\epsilon + 2(\gamma-\epsilon)\omega_T\frac{\Lambda^H}{\Lambda}\right]\left(\sigma_N^H\right)^2\right\}, \tag{9}$$

which remains non-vanishing even when countries have the identical size but different nontraded output volatilities.

Interest rate differential: Country I's short-term interest rate is determined by I's expected SDF growth rate, $r_t^I = -\frac{1}{dt}E_t\left[\frac{dM_{t+dt}^I}{M_t^I}\right]$. The interest rate differential then follows from SDF dynamics (8) and (9),

$$r_t^F - r_t^H = \alpha\gamma\epsilon\omega_N\left(\mu_N^F - \mu_N^H\right) - \frac{1}{2}\alpha^2\gamma^2\epsilon^2\omega_N^2\left((\sigma_{Nt}^F)^2 - (\sigma_{Nt}^H)^2\right) \tag{10}$$
$$-\alpha^2\gamma^2\epsilon(\gamma-\epsilon)\omega_T\omega_N^2\left(\frac{\Lambda^F}{\Lambda}(\sigma_{Nt}^F)^2 - \frac{\Lambda^H}{\Lambda}(\sigma_{Nt}^H)^2\right).$$

When an economy I either is larger or has more volatile nontraded output, it is more difficult for the economy to mitigate the nontraded output growth risk via international trade. The economy's risk-free bond, paying off a basket of nontraded and traded consumption goods, then is highly valuable and associated with a lower interest rate r_t^I. The empirical analysis of Section 2.3.2 examines and presents supporting evidence for this interest rate pattern in the data.

Exchange rate dynamics: In complete markets, the exchange rate S_t, or the number of foreign baskets that buys a home basket at t, equals the SDF ratio, $S_t = \frac{M_t^H}{M_t^F}$. From this equation and SDFs (8) follows the exchange rate growth, defined as the change in the foreign currency value,[5]

$$\frac{S_t}{S_{t+dt}} - 1 = \left[-\gamma\alpha\epsilon\omega_N\left(\mu_N^F - \mu_N^H\right) + \frac{1}{2}\gamma^2\alpha^2\epsilon^2\omega_N^2\left([\sigma_N^H]^2 + [\sigma_N^F]^2\right)\right]dt - \gamma\alpha\epsilon\omega_N\left(\sigma_N^F dZ_{Nt+dt}^F - \sigma_N^H dZ_{Nt+dt}^H\right),$$

$$\text{and,}\qquad \frac{1}{dt}E_t\left[\frac{S_t}{S_{t+dt}} - 1\right] = -\gamma\alpha\epsilon\omega_N\left(\mu_N^F - \mu_N^H\right) + \frac{1}{2}\gamma^2\alpha^2\epsilon^2\omega_N^2\left([\sigma_N^H]^2 + [\sigma_N^F]^2\right). \tag{11}$$

Because the common components of SDF growths (8) cancel out, uncertainties in nontraded consumptions are the only risks in the exchange rate growth, indicating the importance of nontraded output risks in FX markets.

Currency returns and decomposition: Let us consider a net-zero strategy $CR^{-H,+F}$ that borrows home and lends foreign currencies from t to $t+dt$. Home investors earn an (excess) return $CR_{t+dt}^{-H,+F}$ on this strategy thanks to the difference in countries' interest rates (10) and movements

[5]Recall that exchange rate S_t denotes the foreign currency units per home currency unit at t. For convenience, we consider the change in exchange rate (11) because it is the exchange rate growth component in the currency return (12) below.

in the exchange rate (11),

$$CR_{t+dt}^{-H,+F} = \underbrace{(r^F - r^H)dt}_{\text{interest rate differential contribution}} + \underbrace{\left[\frac{S_t}{S_{t+dt}} - 1\right]}_{\text{exchange rate growth contribution}} . \tag{12}$$

Via the exchange rate growth component, home and foreign nontraded growth risks contribute symmetrically to the currency return as captured by the sum $\left([\sigma_{Nt}^H]^2 + [\sigma_{Nt}^F]^2\right)$ in (11). Via the interest rate differential component, these risks enter the currency return in an anti-symmetric pattern as captured by the differences $\left([\sigma_{Nt}^F]^2 - [\sigma_{Nt}^H]^2\right)$ and $\left(\frac{\Lambda^F}{\Lambda}[\sigma_{Nt}^F]^2 - \frac{\Lambda^H}{\Lambda}[\sigma_{Nt}^H]^2\right)$ in (10). This decomposition specifies how different (total and differential) combinations of country-specific nontraded growth risks affect different components of currency returns. The currency risk premium to home investors, which is

$$\frac{1}{dt}E_t\left[CR_{t+dt}^{-H,+F}\right] = -\alpha^2\gamma^2\epsilon\omega_N^2\left\{(\gamma-\epsilon)\omega_T\left[\frac{\Lambda^F}{\Lambda}(\sigma_N^F)^2 - \frac{\Lambda^H}{\Lambda}(\sigma_N^H)^2\right] - \epsilon(\sigma_N^H)^2\right\}, \tag{13}$$

reflects a nontraded risk rationale for the popular carry trade strategy of borrowing low and lending high interest rate currencies. When the home country has volatile nontraded sector (σ_N^H large), home risk-free bonds are valuable as a safe asset for the precautionary saving purpose, and the home interest rates is low (r^H small). At the same time, the strategy of borrowing home and lending foreign currencies $CR^{-H,+F}$ is risky under significant home nontraded risk, hence offering a high expected return. By the same argument, when the foreign nontraded sector is stable (σ_N^F small), the foreign interest rate is high (r^F large), the hedging value of $CR^{-H,+F}$ against the foreign nontraded is lowered, hence also hence offering a high expected return to home investors. Section 2.3.3 below presents an empirical analysis of these implications concerning currency returns.

Linear Factor Analysis: In the current setting, the home consumption aggregator C_t^H (2) and the associated SDF M_t^H (8) are functions of the home traded and nontraded consumptions. The SDF growth therefore is linear in two factors of the traded and nontraded consumption growths

$$dm_{t+dt}^H = \log\frac{M_{t+dt}^H}{M_t^H} = \#dt + b_T f_{Tt+dt}^H + b_N f_{Nt+dt}^H, \tag{14}$$

with $b_T = -\gamma\omega_T$, $b_N = -\gamma\omega_N$, $f_T^H = \frac{dC^H_{Tt+dt}}{C^H_{Tt}}$, $f_N^H = \frac{dC^H_{Nt+dt}}{C^H_{Nt}}$, and the associated factor prices,[6]

$$\lambda_T^H = \gamma\omega_T(\sigma_T)^2 + \alpha^2\gamma(\gamma-\epsilon)^2\omega_T\omega_N^2\frac{(\Lambda^F)^2}{(\Lambda)^2}(\sigma_N^F)^2 - \alpha^2\gamma(\gamma-\epsilon)\omega_N^2\frac{\Lambda^F}{\Lambda}\left[\epsilon + (\gamma-\epsilon)\omega_T\frac{\Lambda^H}{\Lambda}\right](\sigma_N^H)^2,$$

$$\lambda_N^H = \alpha\gamma\omega_N\left[\epsilon + (\gamma-\epsilon)\omega_T\frac{\Lambda^H}{\Lambda}\right](\sigma_N^H)^2 > 0. \tag{15}$$

The home nontraded factor price λ_N^H involves only the H's nontraded output volatility σ_N^H because the nontraded consumption equals nontraded output. This nontraded factor price is unambiguously positive, $\lambda_N^H > 0$, indicating that the uncertainty in the home nontraded output growth is a risk to home investors. In contrast, λ_T^H and its (risk or hedge) characteristic depend on the magnitude of sizes, aggregate traded and country-specific nontraded output volatilities (via international trade in equilibrium). The estimate of λ_T^H is a subject of the empirical determination of Section 2.3.4, which finds a statistically significantly positive λ_T^H using data of currency returns denominated in USD. This estimate indicates that fluctuations in the traded consumption are an important priced risk to the U.S. investors.

2.3 Empirical Analysis

This section presents empirical analysis concerning nontraded growth risks, in line with their conceptual role in international markets discussed above. Section 2.3.1 discusses data aspects and sources, Section 2.3.2 addresses the interest rates, Section 2.3.3 the currency returns, Section 2.3.4 the linear factor prices, Section 2.3.5 the equilibrium traded consumptions. Section 2.3.6 reconciles nontraded growth risks with prominent analysis of the exchange rate literature.

2.3.1 Data

Our quarterly data covers all 36 OECD countries, from 1976 Quarter 1 to 2016 Quarter 4. Individual countries data are dropped from the sample when they join the Economic and Monetary Union (a.k.a the Euro Zone or Euro Area), and replaced by the single Euro series. The OECD countries are chosen because they have reasonably open financial markets and good quality data.

We collect macroeconomics data from the OECD databases, except for the U.S, where we

[6] Factor price λ of a factor f is the risk premium of the asset that perfectly mimics (having unit loading on) the factor, $\lambda = -\frac{1}{dt}Cov_t(M_{t+dt}, f_{t+dt})$. Factor prices (15) follows from this and the definition of factor, equilibrium consumption (6) and SDF (14).

use data from the U.S. Bureau of Economic Analysis (BEA). For interest rate, we use the 3-month interbank data from the OECD Main Economic Indicators (MEI) database. For output and consumption data, we use GDP and (traded and nontraded) components data from the OECD Quarterly National Account (QNA).[7] Population data is also from OECD QNA, and supplemented by World Bank data for some early years when OECD data is not available yet. CPI data is from OECD Key Short-Term Economic Indicators (STEI).

We collect spot exchange rates and 3-month forward rates data from Datastream (now Refinitive Eikon), with quotes from Barclays Bank International and Reuters. Quotes of exchange rates against the USD are available from Quarter 4, 1983. We are able to extend our sample back to Quarter 1, 1976 for the following subset of 14 countries with exchange rates quoted against the GBP (Great British Pound): Austria, Canada, France, Germany, Ireland, Italy, Japan, Netherlands, Norway, Portugal, Spain, Sweden, Switzerland, USA. We convert all data to exchange rates quoted against the USD.

2.3.2 Interest rates

Our empirical analysis starts with examining the key model-implied relationships between interest rate and nontraded output growth characteristics. The interest rate differential equation (10) indicates that, countries associated with higher mean nontraded growths, lower nontraded variances, or lower products of sizes and nontraded variances, tend to have higher interest rates. Taking the U.S. quantities (denoted by H) as the baseline, our empirical test follows Equation (10)'s structure by regressing real interest rate differentials $(r^F - r^H)$ on mean nontraded growth differentials $(\mu^F - \mu^H)$, nontraded variance differentials $([\sigma^F]^2 - [\sigma^H]^2)$, and size and nontraded variance product differentials $\left(\frac{\Lambda^F}{\Lambda}[\sigma^F]^2 - \frac{\Lambda^H}{\Lambda}[\sigma^H]^2\right)$. Table 1 reports the estimation results for this panel regression employing time-series data for the U.S. and OECD countries F, including country and time fixed effects. Values of variables in Table 1 are in real terms, per quarter and per-capita. The time series of means or variances of these variables are constructed at quarterly frequency using rolling windows of past data. Different specifications (columns of Table 1) of this regression control for factors related to inflation, country-specific GDP (e.g, size) characteristics and exchange rate that may have confounding effects (not captured by the model of nontraded growth risks being tested) on the real interest rates.

