Pricing Risks across Currency Denominations*

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Abstract

We document a novel empirical regularity that investors in low interest rate countries earn substantially higher Sharpe ratios on identical carry trade strategies than investors in high interest rate countries. We further document that bilateral exchange rate volatilities are increasing in interest rate differentials between currencies. These observations place new important restrictions on no-arbitrage models of international asset pricing. Our analysis naturally gives rise to a new non-parametric procedure to estimate country-specific stochastic discount factors (SDFs) from exchange rate data. In support of our approach, out-of-sample, the estimated SDFs sort linearly with national output gap fluctuations, and price risks in international equity markets.

JEL-Classification: F31, G15.

Keywords: Currency risks, carry trades, stochastic discount factor, principal components.

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1 Introduction

An investment strategy pays different Sharpe ratios across different currencies of denomination. We document a novel empirical pattern that the well-known currency carry trade strategy to borrow Japanese Yen and lend New Zealand dollars (-JPY/+NZD) pays a higher Sharpe ratio to investors in low than in high interest rate countries.[1] It is straightforward that a Japanese investor earns a higher Sharpe ratio than a New Zealand investor when borrowing JPY and lending NZD. Whenever the NZD appreciates against the JPY the -JPY/+NZD carry trade yields a positive payoff. It is positive measured in any currency, but the return is smaller when measured in NZD than in JPY because the NZD has appreciated against the JPY. On the other hand, when the NZD depreciates the strategy becomes a liability (negative payoff), and the return is more negative when measured in NZD than in JPY because the JPY has appreciated against the NZD. The risk is the same but the return denominated in NZD is always less than the return measured in JPY.[2] This argument is independent of interest rates in New Zealand or Japan; all that matters is that the carry trade involves only the two denomination currencies.

In contrast, without additional structural assumptions it is impossible to explain why higher Sharpe ratios are earned in low interest rate countries (other than Japan) compared to high interest rate countries (other than New Zealand). Moreover, our empirical regularity crucially differs from (and is certainly not implied by) the currency forward premium puzzle. That is, without additional assumptions, there is no theoretical link between the profitability of currency carry trades and our empirical cross-denomination pattern. We illustrate the independence of these two empirical facts in a simple numerical example in Appendix C.5. We provide an explicit example of a 2-factor model which is able to generate reasonable expected carry trade returns earned by a US investor but is inconsistent with our documented cross-denomination regularity.

[1]When we talk about returns to an investor in country I we refer to payoffs denominated in the currency of country I, assuming that the investor cares about the performance of his investment measured in his local currency. We observe the same pattern for other well-known currency carry trades, e.g. currency portfolios sorted based on interest rate differentials (details are in Appendix D Figure 10).

[2]Suppose at time t, one JPY buys E_t NZD, and at time T 1 JPY buys E_T NZD. Gross interest rates are R_J and R_N in JPY and NZD respectively. Borrowing 1 JPY and lending E_t NZD at time t pay $\frac{E_t}{E_T}R_N - R_J$ JPY or $E_tR_N - E_TR_J$ NZD at time T. The carry trade return measured in JPY is $CT^{(JPY)} = \frac{E_t}{E_T}R_N - R_J$ and measured in NZD $CT^{(NZD)} = R_N - \frac{E_T}{E_t}R_J$. Clearly, $CT^{(JPY)} > \frac{E_T}{E_t}CT^{(JPY)} = CT^{(NZD)} > 0$ if $E_t > E_T$ (NZD appreciates against JPY), and $0 > CT^{(JPY)} > \frac{E_T}{E_t}CT^{(JPY)} = CT^{(NZD)}$ if $\frac{R_N}{R_J}E_t < E_T$ (NZD sufficiently depreciates against JPY). Note that only if the NZD depreciates by a tiny amount against the JPY ($E_t < E_T < \frac{R_N}{R_J}E_t$), then $CT^{(NZD)} > CT^{(JPY)}$. However, this case is rather unlikely because exchange rates are sufficiently volatile such that exchange rate changes are often larger than interest rate differentials. Thus, on average $CT^{(JPY)} > CT^{(NZD)}$. Further analysis is given in Appendix C.5.

A special case is a single factor model, which is always consistent with our cross-denomination pattern. The intuition is that in a single factor model all high (low) interest rate countries are essentially identical to New Zealand (Japan). In contrast, a more general multi-factor model requires a specific factor structure to generate our empirical regularity. Accordingly, (i) a single factor is a sufficient but not necessary condition to explain the empirical cross-denomination pattern, and (ii) if we believe that there are multiple factors in FX markets, then the empirical regularity places new restrictions on the factor structure. A large body of research suggests that there are two or more important factors in FX markets (see the section on the related literature below). Moreover, our own empirical results suggest that there are at least two important factors in FX markets. Therefore, our cross-denomination pattern opens a new venue for more empirical research as well as introduces additional, not previously exploited restrictions to discipline structural international asset pricing models.

We further explore the economic mechanism underlying our empirical cross-denomination regularity. Interestingly, the difference in Sharpe ratios earned across currency denominations is closely linked to the covariation of exchange rate growths. Thus, our empirical regularity implies a strong empirical relationship between interest rates and exchange rate growth covariation. Indeed, we document that bilateral exchange rates between countries with small (large) interest rate differentials are relatively smooth (volatile). In turn, this is equivalent to restrictions on the factor structure.

We show that it is possible to map exchange rate fluctuations into pricing factors and we propose a simple procedure using principal component analysis (PCA) and linear regression analysis to construct country-specific SDFs.[3] We find that the first two principal components (PCs) are the most important factors to explain our cross-denomination pattern. Moreover, the same two components carry the largest market prices, and they are the only PCs that are able to explain a significant part of the time-series variation in international stock returns and the cross-sectional variation of average stock returns. Related to our finding the literature has found that a two factor model – dollar and carry factors – is able to price currency carry trade returns (Lustig et al., 2011; Verdelhan, 2015).[4] We show that our first PC is almost identical to the dollar factor. The second

[3]The procedure is similar to the 2-stage Fama-MacBeth regression (Fama and MacBeth, 1973), but in our approach we do not have to estimate time-series regressions in the first stage because of the particular relationship between market prices and exchange rate loadings on risk sources, i.e., the exchange rate is equal to the ratio of SDFs. This reduces estimation errors. Moreover, it ensures that our identified risk sources are always priced risks (in contrast to possibly unpriced risk sources as it is the case with other asset classes).

[4]The dollar factor borrows USD and equally lends in all other currencies, the carry factor borrows in low and lends in high interest rate currencies. The dollar factor explains most of the time-series variation in exchange rate growths, while carry explains most of the cross-section of average carry trade returns. Verdelhan (2015) further shows that

PC is positively correlated with the carry factor but the correlation is significantly less than 1 (between 65% and 74%). Moreover, Maurer et al. (2016) show that a profitable dynamic trading strategy based on our factors has a correlation of only about 40% with the carry factor and is basically uncorrelated to the dollar factor. Interestingly, while the literature shows that dollar and carry have very distinct roles (dollar explains the time-series and carry the cross-section in carry trade returns), we find that the first and the second PCs are both crucial and equally contribute to explain the cross-denomination pattern and the time-series and cross-section in international stock returns.

Our estimation of country-specific SDFs is non-parametric and only uses observable price data as inputs. This is in stark contrast to other methods that either require strong assumptions on preferences and wealth distributions, or need noisy, low frequency macroeconomic data as input. The idea of our approach is that in complete markets (or incomplete markets with completely disentangled risks), all risks in FX markets must be priced by at least one country's SDF. This is in contrast to risks in stock or other asset markets, part of which is idiosyncratic and not priced. FX markets are special because the exposure of an exchange rate to a risk source equals the difference in the two involved countries' prices of risk (i.e., this is because an exchange rate between two currencies is the ratio of the two countries' SDFs). Accordingly, any shock to an exchange rate must be a shock to at least one SDF, which makes FX markets an ideal setting to estimate market prices of risk.[5]

Finally, we look beyond price data into macroeconomic factors to understand which fundamentals of the economy influence the pricing in FX markets. We examine the volatility as well as the covariance of macroeconomic factors with innovations in our constructed SDFs. We find strong evidence that fluctuations in output gap are related to our identified FX market factors. Output gap volatilities and cross-country correlations exhibit significant positive relationships with volatilities and cross-country correlations of our constructed SDFs. Because output gap is an important macroeconomic indicator of business cycles, our findings lend support to the economic intuition that our FX market factors are intrinsically related to marginal utilities.

a factor sorting currencies according to their exposures to the dollar factor is important (in addition to the carry factor) to price currencies in the cross-section.

[5]There is an important limitation to our approach. While all exchange rate risks are priced, we cannot guarantee that all priced risk sources affect FX markets. Consider every country's SDF assigns an identical price to a risk source. The risk is clearly priced, but it is impossible to detect it in FX markets because there is no exchange rate with a non-zero loading on the risk source. Global and undetectable risks exist, and in our view are a crucial factor responsible for the observed "disconnection" between FX and stock markets, in the sense that a risk in the first market is not fully priced in the second, and vice versa.

Finally, Maurer et al. (2016) construct a dynamic trading strategy based on the SDF estimation approach described in this paper with the idea that a strategy which replicates the inverse of the SDF must earn the largest attainable Sharpe ratio. Indeed, the strategy earns a large Sharpe ratio out-of-sample and outperforms popular currency trading strategies, including the carry strategy suggested by Lustig and Verdelhan (2007), across various performance measures and sub-samples. For instance, between 1977-2016 for the set of developed countries the trading strategy earns a Sharpe ratio (after transaction costs) of 0.78 versus 0.56 for the carry strategy by Lustig and Verdelhan (2007).

Related Literature

The framework connecting moments of SDF growth to exchange rates is suggested in Backus et al. (2001). Lustig and Verdelhan (2007, 2011) and Burnside (2011, 2012) discuss the connection between carry trade returns and aggregate consumption growth (CCAPM) and other popular asset pricing factors, which are known to explain the cross-section of stock returns. Recently, a large literature has emerged introducing new currency risk factors: carry factor (Lustig et al., 2011), global volatility factor (Menkhoff et al., 2012a,b), global currency skewness factor (Rafferty, 2012), FX correlation risk factor (Mueller et al., 2013), dollar factor (Lustig et al., 2014; Verdelhan, 2015), downside beta risk factor (Dobrynskaya, 2014; Lettau et al., 2014; Galsband and Nitschka, 2013), FX liquidity risk factor (Mancini et al., 2013), economic size factor (Hassan, 2013), surplus-consumption risk factor (Riddiough, 2014). Habib and Stracca (2012); Cenedese (2012) and Dobrynskaya (2015) link some of these factors to macroeconomic conditions and explore what conditions are associated with "safe haven" properties of currencies. Building on the predictive power of currency volatility risk premia, Della Corte et al. (forthcoming) construct a profitable a carry trade strategy, which outperforms traditional carry trade strategies. Brusa et al. (2015) introduce an international CAPM model with one global equity factor and two currency factors, which does a better job pricing a broad set of international assets than traditional factor models. Daniel et al. (2014) shows that dollar-neutral carry trades and strategies with a dollar exposure are different and the aforementioned factors appear to explain only dollar-neutral returns. Hassan and Mano (2014) explain and quantify the difference between the forward premium puzzle and profitable carry trades. In log-normal setting, the profitable strategy of lending high interest rate (i.e., risky) currencies would imply that those currencies are associated with less volatile SDFs. Gavazzoni et al. (2013)

establish sufficient parametric (i.e., affine) conditions under which high interest rate currencies have more volatile SDFs.[6] Therefore, when such sufficient conditions hold, the log-normal risk setting of international finance may be insufficient to accommodate the observed cross-sectional pattern of currency returns. With respect to this finding, the current paper's setting and estimation approach are non-parametric and rely only on the stationarity of the diffusion-risk setting. Regarding the stationarity assumption, for robustness, we also perform estimations separately for periods of markets' recession and expansion.