[7]Nontraded are defined as the service and constructions sectors, following the standard classification in the literature (see, e.g., Stockman and Tesar (1995)).

Table 1: **Nontraded Risks and Interest Rates**

	(1)	(2)	(3)
dontg $= \left(\mu_{Nt}^F - \mu_{Nt}^H\right)$	0.027	0.143***	0.138***
	(0.032)	(0.033)	(0.034)
dntv $= \left([\sigma_{Nt}^F]^2 - [\sigma_{Nt}^H]^2\right)$	-0.023**	-0.061***	-0.059***
	(0.011)	(0.013)	(0.013)
dntvsize $= \left(\frac{\Lambda_t^F}{\Lambda_t}(\sigma_{Nt}^F)^2 - \frac{\Lambda_t^H}{\Lambda_t}(\sigma_{Nt}^H)^2\right)$	-0.131***	-0.369***	-0.375***
	(0.047)	(0.049)	(0.049)
GDP growth		-0.383***	-0.360***
		(0.064)	(0.074)
Variance of GDP growth		-2.185**	-1.917*
		(0.998)	(1.028)
Variance of inflation		20.154***	19.938***
		(3.744)	(3.717)
Variance of EX growth		0.320***	0.316***
		(0.062)	(0.061)
GDP (USD millions)			0.117
			(0.135)
Constant	0.003**	0.005***	0.004***
	(0.001)	(0.001)	(0.002)
Observations	1,029	978	978
Adjusted R-squared	0.801	0.869	0.869

Notes: Regression: Dependent variable is the difference between real 3-month interest rates of OECD countries and the U.S. Independent variables: "dontg" is the difference between the means of nontraded output growths; "dntv" is the difference between the variances of nontraded output growths; "dntvsize" is the difference between nontraded output variances scaled by sizes, of OECD countries and the U.S. Control variables include each (foreign) country's GDP growth, variance of GDP growth, variance of inflation, variance of exchange rate growth, and GDP measured in USD millions. Both country and time fixed effects are included. Robust standard errors, clustered by country and time, are in parentheses. Significance at the 1%, 5% or 10% level are indicated by ***,** or *. The panel consists of quarterly data for OECD countries from 1976-2016. All variables are measured per quarter. Countries that joined the European Monetary Union are dropped from the sample from the time they joined and are replaced by a single observation for the Euro Zone.

The main findings in Table 1 are as follows. Across all specifications, slope coefficients associated with (i) nontraded variance differentials, and (ii) size and nontraded variance product differentials, are statistically significant at 1% and 5% levels and negative as implied by the model. Column (1)'s estimates pertain exactly to the model-implied specification (10). Relative to the U.S. quantities as the baseline and all else being equal, these (per-quarter) estimates show that a country F with a higher nontraded variance by 1% has a lower interest rate by 2.3 basis points (bps) per quarter. In the same specification, all else being equal, a country F with higher size and nontraded variance product by 1% has a lower interest rate by 13 bps per quarter. These results are consistent with the implications that nontraded growth risks affect interest rate either by themselves or by coupling with economic sizes. Controlling further for GDP growth (i.e., intertemporal consumption smoothing effect), and other country-specific risks such as movements in GDP growth, in inflation and in exchange rate rate growth (Column (2)), does not reduce the statistical significance or change signs of these estimates. Economic sizes as a control variable (Column (3)) do not offer additional explanatory power. These findings indicate that the model-implied relationship (10) between nontraded growth risks and interest rates is robust in the data.

2.3.3 Currency returns

We now examine the model-implied effects of nontraded growth risks on FX markets based on analysis of returns on individual currencies (Tables 2 and 3) and currency portfolios (Tables 4 and 5).

Individual currencies: The model of nontraded growth risk pricing predicts a negative impact of the size and nontraded variance product differentials $\left(\frac{\Lambda_t^F}{\Lambda_t}(\sigma_{Nt}^F)^2 - \frac{\Lambda_t^H}{\Lambda_t}(\sigma_{Nt}^H)^2\right)$ on expected currency returns (13). This is because the underlying strategy of borrowing home and lending foreign currencies is a hedge (a risk) asset against the foreign (home) nontraded risk, offering a low (a high) expected excess return when the foreign (home) nontraded risk is high. Our empirical analysis in this section is relative to the home (U.S.) perspective. Hence, the term $(\sigma_{Nt}^H)^2$ on the right-hand side of (13) is common to all foreign currency risk premia (in the denomination of USD) $E_t\left[CR_{t+dt}^{-H,+F}\right]$, $\forall F$, and does not represent an independent variable in the regression-based test below.

Table 2 reports the regression results for the mean excess return (to U.S. investors) on 3-month currency forward contracts as the dependent variable and includes time and country fixed effects. As we fix USD as the denomination (home) currency, this excess return is determined as the log difference between 3-month forward and spot exchange rates of foreign (OECD) currencies against USD. The regressions employ the same set of control variables as in Table 1 for interest rate differentials. Column (1) considers the size and nontraded variance product differentials as an independent variable. For completeness, Columns (2)-(4) explore the interaction of these product differentials with various country-specific (GDP, inflation, currency) risks as alternative independent variables. Column (1) of Table 2 shows that all else being equal, a country F with a higher size and nontraded variance product by 1% is associated with a lower currency expected returns by 36.5 bps per quarter (1.46% per annum). The interactions with country-specific risks also yield negative and statistically significant estimates (Columns (2) - (4)), indicating that nontraded growth shocks of countries associated with higher country-specific risks tend to have a stronger negative impact on currency returns. For example, note that the historical average GDP growth variance in our sample is 1.5 bps per quarter. At this average level, a 1% increase in the size and nontraded variance product lowers the expected currency returns by $4200 \times 0.015\% \times 1\% = 63$ bps per quarter. However, if the GDP growth variance increases by a standard deviation to 3.35 bps per quarter, the impact of the size and nontraded variance product on currency returns increases more than two folds.[8] With the exception of the Variance of GDP growth and Variance of inflation, control variables are statistically relevant. In their presence, the key estimates remain significant and do not change signs.

Currency excess returns reflect interest rate differential and exchange rate growth components (12). Table 3 illustrates how nontraded growth risks impact the interest rate differential (Column (dr)), the exchange rate growth (Column (iexg)), and the currency returns (Column (cr)) by regressing these variables on nontraded risk characteristics suggested by the model (Equations (10)-(13)). The regressions include both country and time fixed effects and the same set of control variables as in Table 1. The mean nontraded growth differentials $\left(\mu_{Nt}^{F} - \mu_{Nt}^{H}\right)$ affects the interest rate differential and exchange rate growth components in opposite directions, by around 13 bps per quarter for each 1% change in $\left(\mu_{Nt}^{F} - \mu_{Nt}^{H}\right)$. The size and nontraded variance product does not enter the exchange rate growth component (11). An 1% change of this product passes straight from the interest rate differential component to the currency returns, and is associated with a drop of about 37 bps per quarter in currency returns. Table 4 below details the return breakdown of currency risk premium into the interest rate differential and exchange rate growth components.

[8] At this level of the GDP growth variance, for each 1% increase in the size and nontraded variance product, the expected currency return decreases by $4200 \times 0.0335\% \times 1\% = 141$ bps per quarter.

Table 2: **Nontraded Risks and Currency Returns**

	(1)	(2)	(3)	(4)
dntvsize= $\left(\frac{\Lambda_t^F}{\Lambda_t}(\sigma_{Nt}^F)^2 - \frac{\Lambda_t^H}{\Lambda_t}(\sigma_{Nt}^H)^2\right)$	-0.365***			
	(0.101)			
dntvsize × Variance of GDP growth		-4,200.5*		
		(2,428.7)		
dntvsize × Variance of inflation			-1,273.5**	
			(631.6)	
dntvsize × Variance of EX growth				-120.51***
				(14.66)
Variance of GDP growth	-6.131	-9.935	-11.488	-7.990
	(6.517)	(7.013)	(7.292)	(6.598)
Variance of inflation	-15.265***	-13.500***	-12.767***	-14.787***
	(1.374)	(1.896)	(2.105)	(1.385)
Variance of EX growth	0.276	0.148	0.127	0.347
	(0.261)	(0.250)	(0.249)	(0.259)
GDP growth	1.790***	1.735***	1.761***	1.802***
	(0.144)	(0.147)	(0.149)	(0.145)
GDP (USD millions)	2.408***	2.053***	2.126***	2.518***
	(0.308)	(0.311)	(0.311)	(0.315)
Constant	-0.039***	-0.037***	-0.038***	-0.040***
	(0.005)	(0.005)	(0.005)	(0.005)
Observations	982	982	982	982
R-squared	0.865	0.863	0.864	0.867

Notes: Regression: Dependent variable is the mean excess return (denominated in USD) on 3-month currency forward contracts for the set of OECD currencies. Independent variable for the first column is "dntvsize", which is the difference in nontraded output variance scaled by size. Independent variables for the second to fourth columns are the product between "dntvsize" and the variance of GDP growth, of inflation, and of exchange rate respectively. Control variables (for all regressions) include the variance of the nontraded output growth of the US (home country), and each (foreign) country's GDP growth, variance of GDP growth, variance of inflation, variance of exchange rate growth, and GDP measured in USD millions. Both country and time fixed effects are included. Robust standard errors, clustered by country and time, are in parentheses. Significance at the 1%, 5% or 10% level are indicated by ***,** or *. The panel consists of quarterly data for OECD countries from 1976-2016. All variables are measured per quarter. Countries that joined the European Monetary Union are dropped from the sample from the time they joined and are replaced by a single observation for the Euro Zone.

Table 3: **Interest Rate Differentials, Exchange Rates, Currency Returns**

	(dr)	(iexg)	(cr)
dontg $= \left(\mu_{Nt}^{F} - \mu_{Nt}^{H}\right)$	0.138***	-0.120**	
	(0.034)	(0.055)	
dntv $= \left([\sigma_{Nt}^{F}]^2 - [\sigma_{Nt}^{H}]^2\right)$	-0.059***		
	(0.013)		
sntv $= \left([\sigma_{Nt}^{F}]^2 + [\sigma_{Nt}^{H}]^2\right)$		0.049**	
		(0.023)	
dntvsize $= \left(\frac{\Lambda_t^F}{\Lambda_t}(\sigma_{Nt}^{F})^2 - \frac{\Lambda_t^H}{\Lambda_t}(\sigma_{Nt}^{H})^2\right)$	-0.375***		-0.365***
	(0.049)		(0.101)
vontgUS $= [\sigma_{Nt}^{H}]^2$			62.455
			(198.578)
Control variables	Yes	Yes	Yes
Observations	982	1,170	978
Adjusted R-squared	0.8402	0.8578	0.8443

Notes: Regressions: dependent variable for the first column is the difference in real 3-month interest rates between OECD countries and the US, for the second column is the expected exchange rate growth (i.e., $S_t/S_{t+dt} - 1$, with dt denoting a 3-month period), and for the last column is the expected real currency excess return on a 3-month forward contract. Control variables include each country's GDP growth, variance of GDP growth, variance of inflation, variance of exchange rate growth, and GDP measured in USD millions. Both country and time fixed effects are included. Robust standard errors, clustered by country and time, are in parentheses. Significance at the 1%, 5% or 10% level are indicated by ***,** or *. The panel consists of quarterly data for OECD countries from 1976-2016. Countries that joined the European Monetary Union are dropped from the sample from the time they joined and are replaced by a single observation for the Euro Zone.

Currency portfolios: The decomposition analysis above concerns individual currency returns. We next examine how nontraded growth risks are priced in currency portfolios, which help to diversify idiosyncratic risks and their effects on portfolio returns. Fixing the home currency (USD) as the funding currency, the currency risk premium (13) indicates that, lending currency F with a higher size and nontraded variance product is a good hedge asset for home investors, therefore is expensive and offers lower risk premium in the home currency denomination. Guided by this model-implied currency risk premium, Table 4 sorts currencies F into quartiles (portfolios) based on the size and nontraded variance products of associated countries (Panel A), and also on the nontraded growth variance alone (Panel B) and sizes alone (Panel C) for comparison. These variables are constructed in real terms and per-capita at quarterly frequency using past data. Accordingly, the sorting of currencies (and portfolio rebalancing) are implemented on rolling quarterly windows. For each sorting approach, we construct four net-zero equally weighted currency portfolios associated with four quartiles of sorted currencies using currency forwards (against USD).

Table 4 reports the mean excess returns, standard deviations of returns, Sharpe ratios, and the breakdown of mean currency excess returns into interest rate differential and exchange rate growth components for each currency portfolio to U.S. investors. The sorting based on the size and nontraded variance products (Panel A) creates currency portfolios of monotonely decreasing (annual) mean excess returns (ranging from 1.52% for the bottom-quartile to 0.57% for the top-quartile portfolios) and (annual) Sharpe ratios (ranging from 19.87% to 7.65%). The sorting based on either nontraded variances alone (Panel B) or sizes alone (Panel C) does not produce monotone mean excess returns or Sharpe ratios. These results are consistent with the model-implied currency risk premium (13), which depends only on (and decreases with) the size and nontraded variance product of the foreign country $\frac{\Lambda^F}{\Lambda}[\sigma_N^F]^2$ (fixing the home country H to be the U.S.). For all strategies and portfolios in Table 4, the interest rate differential component contributes more to the currency risk premia than the exchange rate growth component in the decomposition (12).

We can further examine the profitability of currency strategies based on the size and nontraded variance product of countries. Table 5 reports the performance of net-zero strategies of taking a long position in the bottom-quartile and a short position in the top-quartile currency portfolios sorted on (i) size and nontraded variance products, and (ii) interest rates (i.e., the usual currency carry strategies). For the set of OECD currencies, the long-short strategy based on size and nontraded variance products earns a mean excess return of 0.94% and a Sharpe ratio of 16.4%. For the set of developed currencies, the corresponding values are 1.45% and a Sharpe ratio of 21.7%.

Table 4: **Currency Portfolios Sorted on Different Characteristics**

Panel A. Sorted by Nontraded Variance x Size				
	Portfolio 1	Portfolio 2	Portfolio 3	Portfolio 4
Mean return (%)	1.52	1.35	0.94	0.57
Std. dev. (%)	7.63	8.41	8.16	7.47
Sharpe ratio (%)	19.87	16.03	11.52	7.65
Int. rate diff. component (%)	1.31	0.62	1.13	0.88
EXrate growth component (%)	0.21	0.73	-0.19	-0.32
Panel B. Sorted by Nontraded Variance				
	Portfolio 1	Portfolio 2	Portfolio 3	Portfolio 4
Mean return (%)	1.35	0.36	0.78	1.16
Std. dev. (%)	8.33	7.05	8.03	8.31
Sharpe ratio (%)	16.24	5.05	9.74	13.96
Int. rate diff. component (%)	0.79	0.63	0.66	1.46
EXrate growth component (%)	0.56	-0.28	0.11	-0.31
Panel C. Sorted by Size				
	Portfolio 1	Portfolio 2	Portfolio 3	Portfolio 4
Mean return (%)	2.11	0.84	1.24	0.19
Std. dev. (%)	7.72	8.6	8.51	7.09
Sharpe ratio (%)	27.34	9.77	14.52	2.70
Int. rate diff. component (%)	1.70	0.88	1.15	0.42
EXrate growth component (%)	0.41	-0.05	0.08	-0.23

Notes: This table presents (annualized) means, standard deviations and Sharpe ratios of the excess returns on four sorted quarterly rebalanced currency portfolios. The currency portfolio return is decomposed into two components: the interest rate differential (against the U.S. interest rate), and the exchange rate growth (against USD). The sample consists of quarterly data series for period 1976-2016. The portfolio are constructed by sorting currencies into four groups at beginning of quarter t based on the value of (1) nontraded real output growth variance $\times$ real GDP's relative size, (2) nontraded real output growth variance, and (3) real GDP's relative size, over the previous 15 quarters. Portfolio 1 contains currencies with the lowest sorted value, portfolio 4 the highest. Due to unbalances in spot and forward exchange rate series, countries' data become available at different times, and number of countries changes over time.

Table 5: **Long-Short Currency Portfolio Returns**

	All OECD countries		Developed countries	
	IntRate	ntvsize	IntRate	ntvsize
LS return (%)	2.296	0.944	1.807	1.450
LS stdev (%)	7.924	5.763	8.841	6.678
LS Sharpe ratio (%)	28.97	16.39	20.44	21.72

Notes: This table presents the characteristics of Long-Short currency portfolios. The sample consists of quarterly data series for period 1976-2016. The portfolio are constructed by: first, sorting currencies into four groups at beginning of each quarter t based on interest rate (IntRate) or real output growth variance multiplied by real GDP's size (ntvsize), forming an equally weighted portfolio from each group, and second, build a Long-Short portfolio by long (short) the portfolio with highest (lowest) IntRate, or the portfolio with the lowest (highest) ntvsize.

For the set of developed currencies,[9] Table 5 shows that strategies based on size and nontraded variance products offer returns and Sharpe ratios comparable to those of the popular currency carry strategies. Whereas, for the set of OECD currencies, the former strategies (based on nontraded growth risks) deliver only a half of returns and Sharpe ratios of the latter strategies (carry trades based on interest rates). To the extent that the performance of currency carry trades constitutes the forward premium puzzle (e.g., Fama (1984)), the pricing of uncertainties in nontraded growths presents a risk-based approach to rationalize the association of a high interest rate r^F with a high risk premium $E_t\left[CR_{t+dt}^{-H,+F}\right]$ (per the discussion below (13)), hence qualitatively mitigating this puzzle.

It is also instructive to examine the composition of the four currencies portfolios constructed by sorting currencies on the size and nontraded variance product of countries (Table 4). Table 6 reports the frequency at which a currency I is sorted into a portfolio P. This frequency is defined as the ratio of the number of quarters during which currency I is in P divided by the total number of quarters of the data sample (i.e., the lifespan of portfolio P). This frequency measure characterizes the time-aggregated prevalence of individual currencies in each of the four currency portfolios. As portfolio 1 contains currencies associated with highest, portfolio 4 with lowest, size and nontraded variance products, currency premia (13) suggests to lend portfolio 1's currencies (investment currencies) and to borrow portfolio 4's currencies (funding currencies) for a model-implied currency strategy of a high expected excess return (bearing and compensating

[9]From the U.S. (home) perspective, developed (foreign) countries are listed as in Lustig et al. (2011), namely, Australia, Belgium, Canada, Denmark, Euro, France, Germany, Italy, Japan, Netherlands, New Zealand, Norway, Sweden, Switzerland, and United Kingdom.

Table 6: **(NT Variance × Size)-Sorted Portfolio Composition**

P_1 (top)		P_2		P_3		P_4 (bottom)	
Country	Freq.	Country	Freq.	Country	Freq.	Country	Freq.
NZL	1.00	CHE	0.44	NOR	0.48	GBR	0.53
HUN	0.47	DNK	0.36	KOR	0.40	JPN	0.50
CZE	0.30	SWE	0.32	AUS	0.27	CHL	0.40
ISR	0.30	AUS	0.29	SWE	0.26	EUR	0.40
AUS	0.29	NOR	0.26	DNK	0.21	CHE	0.36
EST	0.25	LVA	0.22	GBR	0.19	MEX	0.35
LVA	0.14	CZE	0.21	CAN	0.18	TUR	0.35
NOR	0.14	JPN	0.18	SVK	0.17	POL	0.25
SVN	0.11	FRA	0.17	MEX	0.15	DEU	0.24
SWE	0.10	ISR	0.17	FRA	0.11	AUS	0.15
FRA	0.07	POL	0.14	HUN	0.11	DNK	0.15
DEU	0.02	MEX	0.11	EUR	0.10	NOR	0.12
EUR	0.01	KOR	0.08	CHE	0.10	CAN	0.11
KOR	0.01	EUR	0.07	JPN	0.08	CZE	0.11

Notes: This table reports the frequency that the currency of a country is included in four portfolios (P_1-P_4) constructed by sorting currencies on the product of nontraded output growth variance and GDP size. P1 is the portfolio with the lowest product. P4 is the portfolio with the highest product. Frequency of a currency I in portfolio P is measured as the ratio of the number of quarters during which I is in P divided by the total number of quarters of the data sample (i.e., the lifespan of portfolio P). The 14 most frequently occurring currencies (associated countries) for each portfolio are presented.

for high nontraded growth risks from the U.S. investors' perspective). Table 6 shows that NZD (or NZL in the table) is always an investment currency (with a frequency of 1.00 being sorted into portfolio 1), while JPY (JPN), GBP (GBR) and CHF (CHE) are more likely to be funding currencies (respectively with frequencies of 0.50, 0.53 and 0.36 being sorted into portfolio 4).[10] From the interest rate perspective, NZD typically is an investment (high interest rate) currency, while JPY and CHF are funding (low interest rates) currencies. The results of Table 6 concerning NZD, JPY, CHF then show the alignment of two currency sorting approaches (based on the size and nontraded variance products and on interest rates). AUD (AUS), on the other hand, appears with similar frequencies in portfolios P_1, P_2, P_3, and less frequently in P_4 (respectively with frequencies of 0.29, 0.29, 0.27 and 0.15). To the extent that AUD is another typical investment (high interest rate) currency, this pattern shows the some deviation between the two currency sorting approaches. Overall, the currency portfolio compositions in Table 6 enforces Table 5's finding that uncertainties in nontraded growths constitute a risk as well as compensated return for long positions in high interest rate currencies. These results in turn show that the pricing of nontraded growth risks in FX markets helps rationalize the forward premium puzzle.