Another literature employs and examines statistical approaches to build factors. Meese and Rogoff (1983) challenge structural models for exchange rates and show that these models are unable to outperform a simple random walk model. Bakshi and Panayotov (2013) show that time-series predictability of carry trades is significant for dynamic currency portfolios (while being absent in fixed currency pairs). Koedij and Schotman (1989) use PCA to build groups of currencies with similar characteristics and single out four leading currencies: US dollar (USD), Yen (JPY), Deutsche Mark (DM), British Pound (GBP). Similarly, Greenaway-McGrevy et al. (2012) show that the JPY/USD, Euro/USD and GBP/USD exchange rates capture most of the variation in 23 exchange rates. Engel et al. (2007) estimate a factor model which is able to predict exchange rates at long horizons in the sample after 1999 but not in earlier samples. Sarno et al. (2012) estimate an affine multi-currency model with four latent variables which explains exchange rate fluctuations. Dong (2006) estimates a VAR model and finds that inflation and output gap are important to exchange rate dynamics. Rapach and Wohar (2006) and Maasoumi and Bulut (2012) test several exchange rate factor models and conclude that it is hard to consistently outperform a simple random walk model.[7,8]

In contrast to the above literature, we focus on differentials of currency carry trade returns to different base currencies. We document a new and robust empirical fact that there are large return differentials across currency denominations on popular carry trades. This new empirical regularity imposes new constraints for existing asset pricing factor models. Moreover, we show that

[6] On the modeling side, within log-normal and affine setting, Gavazzoni et al. (2013) show that, if countries are driven by either (i) parametrically symmetric risk factors, or (ii) common risk factors, then SDF volatilities increase with interest rate volatilities. On the empirical side, they document that interest rate volatilities increase with interest rate levels. The combination of these features constitutes Gavazzoni et al. (2013)'s conclusion.

[7] See Maasoumi and Bulut (2012) for additional references on structural exchange rate models.

[8] Yet another literature uses option prices to quantify risks of currency crashes and peso events and explain carry trade returns (e.g. Brunnermeier et al. 2008; Burnside et al. 2011; Farhi et al. 2014; Chernov et al. 2013 and Jurek 2014; see Chernov et al. 2013 for a comprehensive literature review on exchange rate crash risks). We do not consider jump risks.

this empirical pattern is closely related to the covariance matrix of exchange rates. Based on this insight we construct factors to proxy for country specific SDFs. An advantage of our approach over other empirical factor models is that we are able to provide a clear theoretical set-up to interpret our identified risk sources. We also link our factors to macroeconomic variables and expected stock returns. Finally, we provide evidence that the first two PCs of exchange rate growths are the most important factors to explain our cross-denomination patter as well as the time-series and the cross-sectional variation in international stock returns.

Brandt et al. (2006) analyze the inconsistency between the strong correlation of countries' SDFs implied by modest exchange rate volatility and the weak correlation of SDFs implied by consumption data. We also connect this correlation to the return differentials that different countries earn on identical carry trade strategies.

Our paper is structured as follows. Section 2 presents two novel empirical results in FX markets to motivate the setup of a diffusion model to study exchange rate risks and their prices. Section 3 makes the case for the PCA on exchange rate data, and proposes a procedure to reconstruct stochastic discount factors from priced risks in FX markets. Section 4 investigates model implications and provides in-sample evidence for a diffusion model for exchange rates. Section 5 investigates additional model implications and provides out-of-sample evidence. Section 6 concludes. The appendices list details on data sources, provide derivations for all theoretical results in the paper, and contain additional empirical results.

2 Diffusion Risks in Foreign Exchange Markets

2.1 Cross-Denomination Pattern

The left panel of Figure 1 plots the Sharpe ratio to an investor in country J against the average interest rate differential between country J and the USA. We fix the carry trade strategy of borrowing JPY and lending NZD, typically the most profitable currency pair, while varying the denomination currency J among 11 developed economies. The graph shows a striking negative relationship with a correlation of -91%: investors in countries of lower interest rates tend to earn substantially higher Sharpe ratios on the same carry trade strategy.[9] The relationship is not only

[9]The correlation is significantly negative and close to one for both the samples 1999-2014 and 1984-2014. More sophisticated portfolio strategies (e.g., High minus Low interest rate carry trade portfolios studied by Lustig and Verdelhan (2007, 2011)) largely involve borrowing JPY and lending NZD. As a result we also observe similar negative

highly statistically significant but also economically large. Between 1999 and 2014, a Japanese investor has earned a Sharpe ratio of 53% which is about 50% larger than the Sharpe ratio of 36% of a New Zealand investor.[10]

While we emphasize our results for the era after the Euro's inception, we repeat our analysis using data from 1984 to 2014 to check for robustness.[11] In addition, we split our sample into two subsamples – good versus bad times – and repeat our tests for each subsample separately.[12] All our empirical findings are qualitatively the same whether we use the sample 19984-2014, 1999-2014, or only subsamples of good or bad times (Tables 5, E.2, E.3, E.7 and E.8).

To the best of our knowledge, we are the first to document this striking inverse relationship between return differentials across currency denominations on a carry trade strategy and associated interest rates. We emphasize that our new empirical cross-denomination finding (Sharpe ratios of currency carry trades are larger when denominated in lower interest rate currencies) crucially differs from (and is certainly not implied by) the well-known profitability of currency carry trades (borrowing low and lending high interest rate currencies). We illustrate the independence of these two empirical facts in a simple numerical example in Appendix C.5. In particular, we provide two versions of 2-factor models, which are both able to generate reasonable carry trade returns (when denominated in USD), but only one of the models is consistent with our cross-denomination pattern. Thus, this example demonstrates that our empirical cross-denomination pattern implies additional restrictions on the factor structure beyond the restrictions implied by the carry trade profitability pattern.

To place our observation in perspective, the right panel of Figure 1 plots the average returns to a US investor of borrowing USD and lending various currencies J against the interest rates in countries J. The relationship is well-known to be positive.[13] Results in Tables E.2 and E.3 in the Appendix suggest that our cross-denomination finding (left panel in Figure 1) is stronger and more robust than the well-known carry trade and interest rate relationship (right panel in Figure 1).

As we explain in section 4.1, the difference in expected returns earned across denomination currencies is linked to the covariation of country specific SDFs and exchange rates. Intuitively,

relationships for these profitable carry trade strategies (see Figure 10 in Appendix D).

[10]Tables 5, E.2, E.3, E.7 and E.8 provide additional empirical results including various robustness checks.

[11]We use German data to backfill data for the EU before 1999.

[12]Our results are essentially the same whether we define bad times according to NBER economic cycles or simply label the three biggest crises since 1984 as bad times, namely the Asian financial crisis, the dot-com bubble, and the financial crisis of 2008.

[13]See Tables 5, E.2 and E.3 for statistical tests corresponding to Figure 1.

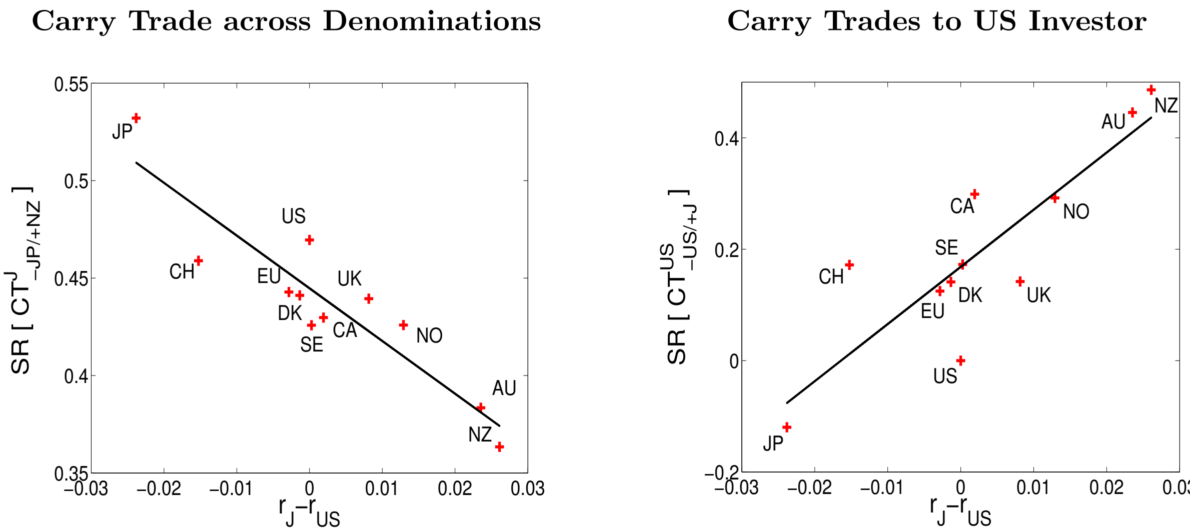

Figure 1: Left Panel: Negative relationship between the Sharpe ratio of the carry trade return of borrowing JPY and investing in NZD from the perspective of an investor in country J and the time series average of the interest rate differential between country J and the USA. Right Panel: Positive relationship between the Sharpe ratio of the carry trade return of borrowing USD and investing in currency J from the perspective of a US investor and the time series average of the interest rate differential between country J and the USA. Red crosses indicate data points and the black line is the best linear fit of the data according to an OLS estimation. Sample: 1999-2014. For statistical test of the illustrated relations see Tables 5, E.2, E.3, E.7 and E.8. We document similar results for High minus Low interest rate carry trade portfolios (Lustig and Verdelhan, 2007, 2011) (see10 in Appendix D). We describe carry trade return calculations in section 2.2.

our empirical regularity (left panel in Figure 1) suggests that countries with similar interest rates price risks similarly (market prices are similar), and thus, the SDFs of these countries should be highly correlated.[14] Consequently, since exchange rates are ratios of country specific SDFs, we expect to find relatively smooth exchange rates between countries with similar interest rates, and relatively volatile exchange rates between countries with large interest rate differentials.[15] Figure 2 documents this positive relationship between bilateral exchange rate growth volatilities and interest rate differentials. The correlation coefficient for the sample 1999-2014 is 70% and statistically significant. Similar results are obtained for the sample period 1984-2014 and the finding is robust to separating our sample into subsamples of good and bad times (Table E.4 in the Appendix).

[14]Technically, these SDFs might still have a low correlation if they assign very different market prices to some risk sources which are irrelevant for the -JPY/+NZD currency carry trade. But, we argue that profitable currency carry trades such as the -JPY/+NZD strategy strongly load on the most important risk sources (section 4.1).

[15]Using pairwise exchange rate growth volatilities to quantify comovements in country specific SDFs is arguably better than investigating covariances between exchange rate growths. This is because in latter case we need to choose a base currency (e.g. USD) against which all exchange rates are defined. But, a change in the base currency may lead to crucial changes in the exchange rate covariance matrix. In a sense, our measure is "base-free", and thus, more desirable.