2.3.4 Linear factor analysis

The theoretical analysis of Section 2.2 shows that the nontraded and traded consumption risks can be viewed as the two pricing factors. This section estimates these two factor prices empirically. Currency strategies whose payoffs correlate with these consumption risk factors are priced, resulting in currency risk premia. Accordingly, we employ currency portfolios as test assets in the estimation of the factor prices. We discuss, in order, the estimation procedure, data, and estimation results.

For each country H, we define the traded and nontraded consumption growths as risk factors for their local investors: $f^H_{T,t+dt} = \frac{C^H_{T,t+dt} - C^H_{T,t}}{C^H_{T,t}}$, $f^H_{N,t+dt} = \frac{C^H_{N,t+dt} - C^H_{N,t}}{C^H_{N,t}}$, with dt denoting the time step (3-month in our empirical analysis). We denote the realized excess return at time $t + dt$ on a currency portfolio i by cr^i_{t+dt}, which satisfies the Euler pricing equation $E[m^H_{t+dt} cr^i_{t+dt}] = 0$, where m^H is H's SDF growth (see (14)).

We use Euler equations and the Generalized Method of Moments (GMM) Hansen (1982) to estimate the model. The factor mean and the factor covariance matrix are also estimated simultaneously with the SDF parameters. Specifically, let F_t be the vector of nontraded and traded

[10] We are taking the USD as the baseline (home) currency and examining foreign nontraded risks and currencies relative to the U.S. nontraded risk and USD. As a result, USD is not among the currencies being sorted.

consumption risk factors, with mean μ_F and covariance Σ_F. Our moment equations for GMM are $E(g(z_t, \theta)) = 0$, where

$$g(z_t, \theta) = \begin{bmatrix} [1 + b'(F_t - \mu_F)]cr_t^i \\ F_t - \mu_F \\ vec((F_t - \mu_F)(F_t - \mu_F)') - vec(\Sigma_F) \end{bmatrix},$$

θ contains the parameters $(b', \mu', vec(\Sigma_F)')'$ and z_t represents the data (cr_t^i, F_t). The currency portfolio used in the Euler equations are 8 well-known currency portfolios, namely, five currency portfolios sorted on interest rates, the HML portfolio, the Dollar portfolio, and the Dynamic Dollar portfolio.[11] We employ a standard two-step GMM procedure, in which the weighting matrix in the first step is an identity matrix. Standard errors are computed based on a heteroscedasticity and autocorrelation consistent (HAC) estimate of the long-run covariance matrix Newey and West (1987). Factor prices are then backed out from the estimated factor loadings and covariance matrix, i.e., $\lambda = -\Sigma_F b$ (Section 2.2 and Footnote 6). The Likelihood Ratio test is used to test for the significance of factor prices, as well as the corresponding factor Sharpe ratios. The procedure is repeated for each OECD country, i.e., each time the home country H (and the currency denomination) is identified with an OECD country (and its currency) under consideration.

Table 7 reports the results when the home country H is taken as each of G10 countries plus the Euro Zone, i.e., the underlying analysis considers various currency denominations. Panel A reports the factor price estimate in currency H of country H's nontraded and traded consumption growth, for every country H in the sample of G10 plus the Euro Zone. The nontraded factor price estimate λ_N^H is positive for all countries where this estimate is statistically significant. The estimate λ_N^H is negative only for Switzerland, but the estimate is statistically insignificant for that country. These estimation results indicate the model's thesis that fluctuations in a country I's nontraded output are a consumptions risk and command a positive factor price $\lambda_N^H > 0$ (15) to local investors because nontraded output is consumed entirely domestically.

The traded factor price estimate λ_T^H, on the other hand, is positive and significant for the U.S., Canada, and Netherlands, but is negative and significant for France, Germany (before entering the Euro Zone), and Italy, as the home country. This indicates that fluctuations in the traded

[11] The Dollar factor is associated with the strategy that borrows home and lends equally foreign currencies, the HML factor borrows equally low-interest rate and lends equally high-interest rate currencies, the Dynamic Dollar portfolio takes a long (short) position in the Dollar portfolio when the median forward discount across all exchange rates is positive (negative). See Lustig et al. (2014) and Section 2.3.6, Equation (20) below.

consumption growth are a risk in some countries, and a hedge in others. In the model, the traded consumption growth factor price λ_T^H (15) can either be positive (risk) or negative (hedge), depending on the traded and home and foreign nontraded output growth volatilities and countries' sizes (via international trade and the substitution of traded and nontraded consumptions in equilibrium). The equilibrium traded consumption C_t^H then determine the marginal utility M_t^H and their correlation $Corr(M_t, H_t)$, whose sign characterizes the risk or hedge nature of the traded consumption price factor and the sign of λ_T^H. For the U.S., Table 7 shows that an additional exposure of one standard deviation to the U.S. traded consumption risk increases the currency strategy's return by 112 bps per quarter on average.

Table 7: **Factor pricing - G10 group (plus Euro)**

	USA	BEL	CAN	FRA	DEU	ITA	JPN	NLD	SWE	CHE	GBR	EUR
	Panel A. Factor price											
λ_N (%)	0.51**	35.19	0.46	0.56***	1.00***	0.71***	0.37**	3.61***	0.19	-215.87	0.54*	0.04
(se) (%)	(0.22)	(79.47)	(0.40)	(0.14)	(0.32)	(0.18)	(0.19)	(0.46)	(0.16)	(217.05)	(0.29)	(0.25)
LR pval	0.02	0.66	0.25	0.00	0.00	0.00	0.05	0.00	0.24	0.32	0.06	0.87
λ_T (%)	1.12*	-2.70	0.90*	-0.49*	-0.70***	-0.36*	1.17	1.34***	0.37	12.71	0.11	-0.42
(se) (%)	(0.63)	(3.42)	(0.54)	(0.28)	(0.27)	(0.20)	(0.87)	(0.32)	(0.34)	(13.12)	(0.39)	(0.42)
LR pval	0.07	0.43	0.09	0.08	0.01	0.08	0.18	0.00	0.26	0.33	0.77	0.32
	Panel B. Factor Sharpe ratio											
SR_N	1.12**	0.09	0.71	0.89***	1.51***	1.56***	0.58**	2.72***	0.43	-1.21	0.66*	0.08
(se)	(0.47)	(0.22)	(0.62)	(0.23)	(0.37)	(0.24)	(0.29)	(0.21)	(0.37)	(1.36)	(0.35)	(0.48)
LR pval	0.02	0.67	0.25	0.00	0.00	0.00	0.04	0.00	0.25	0.38	0.06	0.87
SR_T	0.79*	-0.16	0.95*	-0.38*	-0.40***	-0.27	0.76	1.28***	0.29	0.65	0.09	-0.33
(se)	(0.46)	(0.22)	(0.55)	(0.22)	(0.14)	(0.18)	(0.49)	(0.29)	(0.27)	(0.69)	(0.31)	(0.32)
LR pval	0.09	0.47	0.09	0.08	0.00	0.13	0.12	0.00	0.27	0.35	0.77	0.30
	Panel C. GMM over-identification test											
J-stat	12.02*	3.41	6.45	9.16	5.11	2.61	9.48	3.29	4.22	2.53	5.20	8.22
pval	0.06	0.76	0.38	0.16	0.53	0.76	0.15	0.77	0.65	0.86	0.52	0.22

Notes: GMM estimates for G10 countries, and the European Monetary Union (EMU). Countries that joined the EMU are dropped from the sample from the time they joined and are replaced by a single observation for the Euro Zone. Panel A shows the factor price estimates λ for nontraded (λ_N) and traded (λ_T) growth risks, in percentage points. Panel B shows the corresponding Sharpe ratios SR. Newey-West heteroscedasticity and autocorrelation consistent (HAC) standard errors are in parentheses. "LR pval" shows the p-value of the Likelihood Ratio test that the factor price or the Sharpe ratio estimates are equal to 0. Panel C shows the J-statistics and p-values associated with GMM over-identification test. Significance at the 1%, 5% or 10% level are indicated by ***,** or *.

Panel B of Table 7 reports the corresponding Sharpe ratio for each factors. The Sharpe ratio SR_N for the nontraded consumption risk factor is significant for the U.S., France, Germany (before entering the Euro Zone), Italy, Japan, Netherlands, and the U.K. It is 1.12 for the U.S. per quarter. The Sharpe ratio of the traded consumption risk factor (SR_T) is in general smaller than that of the nontraded consumption risk factor. It is 0.79 for the U.S. per quarter.

The model also has a simple testable implication on the relationship between the nontraded consumption growth factor price, nontraded growth risk, and economic size. Specifically, (15) implies that

$$\frac{\lambda_N^I}{(\sigma_N^I)^2} = \alpha\gamma\omega_N \left[\epsilon + (\gamma - \epsilon)\omega_T \frac{\Lambda^I}{\Lambda}\right]. \tag{16}$$

For a simple evaluation of this linear relationship, we employ the point estimates $\{\lambda_N^I\}$ obtained in Table 7 that are statistically significant to compute the correlation between the nontraded consumption growth factor price per unit of nontraded variance $\frac{\lambda_N^I}{(\sigma_N^I)^2}$ and economic size $\frac{\Lambda^I}{\Lambda}$, obtaining a correlation of 95.8%. This positive and large value of the correlation indicates a model's implication that $\frac{\lambda_N^I}{(\sigma_N^I)^2}$ increases with $\frac{\Lambda^I}{\Lambda}$ (15), i.e., a country with either a more volatile nontraded output or a larger size requires a higher price for its nontraded consumption growth risk factor.

2.3.5 Equilibrium traded consumptions

Nontraded growth risks and their pricing suggest further testable implications for the underlying model. The equilibrium relationship (7) implies that the cross-country difference in traded consumption variances is proportional to the product of total nontraded variance and difference in sizes,[12]

$$\frac{1}{dt}\left(Var_t\left[\frac{C^F_{Tt+dt}}{C^F_{Tt}} - 1\right] - Var_t\left[\frac{C^H_{Tt+dt}}{C^H_{Tt}} - 1\right]\right) = -\left[\alpha(\gamma-\epsilon)\omega_N\right]^2 \left([\sigma_N^H]^2 + [\sigma_N^F]^2\right) \left[\frac{\Lambda^F}{\Lambda} - \frac{\Lambda^H}{\Lambda}\right]. \tag{17}$$

Per the discussion below (7), one country's size amplifies the impact of nontraded growth risks on the other country's traded consumption adjustment, hence also amplifies the latter's traded consumption volatility. The cross-country traded consumption variance differential (17) hence is inversely related to size differential.

Fixing the baseline (home) country H to be the U.S., Table 8 examines this model-implied relationship by regressing F's traded consumption variance differential (with respect to the U.S.)

[12]As a result of the definition $\Lambda \equiv \sum_{I\in\{H,F\}} \Lambda^I$ below (6), we have $\left[\left(\frac{\Lambda^F}{\Lambda}\right)^2 - \left(\frac{\Lambda^H}{\Lambda}\right)^2\right] = \left[\frac{\Lambda^F}{\Lambda} - \frac{\Lambda^H}{\Lambda}\right]$.

on the product of total nontraded variance and size differential (with respect to the U.S.) for a cross section of countries F. These underlying variables are constructed in real terms and per-capita at quarterly frequency using rolling windows of past data. The regressions include both country and time fixed effects and the same set of control variables as in Table 1.

Table 8: **Traded Consumptions and Nontraded Risks**

	(1)	(2)	(3)
sntvsize $= \left([\sigma^F_{Nt}]^2 + [\sigma^H_{Nt}]^2\right)\left(\frac{\Lambda^F_t}{\Lambda_t} - \frac{\Lambda^H_t}{\Lambda_t}\right)$	-0.026**	-0.031**	-0.033**
	(0.013)	(0.015)	(0.014)
vinfl		3.489***	1.444
		(1.117)	(1.176)
gdpg		0.515***	0.592***
		(0.080)	(0.086)
vgdpg		12.751***	14.118***
		(2.164)	(2.216)
viexg		-0.610***	-0.616***
		(0.119)	(0.118)
gdp (mil)			1.273***
			(0.353)
Constant	0.001	-0.007***	-0.014***
	(0.001)	(0.002)	(0.002)
Observations	1,587	1,414	1,414
Adjusted R-squared	0.6363	0.6595	0.6631

Notes: Regression for the difference in the variance of traded factors between each country and that of the US. Control variables: "vinfl" is the variance of inflation; "gdpg" is GDP growth; "vgdpg" is the variance of GDP growth; "viexg" is the variance of exchange rate growth; "gdp (mil)" is GDP measured in USD millions. Both country and time fixed effects are included. Robust standard errors, clustered by country and time, are in parentheses. Significance at the 1%, 5% or 10% level are indicated by ***,** or *. The panel consists of quarterly data for OECD countries from 1976-2016. Countries that joined the European Monetary Union are dropped from the sample from the time they joined and are replaced by a single observation for the Euro Zone.

Across different specifications, the slope coefficient estimate associated with the product of total nontraded variance and size differential (sntvsize) is statistically significant and negative, hence is consistent with the model-implied inverse relationship discussed below (17). Relative to the U.S. quantities as the baseline and all else being equal, a country F with a higher total nontraded variance and size product by 1% (per quarter) is associated with a lower traded consumption variance by 2.6 bps (per quarter). This decrease remains significant in the presence of control variables, indicating the robustness of this inverse relationship in the data.

2.3.6 Discussion: Related findings in the literature

The section discusses the relation between the nontraded growth risk setting adopted in this paper and prominent analysis and findings of Lustig et al. (2011) and Engel (1999).

Lustig et al. (2011)'s reduced-form model: It is instructive to relate the economic factors of traded and nontraded consumption growth risks with the reduced-form factors in the literature. In Lustig et al. (2011)'s (LRV) original reduced-form model, each country-specific SDF is driven by a global shock and a local shock. Whereas, in our setting, both aggregate traded and country-specific nontraded shocks are global as they are also priced by other countries' SDFs (8). To relate the two approaches, we consider the following version of LRV's SDF growth that features multiple global shocks,

$$dm^I_{t+dt} = -\alpha^I - \chi^I_0 z^I_{Nt} - \sum_{J\neq I} \chi^J z^J_{Nt} - \chi^I_T z_{Tt} - \sqrt{\gamma^I_0 z^I_{Nt}}\, u^I_{Nt+dt} - \sum_{J\neq I} \sqrt{\gamma^J z^J_{Nt}}\, u^J_{Nt+dt} - \sqrt{\delta z_{Tt}}\, u_{Tt+dt}, \tag{18}$$

where $dt = 1$ denotes the time step (3-month in our empirical analysis). Adopting LRV's notation, the aggregate (u_{Tt+dt}) and country-specific (u^I_{Nt+dt}) shocks are independent standard normal shocks, and associated with a common (z_{Tt}) and country-specific (z^I_{Nt}) factors. By comparing SDFs (18) and (8), the correspondence between the nontraded risk pricing (left-hand side quantities below) and LRV (right-hand side quantities) frameworks is

$$\begin{aligned} \gamma\omega_T &\longleftrightarrow \delta, & \sigma^2_{Tt} &\longleftrightarrow z_{Tt}, & \left(\sigma^I_{Nt}\right)^2 &\longleftrightarrow z^I_{Nt}, \\ \gamma^2\omega^2_N \left[1-\alpha(\gamma-\epsilon)\omega_T\left(1-\tfrac{\Lambda^I}{\Lambda}\right)\right]^2 &\longleftrightarrow \gamma^I_0, & & & \gamma^2\omega^2_N\alpha^2(\gamma-\epsilon)^2\omega^2_T\left(\tfrac{\Lambda^I}{\Lambda}\right)^2 &\longleftrightarrow \gamma^I. \end{aligned} \tag{19}$$

In particular, the variances of aggregate traded and country-specific nontraded output growths correspond to the common and country-specific factors in LRV's framework. LRV's Dollar factor (borrowing home and lending equally foreign currencies) and HML factor (borrowing equally low-interest rate and lending equally high-interest rate currencies) in the nontraded risk pricing model are

$$\text{Dollar Factor} \sim \sum_{F\neq H} cr^{-H,+F}_{t+dt}, \qquad \text{HML Factor} \sim \sum_{F\in\text{High}} cr^{-H,+F}_{t+dt} - \sum_{F\in\text{Low}} cr^{-H,+F}_{t+dt}, \tag{20}$$

where High (and Low) is portfolio of currencies associated with higher (lower) interest rates than the home interest rate,[13] and $cr^{-H,+F}$ is the long-short currency (log) return (12),

$$cr_{t+dt}^{-H,+F} \equiv r_t^F - r_t^H - (dm_{t+dt}^H - dm_{t+dt}^F) = -\frac{1}{2}\left(\gamma_0^F - \gamma^F\right) z_{Nt}^F + \frac{1}{2}\left(\gamma_0^H - \gamma^H\right) z_{Nt}^H$$
$$- \left(\sqrt{\gamma_0^F} - \sqrt{\gamma^F}\right) \sqrt{z_{Nt}^F} u_{Nt+dt}^F + \left(\sqrt{\gamma_0^H} - \sqrt{\gamma^H}\right) \sqrt{z_{Nt}^H} u_{Nt+dt}^H. \quad (21)$$

The substitution of currency returns (21) into Dollar and HML factors (20) shows that the home nontraded output shock u_{Nt+dt}^H dominates in Dollar factor because it is common to the all return components $\{cr_{t+dt}^{-H,+F}\}$ (associated with all foreign currencies F, fixing the home currency H to be USD). Whereas, this shock u_{Nt+dt}^H is subdued in HML factor because it is in both (hence, canceled from) High and Low portfolios. Independent foreign nontraded shocks $\{u_{Nt+dt}^F\}$ are also diversified in the HLM factor when there are sufficiently many currencies in High and Low portfolios.[14] In summary, the nontraded output risk pricing model presents a concrete structural setting to illustrate and interpret shocks and global pricing factors in LRV's framework.

Engel (1999)'s decomposition: It is also instructive to reconcile a key result of Engel (1999) that movements in the relative traded good price (between countries) account for most of movements in the real exchange rate growth with the prominent role attributed to nontraded growth risks in the current setting. To this end, we first map the traded good price component in Engel (1999)'s decomposition into nontraded growth risks in current setting. The map highlights the significant impact of nontraded growth risks on both traded good prices and real exchange rates, therefore enabling their dominant correlation in equilibrium.

Engel (1999)'s decomposition starts with expressing the number of foreign baskets per home basket, i.e., the real exchange rate S_t (11), in terms of local currencies. Let P_t^I ($I \in \{H, F\}$) denote country I's basket price in units of the traded consumption good, and L_t^I the price of a unit of the traded consumption good in country I's (paper) currency. The real exchange rate then is,

$$S_t = \frac{P_t^H}{P_t^F} = \frac{L_t^F}{L_t^H} \times \frac{L_t^H P_t^H}{L_t^F P_t^F} = S_t^{\text{nom}} \times \frac{L_t^H P_t^H}{L_t^F P_t^F}, \quad (22)$$

where S_t^{nom} denotes the nominal exchange rate (per unit of foreign paper currency). Adopts a

[13] Note that the covered interest rate parity holds in the current setting. As a result, sorting currencies based on the interest rate differential is equivalent to sorting currencies based on the forward exchange rate premium.

[14] Aggregate traded output shock u_{Tt+dt} does not contribute to the two factors because it already drops out from the individual currency returns (21).

parametric (homogeneous of degree one) representation of country I's basket price (in I's paper currency) in terms of I's nontraded (P^I_{Nt}) and traded (P^I_{Tt}) prices; $L^I_t P^I_t = [P^I_{Nt}]^{\beta^I}[P^I_{Tt}]^{1-\beta^I}$, $I \in \{H, F\}$, the decomposition for the log real exchange rate is (where lower cases denote log quantities)

$$s_t = \underbrace{s^{\text{nom}}_t + p^H_{Tt} - p^F_{Tt}}_{\text{relative traded good price component}} + \underbrace{\beta^H \left(p^H_{Nt} - p^H_{Tt}\right) - \beta^F \left(p^F_{Nt} - p^F_{Tt}\right)}_{\text{relative nontraded good price component}}. \tag{23}$$

Engel (1999) finds that, empirically, most of movements in the real exchange rate growth s_t is accounted by the movements in the relative traded good price component of the decomposition (23) for advanced economies.