Exchange Rate Volatilities and Interest Rates

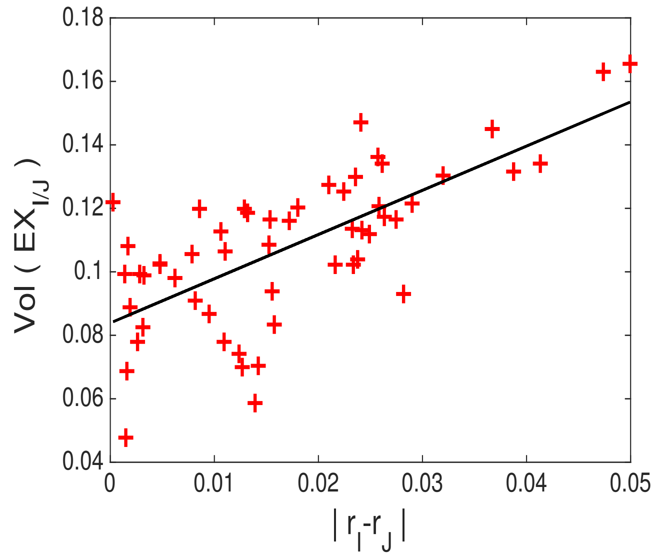

Figure 2: Positive relationship between volatility of exchange rate growth between country I and J and absolute difference in interest rates between countries I and J for every country pair (I, J). Red crosses indicate data points and the black line is the best linear fit of the data according to an OLS estimation. Sample: 1999-2014. For statistical test of the illustrated relations see Table E.4.

We are the first to explicitly document a strong comovement in SDFs of countries with similar interest rates. The main focus in the literature has always been on the relationship between interest rates and *average* rates of currency appreciation/ depreciation. Some papers have pointed out that low interest rate currencies (notably CHF and JPY, the safe-haven currencies) strongly appreciate in bad times (due to flight to quality). But, this evidence of comovements is limited to few currencies and particular economic conditions. In contrast, our analysis includes all available developed currencies and we confirm our findings not only in bad times but also in good times, i.e. our result is not driven by periods of market turmoil. We note that the relationship in Figure 2 is not perfect, which is the basis for the diversification benefit found in portfolio carry trade strategies (Lustig and Verdelhan, 2007, 2011).

2.2 Model Setup and Implications

We model $N + 1$ countries (accordingly, $N + 1$ currencies), with country (currency) index $I \in \{1, \ldots, N + 1\}$. We focus on the diffusion risks in international financial markets. We employ the standard filtered probability space $\{\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, \mathbf{P}\}$, wherein $\{\mathcal{F}_t\}_{t\geq 0}$ is the natural filtration associated with n-dimensional standard Brownian motion Z_t as diffusion risks in the market.

Without loss of generality, components of Z_t are modeled to be independent. Our specification assumptions for the diffusion model of FX market risks are: (A1) no-arbitrage, (A2) complete and frictionless financial markets, (A3) diffusion processes of exchange rates (A4) sufficient stationarity in the exchange rate processes (for the time windows of our study).

Assuming complete markets is only to simplify the exposition, i.e., country specific SDFs are unique and the exchange rate is equal to the ratio of SDFs. Our analysis remains largely unchanged if we assume incomplete markets because we work in a diffusion setting. Maurer and Tran (2015) show that in a pure-diffusion setting there exists a unique exchange rate equal to the ratio of projected SDFs. The assumption of pure-diffusion risks (or actually the weaker assumption that risks are completely disentangled) is, however, important because the introduction of entangled jump risks implies more complex exchange rate dynamics, which are decoupled from SDF dynamics (Maurer and Tran, 2015).

There exists a unique stochastic discount factor (SDF) M_I associated with country I.[16] That is, M_I prices risks from country I's perspective and generates prices in unit of currency I. The SDF follows a diffusion process (below, superscript T denotes the transpose operator),

$$\frac{dM_{I,t}}{M_{I,t}} = -r_{I,t}dt - \eta_{I,t}^T dZ_t. \tag{1}$$

The drift and volatility of SDF growth represents country I's instantaneously risk-free rate $r_{I,t} \in \mathbf{R}$ and the prices of n diffusion risks $\eta_{I,t} \in \mathbf{R}^n$ respectively. The country-specific characteristics of SDFs and their correlations are drivers of differences in pricing across countries. The ratio of SDFs is equal to the exchange rate between currencies by no-arbitrage,

$$EX_{J/I,t} = \frac{M_{I,t}}{M_{J,t}}. \tag{2}$$

Our convention is that, $EX_{J/I,t}$ units of currency J exchanges for one unit of currency I at time t. The dynamics of SDFs (1) then induces the movements in the exchange rate,

$$\frac{dEX_{J/I,t}}{EX_{J/I,t}} = \left[r_{J,t} - r_{I,t} + \eta_{J,t}^T\left(\eta_{J,t} - \eta_{I,t}\right)\right]dt + \left(\eta_{J,t}^T - \eta_{I,t}^T\right)dZ_t. \tag{3}$$

In particular, the exchange rate growth volatilities $\{\eta_{J,t} - \eta_{I,t}\}$ contain key information about differences in pricing of risks across countries as we explain below.

[16] Our convention for M_I is such that, $1 + \frac{dM_{I,t}}{M_{I,t}}$ is the growth in the SDF.

Realized Returns on Currency Trades

To set the notation, we first consider a typical carry trade strategy from the perspective of currency denomination I. At time t, investor I shorts an individual currency B (paying interest rate $r_{B,t}$) and uses the entire proceeds to fund his long position in an individual currency L (earning interest rate $r_{L,t}$). At $t+dt$, the investor liquidates his positions and converts the payoff to the denomination currency I. This is a net-zero investment because the investment is entirely funded by borrowing. The excess return $CT^I_{-B/+L,t+dt}$ (to be realized at $t + dt$) is,[17]

$$CT^I_{-B/+L,t+dt} = \eta^T_{I,t} \left(\eta_{B,t} - \eta_{L,t}\right) dt + \left(\eta^T_{B,t} - \eta^T_{L,t}\right) dZ_t, \tag{4}$$

with an expected excess return of $ECT^I_{-B/+L,t+dt} = \eta^T_{I,t} \left(\eta_{B,t} - \eta_{L,t}\right) dt$ and a Sharpe ratio of,

$$SCT^I_{-B/+L,t} = \frac{||\eta^T_{I,t} \left(\eta_{B,t} - \eta_{L,t}\right)||}{||\eta_{B,t} - \eta_{L,t}||} = ||\eta_{I,t}|| \cos \alpha^I_{-B/+L,t}, \tag{5}$$

where $\alpha^I_{-B/+L,t}$ is the angle between price-of-risk vectors $\eta_{I,t}$ and $(\eta_{B,t} - \eta_{L,t})$. Observe that $CT^I_{-B/+L,t+dt}$ (4) is a simple (i.e., level) realized return, and is related to the log realized returns $rx^I_{-I/+L,t+dt}$, $rx^I_{-I/+B,t+dt}$ often adopted in the literature on the underlying strategies by,[18]

$$CT^I_{-B/+L,t+dt} = \exp\left(rx^I_{-I/+L,t+dt}\right) - \exp\left(rx^I_{-I/+B,t+dt}\right).$$

As elaborated in Appendix C.1, it is the mean of simple return (but not the mean of the log return) that is fully informative about the risk-based compensation (i.e., premium) for a strategy.[19] As the current paper concerns primarily the pricing of carry trade risks from different currency denomination I's perspective, we adopt the simple return as the primary and appropriate convention for returns in our current study. Several further observations concerning return $CT^I_{-B/+L,t+dt}$ are in order.

[17] Detailed derivation is given in Appendix C.1. An analogy to the currency carry trade excess return is the net-zero strategy of issuing risk-free bonds to fund the investment in equity, which earns an (expected) excess return equal to the equity premium.

[18] $rx^I_{-I/+J,t+dt}$ denotes the log return (denominated in currency I) on a strategy of borrowing currency I and lending currency J. See details in Appendix C.1, equation (29).

[19] For a simple illustration, consider a (net) simple return $\frac{X_{t+dt}}{X_t} - 1 = e^{\mu dt + \sigma dZ_t}$ and a SDF $\frac{M_{t+dt}}{M_t} - 1 = -rdt - \eta dZ_t$ given exogenously. The no-arbitrage (Euler) pricing equation $E_t\left[\frac{M_{t+dt}}{M_t}\frac{X_{t+dt}}{X_t}\right] = 1$ implies the premium $\mu + \frac{\sigma^2}{2} - r = \eta\sigma$. Clearly, this premium is the mean of the (excess) simple return $E_t\left[\frac{X_{t+dt}}{X_t} - 1\right] - r$, but not the mean of the (excess) log return $E_t\left[\log\left(\frac{X_{t+dt}}{X_t}\right)\right] - r$. Therefore, indeed only the mean of simple return is fully informative about risk-based pricing of asset X_t.

First, the innovations in the realized excess return (represented by term dZ_t) in (4) are intrinsic to the investment strategy in the sense that they depend only on the funding and investment currencies (B and L, respectively), but not the currency of denomination. No matter in which currency I the investors denominate this carry trade's return, they face identical diffusion risks. Intuitively, this is because in a net-zero investment strategy, long and short positions in the currency of denomination necessarily cancel out, leaving only the diffusion shocks priced by intrinsic SDFs M_B and M_L of the strategy.

Second, these identical innovations are priced differently in different currencies of denomination. This observation is manifested in the presence of the prices of risk η_I in the expected excess return (represented by term dt in (4)). Specifically, in (4), a positive $\eta_{I,t}^T \eta_{B,t}$ contributes to higher expected excess return, while a positive $\eta_{I,t}^T \eta_{L,t}$ has the opposite effect. Intuitively, this is because when the SDF M_I positively correlates with M_B, currency B tends to appreciate (higher M_B) when I's marginal utility increases (higher M_I), and vice versa. As investor I is short currency B in this carry trade, the strategy is risky to I,[20] and offers a compensating expected return in form of a positive component $\eta_{I,t}^T \eta_{B,t}$.[21] Conversely, when the SDF M_I positively correlates with M_L, currency L tends to appreciate (higher M_L) when I's marginal utility increases (higher M_I). As investor I is long currency L in this carry trade, the strategy is a hedge to I, and its expected return is lowered by a negative term $\eta_{I,t}^T \eta_{L,t}$.

Finally, the portfolio strategy of borrowing currencies in set $\mathcal{B}$ and lending currencies in set $\mathcal{L}$ offers the following realized excess return denominated in currency I,[22]

$$CT^I_{-\mathcal{B}/+\mathcal{L},t+dt} = \eta_{I,t}^T \left(\breve{\eta}_{B,t} - \breve{\eta}_{L,t}\right) dt + \left(\breve{\eta}_{B,t}^T - \breve{\eta}_{L,t}^T\right) dZ_t, \tag{6}$$

and the associated Sharpe ratio,

$$SCT^I_{-\mathcal{B}/+\mathcal{L},t} = ||\eta_{I,t}|| \cos \alpha^I_{-\mathcal{B}/+\mathcal{L},t}, \tag{7}$$

where $\alpha^I_{-\mathcal{B}/+\mathcal{L},t}$ is the angle between price-of-risk vectors $\eta_{I,t}$ and $\breve{\eta}_{B,t} - \breve{\eta}_{L,t}$. When Sharpe ratio

[20] That is, investor I tends to lose in this carry trade when he highly values the payoff induced by a high marginal utility.

[21] Conversely, when M_I negatively correlates with M_B, $\eta_{I,t}^T \eta_{B,t}$ is negative and lowers the carry trade's expected return because the strategy then is a hedge to investor I.