In our setting, the real exchange rate equals the ratio of countries' marginal utilities of consuming country-specific baskets (i.e., country-specific SDFs), $S_t = \frac{M^H_t}{M^F_t}$, where $M^I_t = \partial U^I_t / \partial C^I_t$, $I \in \{H, F\}$ (8). A comparison with the real exchange rate (22) implies that a country's marginal utility is proportional to its basket price in equilibrium, $M^I_t \sim P^I_t$. Using utility function U^I_t (2) and the parametric representation of basket price P^I_t specified above, the log linearization approximation of this relationship yields an (approximate) mapping between Engel (1999)'decomposition and the current paper's nontraded growth risk setting,[15]

$$-\gamma\omega_T c^I_{Tt} \longleftrightarrow (1-\beta^I)p^I_{Tt}, \qquad -\gamma\omega_T \delta^I_{Nt} \longleftrightarrow \beta^I p^I_{Nt}. \tag{24}$$

In particular, this mapping shows that the log traded good price p^I_{Tt} is linear in the equilibrium traded consumption c^I_{Tt}. In the current paper's setting, the real exchange growth (11) and the traded consumption growth differential (implied from (6)) are largely driven by nontraded growth risks $\left(\sigma^H_{Nt} dZ^H_{Nt+dt} - \sigma^F_{Nt} dZ^F_{Nt+dt}\right)$. The mapping (24) then implies a significant correlation between the real exchange growth and relative traded good price, in line with Engel (1999)'s decomposition result.[16]

[15]Specifically, the log linearization of marginal utility $M^H_t = \partial U^I_t / \partial C^I_t = e^{-\rho t}[C^I_t]^{-\gamma}$, $I \in \{H, F\}$, produces $\log M^I_t \approx -\rho t - \gamma\omega_T c^I_{Tt} - \gamma\omega_N \delta^I_{Nt}$. Whereas, Engel (1999)'s parametric representation of basket price $L^I_t P^I_t = [P^I_{Nt}]^{\beta^I}[P^I_{Tt}]^{1-\beta^I}$ implies $p^I_t = -l^I_t + (1-\beta^I)p^I_{Tt} + \beta^I p^I_{Nt}$, lower cases denoting log quantities. The linear relationship between m^I_t and p^I_t (arising from the proportional relationship between $M^I_t \sim P^I_t$) then implies the mapping (24).

[16]In practice, the correlation is not perfect because Engel (1999)'s empirical analysis also employs nominal spot exchange rate data (and the heterogeneity of $\{\beta^I\}$ in the representation of country-specific basket prices). Whereas, the current nontraded growth risk model is in a real setting.

3 Equilibrium Economic Sizes

In an international endowment economy framework, endowment streams are the primitives of the setup. Pareto weights $\{\Lambda^I\}$, $I \in \{H, F\}$, which proxy for countries' sizes in previous section, are functions of endowment streams in equilibrium. Taking into account this (endogenous) dependence of Pareto weights on endowments and subsuming effects of equilibrium economic sizes, we analyze the overall impact of nontraded and traded growth risks on asset prices conceptually (Section 3.1) and quantitatively (Section 3.2). Technical derivations are in Online Appendix A.2.

3.1 Conceptual Analysis

Economic sizes: In equilibrium, Pareto weights are related to endowments via budget constraints. This can be seen most intuitively in a two-period setting $\{t, t+dt\}$,[17]

$$C^I_{Tt} + E_t\left[\frac{M_{Tt+dt}}{M_{Tt}} C^I_{Tt+dt}\right] = \Delta^I_{Tt} + E_t\left[\frac{M_{Tt+dt}}{M_{Tt}} \Delta^I_{Tt+dt}\right], \qquad I \in \{H, F\}. \tag{25}$$

Because consumptions (6) and pricing kernel M_{Tt} (5) are functions of sizes, budgets constraints (25) then solve for Pareto weights (or sizes) as functions of primitive endowments in equilibrium.[18] The log linearization yields approximate but intuitive expressions for equilibrium sizes,

$$\begin{aligned} \frac{\Lambda^H}{\Lambda} &= \frac{1}{2} + \frac{\rho}{(\gamma\omega_T + \epsilon\omega_N)\left(g^H_{Tt} - g^F_{Tt}\right) + (\gamma - \epsilon)\omega_N\left(g^H_{Nt} - g^F_{Nt}\right)}, \\ \frac{\Lambda^F}{\Lambda} &= \frac{1}{2} + \frac{\rho}{(\gamma\omega_T + \epsilon\omega_N)\left(g^F_{Tt} - g^H_{Tt}\right) + (\gamma - \epsilon)\omega_N\left(g^F_{Nt} - g^H_{Nt}\right)}, \end{aligned} \tag{26}$$

where g^I_{Tt} and g^I_{Nt} denote country I's expected total traded and nontraded endowment growths (Equation (40), Online Appendix A.2). Expression (26) clearly shows that a country's impact on international asset pricing, characterized by its Pareto weight and FOCs (5), is determined by both current and future (expected) components of the country's endowment stream. The presence of future endowment growths highlights the fact that current outputs or GDPs are only a component of equilibrium economic sizes (Pareto weights) in a rational expectation setup of the international endowment economy. A country H with dominant traded and nontraded endowment growths ($g^H_{Tt} >$

[17] Given that financial markets are complete, Pareto weights are constants and multiple period setting can be mapped into a static setting. Hence, we illustrate the Pareto weight derivation in the static setting for simplicity.

[18] Because nontraded consumptions and endowments are identical (4), they cancel and drop out from the budget constraints.

g^F_{Tt}, $g^H_{Nt} > g^F_{Nt}$) has a dominant international pricing impact, $\left(\frac{\Lambda^H}{\Lambda} > \frac{1}{2} > \frac{\Lambda^F}{\Lambda}\right)$ (26). However, such a dominance decreases with the growth differentials $g^H_{Tt} - g^F_{Tt}$ and $g^H_{Nt} - g^F_{Nt}$ because the (risk-mitigating) international trade between countries constitutes a smaller proportions of H's endowments when the cross-country differences in endowment growths widen.

Equilibrium traded consumptions: The substitution of Pareto weights (26) into expressions for consumptions, SDFs, interest rates and currency returns incorporates and supplants the role of equilibrium economic sizes into the effect of endowment growth risks on these quantities. The traded log consumptions follow from the substitution of (26) into (6) (omitting a pure-time deterministic term),

$$c^H_{Tt} = \delta_{Tt} - \frac{1}{2}\alpha(\gamma-\epsilon)\omega_N\left(\delta^H_{Nt} - \delta^F_{Nt}\right) - \frac{\rho\alpha(\gamma-\epsilon)\omega_N\left(\delta^H_{Nt} - \delta^F_{Nt}\right)}{(\gamma\omega_T + \epsilon\omega_N)\left(g^F_{Tt} - g^H_{Tt}\right) + (\gamma-\epsilon)\omega_N\left(g^F_{Nt} - g^H_{Nt}\right)}, \quad (27)$$

where α is defined below (6). The first two terms, involving only current endowments, show that H's traded consumption increases with aggregate traded endowment and decreases with H's nontraded endowment and reflect roles of international trade and consumption substitutions in mitigating endowment risks. The remaining term, involving both current and future endowment components in g's, shows that such a risk mitigation is limited when H's endowment growths dominate (when $g^H_{Tt} - g^F_{Tt}$, $g^H_{Nt} - g^F_{Nt} > 0$, the last term offsets the second term). This is because dominant endowment growths generate a dominant equilibrium economic size $\frac{\Lambda^H}{\Lambda}$, reducing H's ability to secure sufficient international trade to substitute for its nontraded endowment loss as discussed below (26). The dependence of the home traded consumption on the foreign nontraded endowment captured in (27) follows from a symmetric argument.

Country-specific SDFs: Country-specific log SDF follows from the substitution of the equilibrium sizes (26) into $m^H_t \equiv \log M^H_t$ (8) (see Online Appendix A.2),

$$m^H_t = -\rho t - \gamma\omega_T\delta_{Tt} - \gamma\omega_N\delta^H_{Nt} + \frac{\alpha\gamma(\gamma-\epsilon)\omega_N\omega_T}{2}\left(\delta^H_{Nt} - \delta^F_{Nt}\right)$$

$$+\frac{\rho\alpha\gamma(\gamma-\epsilon)\omega_N\omega_T}{(\gamma\omega_T + \epsilon\omega_N)\left(g^F_{Tt} - g^H_{Tt}\right) + (\gamma-\epsilon)\omega_N\left(g^F_{Nt} - g^H_{Nt}\right)}\left(\delta^H_{Nt} - \delta^F_{Nt}\right). \quad (28)$$

The first three terms characterize standard increases of the home SDF m^H_t with either a higher time discount rate ρ, a smaller traded consumption (due to a decrease in the aggregate traded endowment δ_{Tt}), or smaller nontraded consumption (which equals the nontraded endowment δ^H_{Nt}). The remaining terms, proportional to $\left(\delta^H_{Nt} - \delta^F_{Nt}\right)$, reflect international trade and consumption substitution effects on equilibrium consumption c_{Tt}^H (27), which in turn affects m_t^H. The log SDF differential

$$m_t^H - m_t^F = -\alpha\gamma\epsilon\omega_N \left(\delta_{Nt}^H - \delta_{Nt}^F\right). \tag{29}$$

concerns only countries' nontraded endowments. As the log SDF differential characterizes the relative pricing across currency denominations, expression (29) shows that nontraded endowment growth risks remain key factors the exchange rate dynamics when economic sizes are endogenized in equilibrium.

Interest rate differential: Country-specific interest rates are determined as the expected mean growth rate of the respective SDFs, $r_t^I = -\frac{1}{dt}E_t\left[\frac{dM_{t+dt}^I}{M_t^I}\right]$. Given that equilibrium economic sizes are functions of endowments, the interest rate differential is obtained explicitly by substituting Pareto weights (26) into (10),

$$r^F - r^H = \alpha\gamma\epsilon\omega_N\left(\mu_N^F - \mu_N^H\right) + \frac{1}{2}\alpha\epsilon\gamma^2\omega_N^2\left([\sigma_N^H]^2 - [\sigma_N^F]^2\right) \tag{30}$$

$$+ \frac{\rho\epsilon\alpha^2\gamma^2(\gamma-\epsilon)\omega_N^2\omega_T}{(\gamma\omega_T + \epsilon\omega_N)\left(g_{Tt}^H - g_{Tt}^F\right) + (\gamma-\epsilon)\omega_N\left(g_{Nt}^H - g_{Nt}^F\right)}\left([\sigma_N^H]^2 + [\sigma_N^F]^2\right).$$

The nontraded output growth risks impact the interest rate differential either by their own or by their interaction with expected output growths g's. When the home nontraded growth volatility dominates, $[\sigma_N^H]^2 - [\sigma_N^F]^2 > 0$, the home bond is more valuable as it pays off in terms of the home consumption basket to hedge against the volatile home nontraded output. As a result, the home interest rate tends to be lower (second term in (30)). However, dominant home endowment growths ($g_{Tt}^H > g_{Tt}^F$, $g_{Nt}^H > g_{Nt}^F$) reduce the ability to mitigate the home nontraded output growth risk via international trade and consumption substitution, making the home bond even more valuable and lowering the home interest rate further (third term in (30)). A symmetric argument applies for the effects on the foreign interest rate component in the interest rate differential.

Exchange rate and currency returns: In complete financial markets, the exchange rate is equal to the ratio of SDFs and the log exchange rate $\log S_t$ is given by the log SDF differential $m_t^H - m_t^F$. As a result, the exchange rate dynamics are determined solely by the conditional moments ($\{\mu_{Nt}^I, \sigma_{Nt}^I\}$, $I \in \{H, F\}$) of nontraded endowments as discussed below (29) (also (11)). In contrast, the interest rate differential depends on the expected growths (g's) of entire endowment streams as shown in (30).

Because currency returns are composed of exchange rate growth and interest rate differential components (12), the observations above indicate that economic sizes and the underlying the expected growths (g's) of entire endowment streams affect the currency risk premium exclusively via the interest rate differential component. Substituting Pareto weights (26) into (13) yields an explicit decomposition of the currency risk premium in terms of the expected endowment growths

$$\frac{1}{dt}E_t\left[CR_{t+dt}^{-H,+F}\right] = (r^F - r^H) + \frac{1}{dt}E_t\left[\frac{S_t}{S_{t+dt}} - 1\right]$$

$$= \alpha^2\epsilon^2\gamma^2\omega_N^2[\sigma_N^H]^2 + \frac{1}{2}\alpha^2\epsilon(\gamma-\epsilon)\gamma^2\omega_N^2\omega_T\left([\sigma_N^H]^2 - [\sigma_N^F]^2\right)$$

$$+ \frac{\rho\alpha^2\epsilon(\gamma-\epsilon)\gamma^2\omega_N^2\omega_T}{(\gamma\omega_T + \epsilon\omega_N)\left(g_{Tt}^H - g_{Tt}^F\right) + (\gamma-\epsilon)\omega_N\left(g_{Nt}^H - g_{Nt}^F\right)}\left([\sigma_N^H]^2 + [\sigma_N^F]^2\right). \tag{31}$$

Because borrowing home and lending foreign currencies is a risky (resp., a hedge) strategy with respect to uncertainties in the home (resp., foreign) nontraded output growth, the associated currency risk premium $E_t\left[CR_{t+dt}^{-H,+F}\right]$ tends to increase (resp., decrease) with the home nontraded volatility $|\sigma_N^H|$ (resp., $|\sigma_N^F|$) as seen in the first two terms (resp., the second term) on the right hand side of (31)). However, when the home (resp., foreign) expected endowment growths dominate $g_{Tt}^H > g_{Tt}^F$ and $g_{Nt}^H > g_{Nt}^F > 0$, (resp., $g_{Tt}^H < g_{Tt}^F$ and $g_{Nt}^H < g_{Nt}^F > 0$), the impact of the home (resp., foreign) nontraded risk exacerbates due to limited ability to mitigate this risk via trades and consumption substitution. As a result, the currency strategy $CR^{-H,+F}$ is riskier (resp., less risky), reflected in a positive (resp., negative) last term of (31), i.e., a higher (resp., lower) currency risk premium $E_t\left[CR_{t+dt}^{-H,+F}\right]$.

3.2 Empirical Analysis

In the current setting in which economic sizes are determined in equilibrium, country-specific expected nontraded and traded endowment growths $\{g_{Nt}^I, g_{Tt}^I\}$ are the fundamental inputs to the model and all equilibrium quantities. Specifically, the expected endowment growths are in the numerator of equilibrium sizes (26), consumptions (27), interest rate differentials (30), and currency risk premia (31). Therefore, the expected growths g_{Nt}^I and g_{Tt}^I are non-trivially coupled with each other in these non-linear expressions of equilibrium quantities. Taking g_{Nt}^I and g_{Tt}^I as inputs, our empirical analysis in this section employs the method of moments (GMM) to incorporate this non-linearity in the estimation of model's parameters. In comparison, when sizes are taken as exogenous inputs to the model, $\frac{\Lambda^H}{\Lambda}$ and $\frac{\Lambda^H}{\Lambda}$ are not coupled in the equilibrium quantities (6), (10), and (13).

As a result, linear regressions were employed to examine key model-implied relationships in the setting of exogenous sizes.

Our GMM procedure estimates the four independent parameters of the model, namely, the time discount factor ρ, the inverse of the intertemporal elasticity of consumption (or relative risk aversion) γ, the inverse of the substitution elasticity between traded and nontraded consumptions ϵ, and taste for nontraded consumption ω_N, specified in the preference (2).[19] For moment equations, the estimation employs the currency risk premium (13), the mean of exchange rate growths (11) and the interest rate differential (10),[20] with the traded and nontraded output growths used as the extra instruments. The input expected endowment growths $g_t^I = \delta_t^I + E_t\left[\delta_{t+dt}^I(s)\right] = 2\delta_t^I + \mu^I dt$ are constructed from the traded and nontraded output data.

Table 9: **Equilibrium Sizes - GMM Estimates**

	Estimates	(Standard error)
ρ	0.01	(3.54)
γ	8.41***	(2.07)
ϵ	7.69***	(0.86)
ω_N	0.08**	(0.04)

Notes: GMM estimates for the endogenous model parameters. The moment conditions are the expected value of interest rate, currency return, and exchange rate growth. Two extra instruments are used, namely the traded and nontraded output growth. Bootstrap standard errors are reported in parenthesis.

The GMM procedure is similar to what is used in Section 2.3.4 with Stata 2-step procedure. Table 9 reports the results of this parametric estimation. With the exception of the time discount factor ρ, estimates of three other parameters are positive and statistically significant at the level of 5% and 1%. The point estimates concerning intertemporal elasticity of consumption ($\gamma = 8.41$) and the substitution elasticity between traded and nontraded consumptions ($\epsilon = 7.69$) satisfies $\frac{1}{\epsilon} > \frac{1}{\gamma}$ (3). Since this key parametric inequality presumption underlies the intuition for the pricing of nontraded growth risks in international markets, the estimation results in Table 9 are consistent with, and uphold, the importance of these risks when economic sizes are determined endogenously in equilibrium. The point estimate of the nontraded consumption taste $\omega_N = 0.08$ implies that

[19] Remaining parameters are implied; $\alpha \equiv (\gamma\omega_T + \epsilon\omega_N)^{-1}$ defined below (6), and $\omega_T = 1 - \omega_N$.

[20] Note that our interest rate differentials are calculated directly from the interest rate series, not implied from forward exchange rates via the covered interest rate parity, therefore there is no redundancy in terms of data.

countries place a significant weight on the consumption of traded goods (reflected in a high traded consumption taste of $\omega_T = 1 - \omega_N = 0.92$). This finding is also consistent with a significant factor price estimate of the traded consumption growth risk in the USD denomination (Section 2.3.4, Table 7 and the associated discussion), and in line with Engel (1999)'s finding that the traded good price movements are key to the real exchange rate growth movements (Section 2.3.6, Equation (23) and the associated discussion).

4 Conclusion

This paper studies asset pricing effects of countries' nontraded output growth risks in a setting of international endowment economy. Fluctuations in nontraded output growths are an important risk factor because nontraded outputs are consumed domestically. In interest rate markets, countries of higher nontraded output growth risks are associated with stronger precautionary savings motives and lower interest rates. In currency markets, strategies of higher exposures to nontraded output growth risks offer higher average excess returns.

Nontraded output shocks can be partially mitigated by international trade and an imperfect substitution between nontraded and traded consumptions. In this regard, economic sizes exacerbate the pricing impact of nontraded growth risks because the risk mitigation is less effective for larger economies. However, as economic sizes are functions of outputs in the endowment economy setting, their pricing impact is subsumed into that of output growth risks in equilibrium at a fundamental level.

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Online Appendices

(Not intended for publication)

A Derivations and Proofs

This section presents details and derivations that underlie quantitative results of the main text. Appendix A.1 presents the first-order conditions (FOCs) and associated asset pricing quantities of the international economy considered in the main text when economic sizes are taken as exogenous inputs. Appendix A.2 presents asset pricing quantities when economic sizes are determined from and supplanted by endowments in equilibrium.

A.1 Exogenous Economic Sizes

We consider the international endowment economy with two countries $\{H, F\}$, who derive utilities (2) from consuming a traded (common) good and a respective nontraded (country-specific) good. Let country I have a population of N_I identical individuals, $I \in \{H, F\}$. We assume that financial markets are complete, so the equilibrium can be obtained from solving a static optimization problem of a representative agent. In what follows, for the simplicity of exposition, we further cast this static optimization problem in a two-date setting $\{t, t+dt\}$, where $dt = 1$ denotes the unit of time interval under consideration (e.g., one quarter) and therefore, we can set $t = 0$ and $t + dt = 1$. Different possibilities of state s to be realized at the future time $t + dt = 1$ characterize the uncertainty of the setting perceived at the current time $t = 0$. The representative agent maximizes the following expected utility

$$U^R = \Lambda^H N^H \left(e^{-\rho t} U^H(C_t^H, t) + E_t\left[e^{-\rho(t+dt)} U^H(C_{t+dt}^H, t+dt)\right]\right) \tag{32}$$

$$+\Lambda^F N^F \left(e^{-\rho t} U^F(C_t^F, t) + E_t\left[e^{-\rho(t+dt)} U^F(C_{t+dt}^F, t+dt)\right]\right),$$

subject to resource constraints in nontraded and aggregate traded consumption goods in every state and time,

$$C_{Nt}^H = \Delta_{Nt}^H, \quad C_{Nt}^F = \Delta_{Nt}^F, \quad C_{Nt+dt}^H(s) = \Delta_{Nt+dt}^H(s), \quad C_{Nt+dt}^F(s) = \Delta_{Nt+dt}^F(s), \quad \forall s,$$

$$\sum_{I\in\{H,F\}} N^I C^I_{Tt} = \sum_{I\in\{H,F\}} N^I \Delta^I_{Tt}, \qquad \sum_{I\in\{H,F\}} N^I C^I_{Tt+dt}(s) = \sum_{I\in\{H,F\}} N^I \Delta^I_{Tt+dt}(s), \quad \forall s. \tag{33}$$