[22] In expression (6), θ's denote investment weights; $\sum_{B \in \mathcal{B}} \theta_{B,t} = 1$, $\sum_{L \in \mathcal{L}} \theta_{L,t} = 1$, whereas $\breve{\eta}_{B,t} \equiv \sum_{B \in \mathcal{B}} \theta_{B,t} \eta_{B,t}$, and $\breve{\eta}_{L,t} \equiv \sum_{L \in \mathcal{L}} \theta_{L,t} \eta_{L,t}$ denote the weighted averages of prices of risks associated with the short and long portfolios, respectively. Further details are in Appendix C.1.

is used as a metric for investment performance, the gain from diversification in currency-portfolio trades stems from the flexibility to minimize the angle $\cos\alpha^I_{-\mathcal{B}/+\mathcal{L},t}$ in (7) (compared to $\cos\alpha^I_{-B/+L,t}$ in (5)), when more currencies are at investors' disposal.

Diffusion-Invariant Portfolios

We refer to the class of investments whose realized excess returns exhibit identical diffusion innovations in all currencies of denomination as *diffusion-invariant strategies.* Diffusion-invariant strategies thus offer an ideal setting to examine differences in pricing of same risks by different economic agents in international financial markets. This intuitive link between a strategy's net-zero investment and its diffusion invariance is noted in Brandt et al. (2006), and is formalized to utmost generality as follows (with derivation being relegated to Appendix C.2).

Proposition 1 *Assuming no-arbitrage, a strategy is diffusion-invariant if and only if it is a net-zero investment.*

This result renders the class of diffusion-invariant strategies ubiquitous for empirical studies. Intuitively, by no-arbitrage, a net-zero investment remains a net-zero strategy in any currency of denomination. Thus, when the denomination changes from a currency to another, the realized excess return is simply scaled by the exchange rate of the two currencies.[23] Because this is an excess return, innovations in realized exchange rates necessarily cancel in the long and short positions of the net-zero strategy. In other words, the scaling by the exchange rate does not alter the innovations to the excess return.

We use two empirical tests to quantify how well our diffusion-invariance property holds in the data. First, we estimate correlations between daily carry trade returns $CT^{US}_{-US/+J}$ and $CT^{I}_{-US/+J}$, $\forall I, J$. For our sample of 11 developed countries and the time horizon 1999-2014 all correlation estimates are larger than 99.96%. The result for the sample 1984-2014 is almost identical; all estimates are larger than 99.96%. Second, we estimate the absolute difference between the annualized standard deviations of $CT^{US}_{-US/+J}$ and $CT^{I}_{-US/+J}$, $\forall I, J$. For the sample 1999-2014 the absolute difference is always less than 0.07%, which is negligible in comparison to the annual standard deviation of $CT^{US}_{-US/+J}$, which is about 10%, $\forall J$. Again, the result for the longer sample 1984-2014 is

[23] This is a property intrinsic to net-zero investments. When investments require non-zero initial capitals, it is the realized gross payoffs, not the realized returns, that are scaled by the exchange rate under a change in currency denomination.

almost identical; the absolute difference is always less than 0.06%. We take these empirical findings as strong evidence in favor of the diffusion-invariance property when we work with daily exchange rate data.

Being net-zero investments, all standard carry trade strategies considered in the literature belong to the diffusion-invariant class of investments. But Proposition 1 covers many more, within and beyond FX markets.[24]

3 Principal Component Analysis

In this section we first discuss the working of PCA on FX market data. We then propose a procedure to construct a SDF that prices exchange rate risks. We begin with an important observation that diffusion risks in carry trades (4) are precisely the volatilities in the exchange rates (3). So to study the former risks, we apply PCA apparatus to quantify the latter volatilities. PCA helps us to identify and sort, in the order of statistical importance, all the principal risk factors that drive fluctuations in the exchange rate data.[25] We briefly present our key methodological steps below, and relegate full details to Appendix C.3.

3.1 Identifying Principal Risks in Foreign Exchange Markets

For the concreteness of the analysis, we examine all N exchange rates $\{EX_{J/I}\}$, $J \in \{1, \ldots N\}$, with respect to a single base currency I (in practice, most commonly, this base currency is USD).[26] We first arrange N *demeaned* time series $X_{J/I}$ of exchange rate growths (3) into N columns of a matrix $X = [X_{1/I}; \ldots; X_{N/I}]$ (see (31)). The PCA on the exchange volatilities then centers around diagonalizing the $N \times N$ symmetric matrix $X^T X$ by an orthogonal matrix W, such that,

$$W^T [X^T X] W = \text{Diag}\,[\lambda_1; \ldots; \lambda_N] \,. \tag{8}$$

Without loss of generality, the eigenvalues are arranged in descending order, $\lambda_1 \geq \ldots \geq \lambda_N$. Let $\boldsymbol{\eta}$ be a $n \times N$ matrix $\boldsymbol{\eta} = \left[\eta_{1/I}; \ldots; \eta_{N/I}\right]$, where $\eta_{J/I} \equiv (\eta_J - \eta_I) \in \mathbf{R}^n$, $\forall J \in \{1, N\}$ (31). We then

[24] For instance, net-zero investments in international equity markets can potentially serve as test assets to elucidate the differentials in the prices of risks across countries whose business cycles are non-synchronous, see equation (26).

[25] For the application of the analysis, we assume sufficient stationarity in FX markets for the time windows of our study, and assess the model's implications afterwards, using both in-sample and out-of-sample data.

[26] The explicit reference currency I shows up throughout our exposition, while the specific choice of I has no material effects on the results obtained in the paper.

define N transformed differential prices of risks, $\overline{\boldsymbol{\eta}} = \left[\overline{\eta}_{1/I}, \ldots, \overline{\eta}_{N/I}\right]$ as follows,

$$\overline{\boldsymbol{\eta}} \equiv \boldsymbol{\eta} W \Longleftrightarrow \boldsymbol{\eta} = \overline{\boldsymbol{\eta}} W^T. \tag{9}$$

The columns of this matrix,

$$\overline{\eta}_{J/I} = \sum_{K=1}^{N} W_{K,J} \eta_{K/I}; \qquad \eta_{J/I} = \sum_{K=1}^{N} W_{J,K} \overline{\eta}_{K/I}; \qquad J \in \{1, \ldots, N\}, \tag{10}$$

are pair-wise orthogonal (see (32)),

$$\overline{\boldsymbol{\eta}}^T \overline{\boldsymbol{\eta}} = \frac{1}{s \times dt} \mathrm{Diag}\left[\lambda_1; \ldots; \lambda_N\right]. \tag{11}$$

Principal risks and notations: following PCA literature and to sum up, columns of the transformed $s \times N$ matrix $\Pi \equiv XW$ are referred to as *principal components* (PCs). Elements of each PC are commonly referred to as *scores*, and elements $\{W_{1,J}, \ldots, W_{N,J}\}$ of J-th column of matrix W as the *loadings* on J-th PC. Conventionally, we also refer to J-th transformed vector $\overline{\eta}_{J/I} \in \mathbf{R}^n$ as J-th *principal price-of-risk vector* (PPoR), and innovations carried by J-th PC $\Pi_{t,J}$ as J-th *principal risk* in FX markets (see (12) below).

Combining (30) and (10) yields a concise expression for the scores in terms of the PPoRs and fundamental shocks in FX markets,

$$\Pi_{t,J} = \sum_{K} X_{t,K} W_{K,J} = \left(\sum_{K} W_{K,J} \eta_{K/I}^T \right) dZ_t = \overline{\eta}_{J/I}^T dZ_t, \quad \forall t, J. \tag{12}$$

Equation (10) in the PCA indicates that each $n \times 1$ PPoR vector $\overline{\eta}_{J/I}$ is a linear combination of all original price-of-risk vectors $\eta_{K/I}$, with the weights being the respective loadings $\{W_{K,J}\}_{K=1}^{N}$. We will employ these key relationships between the loadings W's, the scores Π's, and the eigenvalues λ's to construct portfolios mimicking the principal risks in FX markets.

Scores associated with J-th PC represent a fraction $\frac{\lambda_J}{\sum_{K=1}^{N} \lambda_K}$ of the fluctuation in the exchange rate data (Appendix C.3). We earlier adopted the convention to arrange the eigenvalues λ's in descending order, clearly the first PPoR vector $\overline{\eta}_{1/I} \in \mathbf{R}^n$ captures more priced exchange rate risks than the second PPoR vector $\overline{\eta}_{2/I} \in \mathbf{R}^n$, and so on. Although vectors $\overline{\eta}_{J/I}$ are unobservable, we do observe $N \times N$ loading matrix W, the $s \times N$ score matrix Π, and N eigenvalues λ's via the diagonalization process of the observable covariance matrix $X^T X$ of the exchange rate growths.

The return differential on the same net-zero carry trade strategy of borrowing currency B and lending currency L in two currencies of denomination J and H reads (see (33) in Appendix C.3),

$$ECT^J_{-B/+L,t} - ECT^H_{-B/+L,t} = \frac{1}{s \times dt} \sum_K^N \lambda_K \left(W_{J,K} - W_{H,K}\right)\left(W_{B,K} - W_{L,K}\right). \tag{13}$$

Section 4 below presents detailed empirical analysis of this pricing equation. For now, we note that the knowledge of the loadings $\{W_{J,K}\}$ and $\{\lambda_J\}$ can fully reproduce the differential returns. The contribution of K-th PC is weighted by λ_K, so that when the eigenvalue sequence λ's exhibit a clear hierarchy, the first few components replicate most of the differential returns. In this circumstance, the risks in FX markets are captured well by the first few PCs, while each of them features a special risk factor of interest. We then just need to construct the currency portfolios, whose returns mimic the PCs, to visualize and study these risk factors. We relegate details to Appendix C.4.

3.2 Constructing Prices of Risks from FX Data

The PCA on the FX market data concerns the diagonalization of the covariance matrix of exchange rates. Consequently, the PCA restructures the fluctuations in exchange rates into N orthogonal and quantifiable risk factors in a space spanned by the price-of-risk differentials $\{\eta_{J/I} \equiv \eta_J - \eta_I\}$. Naturally, we also want to quantify the risks spanned by the original price-of-risk vectors $\{\eta_I\}$. To this end, we first note that in complete financial markets, the set of prices of risk $\{\eta_I\}$ is unique, and each η_I faithfully captures all risks priced by the respective country I, for all I. Any residual risk coherent in η_I but not priced in the carry trade returns (4) must satisfy *both* of the following conditions. First, the residual risks have identical prices in all countries, so that they are canceled and do not affect in the exchange rate volatility risk (i.e., diffusion term dZ_t in (4)). Second, the residual risks are orthogonal to the risks revealed in the exchange rate volatilities, so that the former do not affect the carry trades' expected returns (i.e., drift term dt in (4)). In other words, the residual risks are orthogonal to each and every PC $\overline{\eta}_{J/I}$. We formalize the rationals of these conditions next.

Let η_I^* denote country I's market prices of the residual risks. The full market prices of risks

$\eta_I + \eta_I^*$ generate identical carry trade returns, and thus is indistinguishable from η_I, if and only if,

$$\begin{cases} \eta_I^* = \eta_J^* \equiv \eta^*, & \forall I \in \{1, \ldots, N+1\}, \\ & \\ (\eta^*)^T (\eta_J - \eta_I) = 0, & \forall I, J \in \{1, \ldots, N+1\}. \end{cases} \tag{14}$$

Without loss of generality, the full prices of risks of any country I can be decomposed into two orthogonal components: a country-specific component η_I that is fully revealed by FX market data, and a global component η^* that is identical for all countries,[27]

$$\eta_I^{\text{Full}} = \eta_I + \eta^*, \quad \forall I \in \{1, \ldots, N\}. \tag{15}$$

Our focus remains on the risks affecting FX markets. We now employ a regression-based approach to construct the prices of risks $\{\eta_I\}$ revealed by the exchange rate fluctuations for every country I. That is, we look for an explicit representation of $\{\eta_I\}$ as a linear combination of the PPoR of the form,

$$\widehat{\eta}_I = \sum_{K=1}^{G} \widehat{\beta}_K^I \overline{\eta}_{K/I}; \qquad G \leq N, \tag{16}$$

where G is the number of PCs we want to retain in the construction of η_I, and the solution for coefficients $\widehat{\beta}$'s is given below in (20). Hence, $\widehat{\eta}_I$ is the image of the original price-of-risk vector η_I projected onto the risk space spanned by the first G PCs. In the linear regression picture associated with this projection, $\widehat{\beta}$'s are the slope coefficients.