In the equations above, U^I, C^I_N's, C^I_T's, Δ^I_N's and Δ^I_T's denote per-capital quantities (utility, nontraded and traded consumptions and endowments) of country I, $I \in \{H, F\}$, and Λ^I the associated Pareto weight. First-order conditions (FOCs) associated with the variation of traded consumptions at time t, and time $t + dt$ and state s, imply equalities of marginal utilities with respect to traded consumptions, which also determine the pricing kernel M_{Tt} associated with the traded consumption good numeraire

$$\frac{\Lambda^H}{\Lambda} e^{-\rho t} \frac{\partial U^H(C^H_{Tt}, \Delta^H_{Nt})}{\partial C^H_{Tt}} = \frac{\Lambda^F}{\Lambda} e^{-\rho t} \frac{\partial U^F(C^H_{Tt}, \Delta^F_{Nt})}{\partial C^F_{Tt}} \equiv M_{Tt},$$

$$\frac{\Lambda^H}{\Lambda} e^{-\rho(t+dt)} \; \frac{\partial U^H(C^H_{Tt+dt}(s), \Delta^H_{Nt+dt}(s))}{\partial C^H_{Tt+dt}(s)} = \frac{\Lambda^F}{\Lambda} e^{-\rho(t+dt)} \; \frac{\partial U^F(C^H_{Tt+dt}(s), \Delta^F_{Nt+dt}(s))}{\partial C^F_{Tt+dt}(s)} \equiv M_{Tt+dt}(s), \quad \forall s.$$

Without loss of generality and for convenience, we have rescaled Pareto weights by their sum, $\Lambda \equiv \Lambda^H + \Lambda^F$. Therefore,

$$\frac{\Lambda^H}{\Lambda} + \frac{\Lambda^F}{\Lambda} = 1. \tag{34}$$

Taken the sizes of economies (proxied by Pareto weights $\frac{\Lambda^H}{\Lambda}$, $\frac{\Lambda^F}{\Lambda}$) as given, the equilibrium of the economy is determined by the equation system of the resource constraints and FOCs. Because these equations are non-linear, we log-linearize the equation system to obtain approximate and tractable equilibrium consumption expressions for an intuitive analysis. We use lower cases, $q \equiv \log Q$, to denote log quantities throughout. Taking log of the FOCs relates country I's log consumptions with the log SDF m_T (associated with the traded consumption numeraire)

$$\lambda^I - \rho t - (\gamma - \epsilon)\left(\omega_T c^I_{Tt} + \omega_N \delta^I_{Nt}\right) - \epsilon c^I_{Tt} + \log \omega_T = m_{Tt}, \quad I \in \{H, F\} \tag{35}$$

$$\lambda^I - \rho(t+dt) - (\gamma-\epsilon)\left(\omega_T c^I_{Tt+dt}(s) + \omega_N \delta^I_{Nt+dt}(s)\right) - \epsilon c^I_{Tt+dt}(s) + \log \omega_T = m_{Tt+dt}(s), \quad \forall s, \;\; I \in \{H, F\}.$$

The log linearization of resource constraints produces,[21]

$$\frac{\Lambda^H}{\Lambda} c^H_{Tt} + \frac{\Lambda^F}{\Lambda} c^F_{Tt} = \frac{\Lambda^H}{\Lambda} \delta^H_{Tt} + \frac{\Lambda^F}{\Lambda} \delta^F_{Tt} \equiv \delta_{Tt},$$

$$\frac{\Lambda^H}{\Lambda} c^H_{Tt+dt}(s) + \frac{\Lambda^F}{\Lambda} c^F_{Tt+dt}(s) = \frac{\Lambda^H}{\Lambda} \delta^H_{Tt+dt}(s) + \frac{\Lambda^F}{\Lambda} \delta^F_{Tt+dt}(s) \equiv \delta_{Tt+dt}(s), \qquad \forall s, \tag{36}$$

where δ_{Tt} and $\delta_{Tt+dt}(s)$ defined in the equations above are loosely referred to as aggregate (log) traded endowments. By first computing and expressing c^I_{Tt}, $c^I_{Tt+dt}(s)$, $I \in \{H, F\}$, in terms of log SDFs m_{Tt}, $m_{Tt+dt}(s)$ from (35), and then substituting these expressions into (36) we obtain the solution (8) for the log SDFs m_{Tt}, $m_{Tt+dt}(s)$ associated with the traded consumption numeraire. In turn, this solution and the log FOCs (35) imply the solution (6) for the log traded consumption c^I_{Tt} and a similar solution for $c^I_{Tt+dt}(s)$ (by replacing the current time index t in (6) by the future time state indices $(t + dt, s)$).

The country-specific log SDFs (associated with country I's consumption basket numeraire) $m^I_t = \log \frac{\partial U^I_t}{\partial C^I_t}$ and $m^I_{t+dt}(s) = \log \frac{\partial U^I_{t+dt}}{\partial C^I_{t+dt}(s)}$ (8) then are obtained from substituting the log traded consumptions c^I_{Tt}, $c^I_{Tt+dt}(s)$ obtained above into country I's log-linearized consumption baskets $c^I_t \approx \omega_T c^I_{Tt} + \omega_N c^I_{Nt}$ and $c^I_{t+dt}(s) \approx \omega_T c^I_{Tt+dt}(s) + \omega_N c^I_{Nt+dt}(s)$, where $c^I_{Nt} = \delta^I_{Nt}$ and $c^I_{Nt+dt}(s) = \delta^I_{Nt+dt}(s)$.

A.2 Equilibrium Economic Sizes

We note that the traded consumption (6) and country-specific SDF (8) are functions of economic sizes because the Pareto weights are taken as given (Appendix A.1). However, Pareto weights are functions of model's primitive endowment inputs in equilibrium. Therefore equilibrium consumptions and SDFs can be expressed as functions of primitive endowments only by substituting and supplanting the Pareto weights. To derive the equilibrium economic sizes (or Pareto weights) from endowments, we first consider the budget constraints for individuals in country $I \in \{H, F\}$ (note that because nontraded consumptions equal nontraded endowments, they cancel out and drop from the budget constraints)

$$C^I_{Tt} + E_t \left[\frac{M_{Tt+dt}(s)}{M_{Tt}} C^I_{Tt+dt}(s) \right] = \Delta^I_{Tt} + E_t \left[\frac{M_{Tt+dt}(s)}{M_{Tt}} \Delta^I_{Tt+dt}(s) \right], \qquad I \in \{H, F\}. \tag{37}$$

[21] These equations arise from first rewriting resource constraints (33) as $\frac{\Lambda^H}{\Lambda}\left(\frac{N^H \Lambda}{\Lambda^H} C^H_{Tt}\right) + \frac{\Lambda^F}{\Lambda}\left(\frac{N^F \Lambda}{\Lambda^F} C^F_{Tt}\right) = \frac{\Lambda^H}{\Lambda}\left(\frac{N^H \Lambda}{\Lambda^H} \Delta^H_{Tt}\right) + \frac{\Lambda^F}{\Lambda}\left(\frac{N^F \Lambda}{\Lambda^F} \Delta^F_{Tt}\right)$, and then approximating the log and canceling common terms of both sides of this equation.

The log linearization of budget constraints produces,[22]

$$c^I_{Tt} + E_t\left[c^I_{Tt+dt}\right] = \delta^I_{Tt} + E_t\left[\delta^I_{Tt+dt}\right], \qquad I \in \{H, F\}.$$

Substituting the log traded consumptions c^H_{Tt} and $c^H_{Tt+dt}(s)$ obtained in Appendix A.1 into the log budget constraints above for country H yields ($dt = 1$ being one unit of time interval),

$$\begin{aligned}(\delta_{Tt} + E_t\left[\delta_{Tt+dt}(s)\right]) - \alpha(\gamma-\epsilon)\omega_N \frac{\Lambda^F}{\Lambda}\left(\delta^H_{Nt} + E_t\left[\delta^H_{Nt+dt}(s)\right] - \delta^F_{Nt} - E_t\left[\delta^F_{Nt+dt}(s)\right]\right) - \alpha\rho \\ = \delta^H_{Tt} + E_t\left[\delta^H_{Tt+dt}(s)\right] \qquad (38)\end{aligned}$$

Using the definition of δ_{Tt} (36) and the normalization (34), we have $\delta_{Tt} - \delta^H_{Tt} = \frac{\Lambda^H}{\Lambda}\delta^H_{Tt} + \frac{\Lambda^F}{\Lambda}\delta^F_{Tt} - \delta^H_{Tt} = \frac{\Lambda^F}{\Lambda}\left(\delta^F_{Tt} - \delta^H_{Tt}\right)$. Similarly, $\delta_{Tt+dt}(s) - \delta^H_{Tt+dt}(s) = \frac{\Lambda^F}{\Lambda}\left(\delta^F_{Tt+dt}(s) - \delta^H_{Tt+dt}(s)\right)$. Therefore, we can rewrite (38) as

$$\begin{aligned}\frac{\Lambda^F}{\Lambda}\left\{\left(\delta^F_{Tt} + E_t\left[\delta^F_{Tt+dt}(s)\right]\right) - \left(\delta^H_{Tt} + E_t\left[\delta^H_{Tt+dt}(s)\right]\right)\right\} \qquad (39) \\ + \frac{\Lambda^F}{\Lambda}\alpha(\gamma-\epsilon)\omega_N\left\{\left(\delta^F_{Nt} + E_t\left[\delta^F_{Nt+dt}(s)\right]\right) - \left(\delta^H_{Nt} + E_t\left[\delta^H_{Nt+dt}(s)\right]\right)\right\} = \alpha\rho.\end{aligned}$$

Let us introduce the country-specific expected growths (i.e., logs) of per-capita traded and non-traded endowments

$$g^I_{Nt} = \delta^I_{Nt} + E_t\left[\delta^I_{Nt+dt}(s)\right], \qquad g^I_{Tt} = \delta^I_{Tt} + E_t\left[\delta^I_{Tt+dt}(s)\right], \qquad I \in \{H, F\}, \qquad (40)$$

which have both a current component (i.e., endowments in place) and a future expected component (i.e., endowment growth potentials). We can now rewrite (39) as

$$\frac{\Lambda^F}{\Lambda}\left[\left(g^F_{Tt} - g^H_{Tt}\right) + \alpha(\gamma-\epsilon)\omega_N\left(g^F_{Nt} - g^H_{Nt}\right)\right] = \alpha\rho. \qquad (41)$$

A similar derivation yields

$$\frac{\Lambda^H}{\Lambda}\left[\left(g^H_{Tt} - g^F_{Tt}\right) + \alpha(\gamma-\epsilon)\omega_N\left(g^H_{Nt} - g^F_{Nt}\right)\right] = \alpha\rho. \qquad (42)$$

Solving the linear system of two equations (41) and (42) and two unknowns $\left\{\frac{\Lambda^H}{\Lambda}, \frac{\Lambda^F}{\Lambda}\right\}$ yields the solutions (26) for the Pareto weights (i.e., sizes) as endogenous functions of endowment inputs.

[22]This equation arises from approximating the log of both sides of the budget constraints, $c^I_{Tt} + E_t\left[m_{Tt+dt}(s) - m_{Tt} + c^I_{Tt+dt}(s)\right] \approx \delta^I_{Tt} + E_t\left[m_{Tt+dt}(s) - m_{Tt} + \delta^I_{Tt+dt}(s)\right]$, and canceling common terms.