When $G = N$, the above representation is exact, $\widehat{\eta}_I = \eta_I$, since by design (15), η_I is in the space of the exchange rate risks spanned by all PPoRs $\{\overline{\eta}_{K/I}\}$. When the spectrum of eigenvalues $\{\lambda\}$ exhibits a significant hierarchy, one may opt for $G < N$. In this circumstance, the representation (16) is a founded approximation. The representation (16) for a single country I suffices to immediately yield the representation $\widehat{\eta}_J$, linear in $\{\overline{\eta}_{K/I}\}$, for any other country J,

$$\widehat{\eta}_J = \widehat{\eta}_I + (\widehat{\eta}_J - \widehat{\eta}_I) = \widehat{\eta}_I + \eta_{J/I} = \widehat{\eta}_I + \sum_{K=1}^{G} W_{J,K} \overline{\eta}_{K/I} \equiv \sum_{K=1}^{G} \widehat{\beta}_K^J \overline{\eta}_{K/I}, \tag{17}$$

[27] Technically, we need to augment the dimension of the diffusion space to accommodate the global risks (associated with η^*) that are not priced in FX markets,

$$\mathbf{R}^{n+m} \ni \left(\eta_I^{\text{Full}}\right)^T = \left[\eta_I^T; (\eta^*)^T\right] = [\underbrace{\eta_{I,1}, \ldots \eta_{I,n}}_{\in \mathbf{R}^n}; \underbrace{\eta_{n+1}^*, \ldots, \eta_{n+m}^*}_{\in \mathbf{R}^m}].$$

Technical details can be found in Appendix C.

$$\text{with:}\qquad \widehat{\beta}^J_K = \widehat{\beta}^I_K + W_{J,K},\quad \forall J \in \{1,\ldots,N\};\, K \in \{1,\ldots,G\}.$$

where we have used representations (10) and (16) to arrive at the expression for $\widehat{\beta}^J_K$. Equation (17) shows that country J's price of K's principal exchange risk differs from that of the reference country I by a the loading $W_{J,K}$. Intuitively, $W_{J,K}$ represents the loading of country J to K-th principal price of risks $\overline{\eta}_{K/I}$ when I is the reference currency,[28] i.e., on top of I's loading on that risk. Overall J's loadings on K-th PPoR must be the sum of these two components.

Equation (17) holds at the precision order of G PCs retained in the approximation, for any G of choice.[29] Consequently, using the orthogonality of PPoRs $\overline{\eta}_{J/I}$ (10), the variance of a country's SDF growth *implied* by the priced exchange rate risks is,

$$||\widehat{\eta}_J||^2 = \frac{1}{s\times dt}\sum_K^G \lambda_K \left(\widehat{\beta}^J_K\right)^2 = ||\widehat{\eta}_I||^2 + \frac{1}{s\times dt}\sum_K^G \lambda_K W_{J,K}\left(2\widehat{\beta}^I_K + W_{J,K}\right),\quad \forall J, \tag{18}$$

where the terms of order W^2 capture the fraction of the exchange rate volatility contribution to the volatility of J's SDF growth, and the terms of order $W\widehat{\beta}$ capture the fraction of the expected carry trade return's contribution.[30]

To obtain $\widehat{\beta}$'s in (16), we project the carry trade's mean return $ECT^I_{-I/+J}$ in (4) into the space spanned by the first G PCs, using (16) and the orthogonality (10),

$$ECT^I_{-I/+J} = \eta_I^T\left(\eta_I - \eta_J\right) = -\eta_I^T\eta_{J/I} = -\frac{1}{s\times dt}\sum_{K=1}^G W_{J,K}\lambda_K\widehat{\beta}^I_K. \tag{19}$$

By defining $N\times G$ matrix $V \equiv -\frac{1}{s\times dt}\widehat{W}\,\mathrm{Diag}\left[\lambda_1,\ldots,\lambda_F\right]$ we can assemble[31] all N such equations (for $J\in\{1,\ldots,N\}$) into a system of N linear constraints on G unknowns $\{\widehat{\beta}^I_K\}$ ($K\in\{1,\ldots,G\}$). The solution then can be obtained by the standard procedure of least-square fitting,

$$\widehat{\beta}^I = \left(V^TV\right)^{-1}V^T ECT^I_{-I}, \tag{20}$$

where $\widehat{\beta}^I$ denotes the $G\times 1$ vector of coefficients $\left[\widehat{\beta}^I_1,\ldots,\widehat{\beta}^I_G\right]^T$, and ECT^I_{-I} denotes the $N\times$

[28]Exchange rate data X_t fed into the PCA are exchange rates with respect to currency I. Then the K-th PPoR $\overline{\eta}_{K/I}$, associated with K-th PC, is a linear combination of $\{\eta_{J/I}\}$, see (10): $\overline{\eta}_{K/I} = \sum_J W_{J,K}\eta_{J/I}$.

[29]By construction, each and every $\widehat{\eta}_J$ is in the space spanned by the PPoRs $\{\overline{\eta}_{K/I}\}$, so $\widehat{\eta}_J - \widehat{\eta}_I$ is also in this space. Thus, $\widehat{\eta}_J - \widehat{\eta}_I = \overline{\eta}_J - \overline{\eta}_I \equiv \overline{\eta}_{J/I}$.

[30]This stems from the identity $||\eta_J||^2 - ||\eta_I||^2 = ||\eta_I - \eta_J||^2 + 2\eta_I^T\left(\eta_J - \eta_I\right)$. The use of PCA approach is that we can capture the prices of risk η_J to an order of approximation of choice.

[31]Where $N\times G$ matrix $\widehat{W}$ retains the first G columns of the full $N\times N$ loading matrix W. See (34) for further properties of matrix V.

1 vector of mean carry trade excess returns $\left[ECT^{I}_{-I/+1}, \ldots, ECT^{I}_{-I/+N}\right]^T$ resulted from N net-zero strategies of borrowing I, lending respectively one of the other N currencies, with proceeds denominated in currency I.[32] Further details can be found in equation (35) in Appendix C.

Finally, we can also construct the portion of the SDF growth $\frac{dM_J}{M_J}$ for any country J, that is sensitive to, and thus prices, the risks in FX markets. Combining (1) with (17) yields the *time series* of SDF growth,

$$\frac{dM_J}{M_J} = -r_{J,t}dt - \sum_{K=1}^{G} \widehat{\beta}^J_K \overline{\eta}^T_{K/I} dZ_t = -r_J dt - \sum_{K=1}^{G} \widehat{\beta}^J_K \Pi_{t,K}, \quad \forall J, \tag{21}$$

where, for each K, $\Pi_{t,K}$ is the observable time-series of scores (12) associated with K-th PC. Clearly, equation (21) indicates that innovations in the SDF growth (the part that correlates with risks in FX markets) can be observed and constructed from the PCA's scores Π and our regression procedure to determine $\widehat{\beta}$'s, for every country J.

4 Model Implied Results

We apply the methodology introduced above to the data to estimate a diffusion model and present in- and out-of-sample evidence to examine the validity and practicability of our new approach. We show that our estimated model is able to explain many interesting empirical patterns in the data. Moreover, we show that the first two PCs are the the most important factors.

We use daily exchange rates between 11 developed countries: Australia, Canada, Denmark, Eurozone[33], Japan, New Zealand, Norway, Sweden, Switzerland, UK and USA. We define all exchange rates against the USD, and our data sample consists of $N = 10$ exchange rate return time series. We run our analysis for two data samples: era after the Euro was introduced (1999-2014), and a longer sample of 1984-2014. Since our PCA requires a complete exchange rate time series for each country and the number of countries should not change through time, we use German data to proxy for Euro data before 1999. We limit our focus to the developed world because data for emerging countries is less reliable and more noisy. More details about the data can be found in the Appendix.

[32] In the implementation of the estimation (20) in later sections, to proxy for $ECT^{I}_{-I/+J}$'s, we use the average of past realized carry trade returns.

[33] For simplicity we refer to the Eurozone simply as Euro or EU, although not all countries in the EU use the Euro.

4.1 Principal Component Analysis

We construct matrix X (equation (30)) by demeaning daily exchange rate returns. We compute the 10×10 matrix of eigenvectors W (equation (8)), the respective eigenvalues $\lambda_1 \geq ... \geq \lambda_{10}$ (equation (8)), and the PCs Π (equation (12)). For the data sample 1999-2014 (1984-2014), we find that the first PC explains 60% (56%) and the second component captures 14% (17%) of the variation in exchange rate changes in the developed world.

We estimate correlations to illustrate the relationship of our first two PCs to the dollar and carry factors.[34] For the post Euro sample (1999-2014) we find that the first PC has a correlation of 99.5% with the dollar factor and a correlation of 49.3% with carry. The correlations between the second PC and dollar and carry are -3.7% and 73.8%, respectively. Similarly, for the longer sample (1983-2014) we find again that the first PC has a correlation of 98.8% with dollar and 16.8% with carry. The second PC has a correlation of -5.4% with dollar and 64.7% with carry. This is similar to the findings by Lustig et al. (2011) who show that dollar and carry are almost identical to the first two PCs on five interest rate sorted portfolios. Notice though that the correlation between our second PC and carry is significantly less than 1. This is due to the fact that we perform PCA on all individual exchange rate growths (defined against USD), while Lustig et al. (2011) perform PCA on five interest rate sorted portfolios. Nevertheless, the first (second) PC captures most of the variation in the dollar (carry) factor.

Figure 3 illustrates FX risks a currency trader is exposed to and explains why investors based in different countries do not earn identical returns on the same carry trade strategy. The figure is constructed using data from 1999 to 2014.[35] We plot $W_{J,1}\sqrt{\lambda_1}$ on the vertical axis against $W_{J,2}\sqrt{\lambda_2}$ on the horizontal axis for each country J. Remember that loading $W_{J,K}$ captures the importance of exchange rate returns J/USD for the PC K. By definition the USA is located at the origin because our input data in the PCA are exchange rates against the USD. The black dotted line labeled by '**a**' connects Japan with New Zealand. The other black dotted line labeled with '**b**' is orthogonal to line '**a**' and runs through the origin.

The plot illustrates the risks an investor faces in currency markets.[36] For instance, borrowing money in Switzerland and lending in either Australia or New Zealand will strongly load on the first

[34] The dollar factor borrows USD and equally lends in all other currencies, the carry factor borrows in low and lends in high interest rate currencies.

[35] Similar results are obtained for the sample 1984-2014.

[36] Though, the figure only considers risks captured by the first two PCs, as documented in the previous paragraphs these two components contain an important fraction of the total risk in exchange rate returns.

First and Second Principal Components

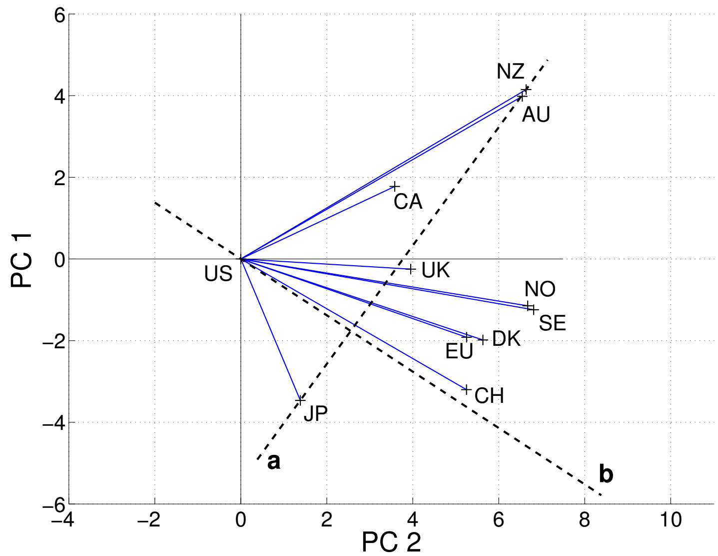

Figure 3: Plot of $W_{J,1}\sqrt{\lambda_1}$ on the vertical axis against $W_{J,2}\sqrt{\lambda_2}$ on the horizontal axis for each country J. Solid blue lines connect the plotted points to the origin. Black dotted line labeled by '**a**' connects JP with NZ and line labeled with '**b**' is orthogonal to '**a**' and runs through the origin.

PC and have little exposure to the second component. In contrast, to get an exposure to mostly the second PC an investor may borrow in Japan and lend in Switzerland. Accordingly, the profitable carry trade strategy of borrowing in Japan and lending in New Zealand (or Australia) comes with a large exposure to both the first and the second PC.

Figure 3 further provides an intuitive graphical representation of the cross-country relationship of expected carry trade returns in equation (13). Consider the carry trade strategy of borrowing JPY and lending NZD. Equation (13) states that an investor in country J earns a larger expected return than the US investor if and only if the angle between the vector from NZ to JP (on line '**a**') and the vector from the origin (US) to J is less than 90°. In Figure 3, these are countries plotted on the lower left side of the black dotted line '**b**'. The only country satisfying this condition is Japan. Indeed, the Japanese investor is the only one who has earned a higher average annual return (8.8%) than the US investor (7.8%) on the carry trade −JPY/+NZD from 1999-2014 (see Table E.6, Appendix E). Following this argument we can create a ranking of $E\left[CT^{J}_{-JP/+NZ}\right]$ for all countries J located on the upper right side of the black dotted line '**b**' according to the (shortest) distance between line '**b**' and country J. In particular, $E\left[CT^{J}_{-JP/+NZ}\right] > E\left[CT^{I}_{-JP/+NZ}\right]$ if and only if I lies farther apart from line '**b**' than J. This ranking is not perfect because Figure 3 only considers the first two PCs. Nevertheless, Figure 3 captures most of the cross-country pattern of $E\left[CT^{J}_{-JP/+NZ}\right]$ observed in the data (see Table E.6, Appendix E).

It is important to notice that we need a combination of both the first and the second PC to explain the cross-denomination pattern of $E\left[CT^{J}_{-JP/+NZ}\right]$. That is, if we only focus on the first PC, we would conclude that all denomination countries except for AU, CA and NZ earn a larger average carry trade return than the US. This is clearly not true in the data (Table E.6, Appendix E). Similarly, if we only consider the second PC, we would conclude that the US earns the largest average carry trade return and most other countries (i.e., CH, DK, EU, NO, SE) earn the same average carry trade return as AU and NZ. Again, this is clearly different than what we observe in the data (Table E.6, Appendix E). Thus, only a combination of both PCs is able to explain the entire pattern in the data. This is in contrast to the carry trade literature which assigns separate rolls to the dollar and carry factors: dollar explains the time-series variation in exchange rate growths, while carry explains the cross-section of average carry trade returns.

4.2 Estimation of SDF

We use the regression design proposed in equation (19) to estimate $\widehat{\beta}^{I}_{K}$ and construct country J's SDF according to equation (21). Figure 4 plots the time series for all 11 countries' SDFs. There is a strong co-movement of the SDFs across all countries. In recent years the spread between the SDF in the USA and the SDFs in basically all European countries except for the UK has widened a little. We estimate correlations of daily changes in the SDFs between any country pair in our sample and find that all estimates are above 97%. The observation of highly correlated SDFs is consistent with the finding of Brandt et al. (2006), who conclude that the correlation between the SDFs in the USA and UK has to be close to one to match the smooth exchange rate process in the data.[37]

Figure 4 suggests that the SDF increased during the burst of the dotcom bubble in the early 2000s, and there was a sharp increase in late 2008 in the days after Lehman Brothers collapsed. The widening in the gap between the SDF in the USA and the SDFs in the European countries in recent years may be attributed to the sovereign debt crisis in Europe and Europe's slow recovery relative to the USA. Although we do not have a formal test to analyze these events and the time

[37]We do not take a stance on how SDFs are related to macroeconomic fundamentals and do not address the international correlation puzzle.

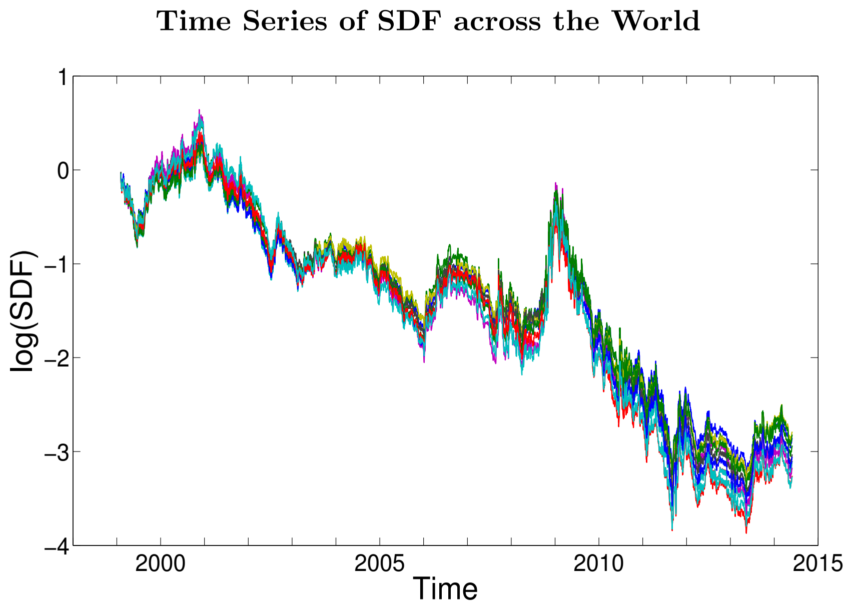

Figure 4: Time Series of log of SDF of 11 developed countries estimated according to (21).

series pattern, we interpret it as additional evidence in favor of our estimates.

We estimate the SDF volatility according to equation (18). The variation in SDF volatilities across countries is economically large. For the sample 1999-2014 the SDF volatilities in Australia and New Zealand are only 52% and 51%, while volatilities in Japan and the USA are 64% and 61%. We find a similar spread between the largest and smallest SDF volatilities in the sample 1984-2014; New Zealand has volatility of 56%, while we estimate volatilities of 65% and 66% in Japan and the USA (see Table E.6, Appendix E).

Figure 5 documents a striking negative relationship between the interest rate in country J and its SDF volatility. Column (2) in Table E.1 (Appendix E) reports that the relationship is highly statistically significant. A common perception is that volatility in the SDF induces precautionary savings. Based on that a large (small) SDF volatility indeed implies much (little) precautionary savings and a relatively low (high) interest rate in equilibrium. Though, such an argument requires much stronger assumptions on preferences and the risk sources in the economy than what we are assuming in this paper.

Our results differ from Gavazzoni et al. (2013) who show, in an affine diffusion model, that interest rates and market prices of risk are tightly linked. In particular, they show that under certain parametric assumptions the volatility of the SDF is proportional to the volatility of the interest

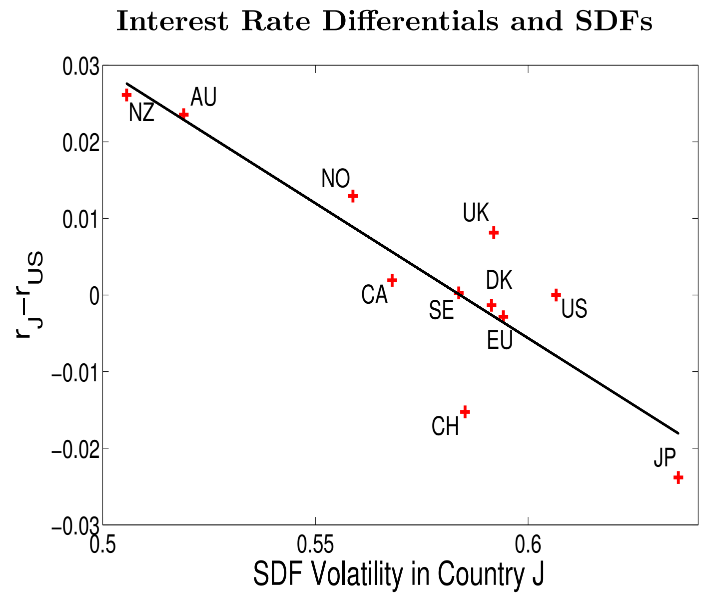

Figure 5: Negative relationship between interest rate and volatility of SDF as defined in equation (18).

rate. They further document empirically that high interest rates tend to be more volatile, and therefore, are associated with more volatile SDFs under their modeling assumptions. In contrast, our estimates imply a negative relation between interest rates and SDF volatilities. The difference arises because our estimation is non-parametric and does not make any assumptions (such as an affine structure) on the relationship between interest rates and market prices of risks.[38]

4.3 Denomination Currencies vs. SDF Volatilities

In the motivation of the paper we have documented that the average of $CT^{US}_{-US/+J}$ varies substantially across countries J while the variance of $CT^{US}_{-US/+J}$ hardly changes (Figure 6). Given this empirical pattern we can show in the context of a diffusion model that there must be a strong relationship between the expected carry trade return −USD/+J and the volatility of country J's

[38] In light of Gavazzoni et al. (2013), we can conclude that our estimated SDFs do not fit into parametric restrictions, which they need to impose on their affine risk setting. For instance, it is important in Gavazzoni et al. (2013) that interest rate volatilities sort the same way as SDF volatilities do in the cross section – which is a parametric assumption. Our procedure aims to estimate SDF volatilities from asset prices, and makes no assumption on the pattern of cross-sectional variation of interest rate volatilities a priori.

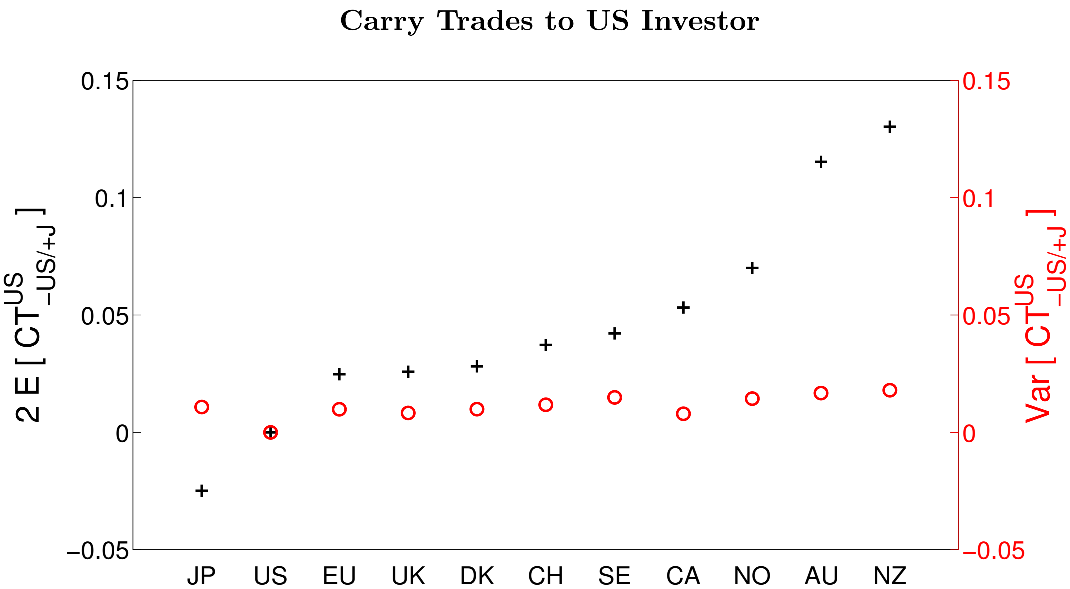

Figure 6: Carry trade strategies of borrowing USD and lending currency J from the perspective of a US investor. Left vertical axis–black crosses: Cross-sectional variation in 2×the average carry trade returns. Right axis–red circles: Cross-sectional variation in the variance of carry trade returns.

SDF. Indeed,

$$\begin{aligned} ||\eta_J||^2 &= ||\eta_{US}||^2 - 2\eta_{US}^T(\eta_{US} - \eta_J) + ||\eta_{US} - \eta_J||^2 \\ &= ||\eta_{US}||^2 - 2E\left[CT_{-US/+J}^{US}\right] + Var\left[CT_{-US/+J}^{US}\right]. \end{aligned}$$

$||\eta_I||^2 - ||\eta_{US}||^2$ is the difference between red circles and black crosses in Figure 6. It is apparent from Figure 6 that $Var\left[CT_{-US/+I}^{US}\right]$ is small and its cross-country variation is almost zero. We get the empirical relationship,

$$||\eta_I||^2 - ||\eta_J||^2 \approx 2E\left[CT_{-US/+J}^{US} - CT_{-US/+I}^{US}\right] = 2E\left[CT_{-I/+J}^{US}\right]. \tag{22}$$

Relationship (22) is similar to equation (4) in Verdelhan (2010), but there are some key differences. Verdelhan (2010) derives his equation (4) for the expected log-return instead of the expected (continuously compounded) return an investor earns. While co-variations between SDFs across countries are unimportant in his analysis, they are a vital piece when modeling risks in FX markets. We show that a version of Verdelhan (2010)'s equation (4) can be recovered if we assume that SDFs across countries feature a correlation close to one. This is important because a high

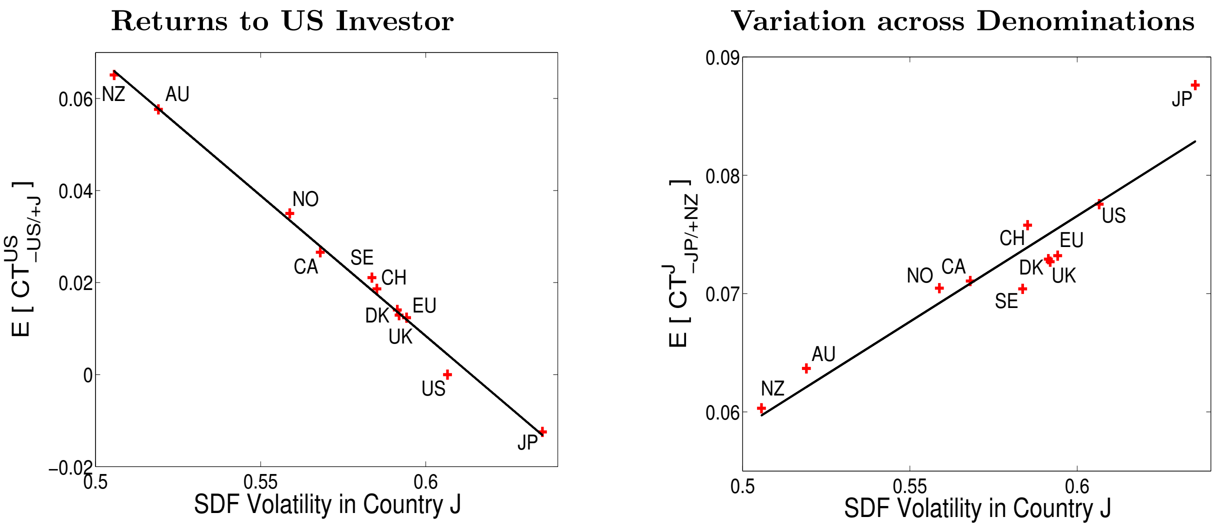

Figure 7: Left: Negative relationship between estimated SDF volatility in country J and average carry trade return of borrowing USD and lending currency J earned by US investor. Right: Positive relationship between SDF volatility in country J and average carry trade return of borrowing JPY and lending NZD earned by an investor in country J.

correlation in SDFs across countries is key to match the low exchange rate volatilities in the data.

The left plot in Figure 7 shows that our estimated model matches the relationship in equation (22) very well. Indeed, regression (20) in our estimation methodology by construction implies this strong relationship (provided that the model matches exchange rate volatilities and average carry trade returns in the data).

The plot on the right in Figure 7 shows a strong positive relationship between the average carry trade return −JPY/+NZD earned by investor J and the volatility of the SDF in country J. To interpret this finding it is useful to rewrite equation (4),

$$E\left[CT^J_{-JPY/+NZD}\right] \quad = \quad ||\eta_J|| \cos\left(\alpha^J_{-JPY/+NZD}\right) ||\eta_{JP} - \eta_{NZ}||,$$

where $\alpha^J_{-JPY/+NZD}$ is the angle between vector η_J and $\eta_{JP} - \eta_{NZ}$. Ceteris paribus, a more (less) volatile SDF in country J implies a larger (lower) expected return and Sharpe ratio for an investment strategy denominated in currency J. This argument assumes that the angle $\alpha^J_{-JPY/+NZD}$ does not vary across denominations J. Our theory does not provide any guidance, but Figure 7 provides strong empirical evidence that $\alpha^J_{-JPY/+NZD}$ is constant across all currency denominations J.

5 Out-of-Sample Results

We examine the diffusion risk prices backed out from FX markets against other asset prices and macroeconomic risks. These include countries' risk free rates, stock market returns and output gap volatilities, which are not inputs to our statistical procedure. Thus, the tests below have out-of-sample nature.

5.1 International Stock Returns

Assume that country J's stock market return follows a diffusion process,[39]

$$\frac{dP_{J,t} + D_{J,t}dt}{P_{J,t}} = \mu_{J,t}dt + \sigma_{J,t}^T dZ_t. \tag{23}$$

In the absence of arbitrage, the expected excess return on J's stock (i.e., equity premium) measured in its home currency J is (see (25)),

$$\begin{aligned} ER_{J,t}^{\text{implied}} &= \mu_{J,t} - r_{J,t} = -\frac{1}{dt}\text{Cov}_t\left(\frac{dM_{J,t}}{M_{J,t}}, \frac{dP_{J,t} + D_{J,t}dt}{P_{J,t}}\right) \\ &= \frac{1}{dt}\text{Cov}_t\left(\sum_{K=1}^{G} \widehat{\beta}_K^J \Pi_{t,K}\,,\, \sigma_{J,t}^T dZ_t\right) \end{aligned} \tag{24}$$

where the scores Π's (21) are obtained from the PCA, and coefficients $\widehat{\beta}$'s are results of regression (16) which constructs prices of risks. For these reasons, (24) presents an expression for the equity premium *implied* by our constructed prices of risks in FX markets. We provide support for our estimated country-specific SDFs in three empirical tests.

Table 1 investigates the relationship between average realized stock excess returns and implied equity premia (24) for 11 developed economies. Realized stock returns are computed from the monthly MSCI Total Return Index series. The implied equity premia are constructed purely from priced risks in FX markets. In particular, the equity premia are computed by retaining all ($G = 10$) PCs in (24).[40] We find a positive relationship (though only significant in the longer data sample 1984-2014), providing evidence that risks in exchange rates also price stock markets. The positive correlation lends support to our construction of country-specific SDFs (21). It is reasonable to

[39]For country J, $P_{J,t}$ denotes ex-dividend stock price, and $D_{J,t}$ denotes the associated dividend process.

[40]For robustness, we also compute the implied equity premium by retaining G first PCs, for each value of $G \in \{2, \ldots, 9\}$. The results do not change either qualitatively or quantitatively, indicating that the first two PCs capture most of the variation in exchange rate movements.

Table 1: Cross-country Regressions of Average Stock Excess Returns on Implied Equity Premia

(1)	(2) Average Stock Excess Return in Country J from 1999 to 2014	(3) Average Stock Excess Return in Country J from 1984 to 2014
Implied Equity Premium in Country J	0.67	2.51*
	(0.86)	(2.10)
constant	Yes	Yes
R^2	7%	30%

Notes: OLS regression $AveExReturn_J = \alpha + \beta ER_J^{\text{implied}} + \epsilon_J$ where $AveExReturn_J$ is the average excess return in country J and $ER_{J,t}^{\text{implied}}$ is the implied equity premium as defined in equation (24). Values in parentheses below each regression coefficient are t-statistics. We have 11 observations. 10%, 5%, 1% significance levels of two sided t-statistic are indicated by *, ** and ***, respectively.

expect the statistical significance to be moderate. Theoretically, there is a global uniform risk component η^* (15) that is not priced in FX markets. Empirically, the average realized stock excess return is an imprecise measurement of the expected return. Latter problem of estimation errors due to a short data sample is consistent with the finding that our estimate is weaker in the shorter sample with only 16 years of data (1999-2014). Adding another 15 years of data (1984-2014) appears to partly mitigate this issue and our estimates are indeed statistically significant with a p-value = 6.5% of a two sided t-test.

To further quantify the relationship between FX risks and stock returns, for each country we regress monthly realized excess stock returns on the monthly PCs, i.e. on each column of the $s \times N$ score matrix Π.[41] The R-squared of this regression gives a measure of how much time-series variation in a country's realized stock return can be explained by movements in FX markets. The results, along with numerical values for the implied equity premium (24), are reported in Table 2 below.

On average, the model-implied stock excess returns (in column 4) account for a sizable fraction of the stock excess returns in the data (in column 2), for either 1999-2014 or 1984-2014 periods.

[41] We recall that K-th PC is a $s \times 1$ column vector of scores, $[\Pi_{t,K}]$ for $t = \{1, \ldots, s\}$.

Table 2: Monthly country-specific stock excess returns on implied SDF growth

	Panel A: 1999-2014			
(1)	(2)	(3)	(4)	(5)
Country	Ave. stock exs. return	Volatility of stock exs. ret.	Ave. implied stock exs. ret.	R-squared
Australia	0.053	0.140	0.025	0.432
Canada	0.074	0.174	0.030	0.447
Denmark	0.113	0.219	0.039	0.437
Euro	0.053	0.231	0.042	0.342
Japan	0.039	0.212	0.056	0.448
New Zealand	0.030	0.154	0.017	0.246
Norway	0.094	0.244	0.049	0.484
Sweden	0.088	0.250	0.044	0.391
Switzerland	0.045	0.162	0.033	0.473
UK	0.029	0.167	0.038	0.538
USA	0.042	0.170	0.041	0.496

	Panel B: 1984-2014			
(1)	(2)	(3)	(4)	(5)
	Ave. stock exs. return	Volatility of stock exs. ret.	Ave. implied stock exs. ret.	R-squared
Australia	0.062	0.177	0.013	0.121
Canada	0.062	0.164	0.014	0.203
Denmark	0.090	0.201	0.021	0.292
Euro	0.082	0.231	0.017	0.129
Japan	0.103	0.245	0.034	0.147
New Zealand	0.037	0.200	0.009	0.229
Norway	0.026	0.185	0.019	0.096
Sweden	0.078	0.251	0.025	0.210
Switzerland	0.144	0.247	0.021	0.148
UK	0.059	0.185	0.011	0.158
USA	0.085	0.165	0.018	0.210

Notes: Country-specific OLS time-series regression $R_{J,t}^{\text{excess}} = \alpha_J + \sum_K^F \beta_{J,K}\Pi_{t,K} + \epsilon_{J,t}$ to examine the effects of exchange rate risks (captured in the PCA scores $\{\Pi_{t,K}\}$) on country J's stock excess returns $R_{J,t}^{\text{excess}}$. Panel A uses data for period 1999-2014, Panel B for period 1984-2014. Input time series are at monthly frequency. Columns 2 and 3 respectively reports the mean and volatility of excess stock return in the data, column 4 reports the countries' implied equity premium computed from (24), column 5 reports the fraction of the variation in excess stock return explained by variation in global exchange rates (i.e., R-squared of above regression). Excess stock returns are computed from monthly country-specific MSCI Total Return Index series. The stock excess return $R_{J,t}^{\text{excess}}$ is the difference between realized return (23) and the respective risk free rate $r_{J,t}$. The regression retains all $F = 10$ PCs. All reported returns and volatilities are annualized.

Furthermore, the variation in FX markets explains from 25% (New Zealand) to 54% (UK) of the time-series variation of stock returns in the data for the period of 1999-2014, and from 10% (New Zealand) to 30% (Denmark) the period of 1984-2014. These results justify the importance of the diffusion risk model of global exchange rates in explaining the time-series variation and pricing the cross-section of stock returns.

We repeat this regression for the US for which the data is available at daily frequency. The results are reported in Table 3. Both value-weighted US stock market returns (sourced from the Center for Research in Security Prices – CRSP) and the PCs are at daily frequency. We compute the implied US equity premium (24) for investment horizons (i.e., holding periods) of 1, 5, 10, 20, 60, 125, and 250 trading days. We use overlapping windows in our analysis, but results for non-overlapping windows (not reported) are very similar. Again, on average, the model-implied stock excess returns (in columns 2 and 5) account for a sizable fraction of the US stock excess returns in the data (listed at the top of each panel A and B), for 1999-2014 or 1984-2014 periods, and for different investment horizons. Moreover, our PCs explain a considerable fraction of the time-series variation in stock excess returns, as reflected by R-squared statistics (in column 5). The fraction of the variation explained is increasing with the length of investment horizon. A reason is that the aggregation of daily returns over longer horizons better eliminates noise in realized returns. Longer horizons also overcome the non-synchronous trading times of stock and FX markets. The FX diffusion risk model and its implied statistics match the data clearly better for the period 1999-2014 (than for 1984-2014). A possible reason is that the stationarity assumption in our statistical model is less robust for a longer period of time. Overall, these empirical investigations show that our constructed SDFs explain a substantial part of the time-series variation in stock returns and price nontrivial parts of risks in both monthly stock returns of the advanced economies and daily US stock returns.

Table 4 illustrates how much each of our 10 PCs (PCA scores extracted from exchange rate data) on its own matters for pricing the US stock market.[42] Interestingly, the first two PCs, which explain over 75% of the variation in exchange rate changes (column (2) in table 4), are also the most important risk sources driving stock returns. In the sample 1999-2014 the first two PCs explain 15% respectively 18% of the time-series variation in the US stock market. In comparison, the remaining 8 PCs together explain only 14% (column (3) in table 4). This finding is mirrored

[42]Our findings for other countries are similar. Table 4 only reports results for returns over a 20 trading-day investment horizon; results for other investment horizons are similar. We only report our findings for US stock returns for brevity.

Table 3: US stock excess returns on implied SDF growth

Panel A: 1999-2014

(1) Horizon	Empirical US stock ave. excess return 0.054; volatility 0.205		
	(2) Ave. implied stock excess return	(3) Confidence interval	(4) R-squared
1-day	0.029	(0.024, 0.035)	0.178
5-day	0.044	(0.034, 0.053)	0.403
10-day	0.041	(0.029, 0.053)	0.455
20-day	0.039	(0.021, 0.054)	0.473
60-day	0.044	(0.013, 0.067)	0.640
125-day	0.046	(-0.004, 0.080)	0.739
250-day	0.039	(-0.039, 0.089)	0.820

Panel B: 1984-2014

(1) Horizon	Empirical US stock ave. excess return 0.083 ; volatility 0.177		
	(2) Ave. implied stock excess return	(3) Confidence interval	(4) R-squared
1-day	0.016	(0.012, 0.019)	0.105
5-day	0.022	(0.014, 0.029)	0.242
10-day	0.020	(0.010, 0.030)	0.269
20-day	0.018	(0.004, 0.031)	0.290
60-day	0.018	(-0.009, 0.042)	0.343
125-day	0.024	(-0.019, 0.062)	0.363
250-day	0.017	(-0.045, 0.072)	0.360

Notes: US stock return OLS time-series regression $R^{\text{excess}}_{US,t} = \alpha_{US} + \sum_K^F \beta_{US,K}\Pi_{t,K} + \epsilon_{US,t}$ to examine the effects of exchange rate risks (captured in the PCA scores $\{\Pi_{t,K}\}$) on US stock excess returns $R^{\text{excess}}_{US,t}$. Panel A uses data for period 1999-2014, Panel B for period 1984-2014. Input time series are at daily frequency, the return is value-weighted US stock market return. Windows indicate the investment horizon (holding period) used to compute the stock returns. Column 2 reports the US implied equity premium computed from (24), column 3 reports the 99%–confidence interval of the estimates, column 4 reports the fraction of the variation in excess stock return explained by variation in global exchange rates (i.e., R-squared of above regression). US excess stock returns are sourced from daily MSCI Total Return Index series for US. The stock realized excess return $R^{\text{excess}}_{US,t}$ is the difference between realized return (23) and the respective risk free rate $r_{US,t}$. The regression retains all $F = 10$ PCs. Reported results are for overlapping windows. All reported returns and volatilities are annualized.

Table 4: US stock excess returns and individual principal components (Π_K)

	Panel A: 1999-2014				
	Empirical US stock ave. excess return 0.054; volatility 0.205				
(1) Π_K	(2) Exchange Rate Var explained by Π_K	(3) US stock return Var explained by Π_K	(4) Market price of Π_K: $\eta_{US,K}$	(5) US stock return loading on Π_K: $\sigma_{US,K}$	(6) US equity premium implied by risk Π_K
Π_1	0.62	0.15	-0.334	-0.713	0.0238
Π_2	0.14	0.18	-0.375	-0.789	0.0296
Π_3	0.08	0.00	-0.047	-0.021	-0.0010
Π_4	0.04	0.04	-0.038	-0.033	0.0012
Π_5	0.04	0.03	0.133	-0.037	-0.0049
Π_6	0.03	0.00	-0.066	0.010	-0.0007
Π_7	0.02	0.01	0.098	-0.014	-0.0013
Π_8	0.02	0.05	0.227	-0.041	-0.0094
Π_9	0.02	0.00	-0.067	0.013	-0.0009
Π_{10}	0.01	0.01	0.157	0.025	0.0039

	Panel B: 1984-2014				
	Empirical US stock ave. excess return 0.083 ; volatility 0.177				
(1) Π_K	(2) Exchange Rate Var explained by Π_K	(3) US stock return Var explained by Π_K	(4) Market price of Π_K: $\eta_{US,K}$	(5) US stock return loading on Π_K: $\sigma_{US,K}$	(6) US equity premium implied by risk Π_K
Π_1	0.56	0.02	-0.420	-0.026	0.0107
Π_2	0.17	0.15	-0.292	-0.061	0.0180
Π_3	0.08	0.01	-0.046	0.019	-0.0009
Π_4	0.04	0.03	0.161	-0.019	-0.0031
Π_5	0.04	0.02	0.200	-0.021	-0.0041
Π_6	0.03	0.03	-0.132	-0.036	0.0048
Π_7	0.03	0.01	-0.059	0.013	-0.0008
Π_8	0.02	0.00	-0.125	0.009	-0.0011
Π_9	0.02	0.02	-0.008	0.028	-0.0002
Π_{10}	0.01	0.01	0.268	-0.025	-0.0066

Notes: US stock return OLS time-series regression $R^{\text{excess}}_{US,t} = \alpha_{US} + \sum_K^F \beta_{US,K}\Pi_{t,K} + \epsilon_{US,t}$ to examine the effects of each individual PC K extracted from exchange rate data ($\{\Pi_{t,K}\}$) on US stock excess returns $R^{\text{excess}}_{US,t}$. Panel A uses data for period 1999-2014, Panel B for period 1984-2014. Input time series are 20 trading-day returns of a value-weighted US stock market portfolio (data form CRSP); results are similar for other holding periods. Column 2 reports the percentage of variation in exchange rates explained by score $\{\Pi_{t,K}\}$, column 3 reports the percentage of variation in US stock excess returns explained by score $\{\Pi_{t,K}\}$, column 4 reports the market price of risk attached to the risk source $\{\Pi_{t,K}\}$, column 5 reports the loading on US stock excess returns on the risk source $\{\Pi_{t,K}\}$, and column 6 reports the US implied equity premium by risk source $\{\Pi_{t,K}\}$ computed according to (24). US excess stock returns are sourced from daily MSCI Total Return Index series for US. The stock realized excess return $R^{\text{excess}}_{US,t}$ is the difference between realized return (23) and the respective risk free rate $r_{US,t}$. All reported returns and volatilities are annualized.

in column (5) in table 4: the loadings of the US stock market on the risk sources described by PCs 1 and 2 is substantially larger than the exposure to any of the other 8 components/ risk sources. Moreover, market prices of risk associated with the first two PCs are significantly larger than the prices of the other 8 components (column (4) in table 4). Finally, most of the US equity premium is a compensation for exposure to risks described by the first two PCs, while all of the other 8 components hardly affect the risk premium (column (6) in table 4). Interestingly, the US stock market appears to be somewhat a hedge (though the correlation is close to zero) to several PCs (namely the risks described by components 3, 5, 6, 7, 8 and 9).

In summary, we conclude that the first two PCs explain a significant part of the time-series variation in US stock returns and the equity premium is mostly a compensation for exposure to the first two PCs. Interestingly, both the first and the second PC appear equally important to explain both the time-series variation and the equity premium. This is in comparison to the finding that the dollar factor (which is similar to our first PC) is mostly explaining the time-series variation and the carry factor (which is similar to our second PC) is mostly explaining the cross-section of carry trade returns. Thus, these factors appear to have different roles in different asset markets (FX versus equity).

5.2 Denomination Currencies versus Interest Rates

In the Figure 1 in section 2.1, we document that in the cross section, the same carry trade strategies tend to offer *higher* returns to investors denominated in currencies with *lower* interest rates. In this section we undertake a more careful empirical investigation into this novel observation. We fix the same net-zero carry trade strategy of borrowing JPY and lending NZD. We run a feasible general least square (FGLS) panel regression of the next-period (realized) carry trade return differential $CT^{J}_{-JP/+NZ,t+dt} - CT^{US}_{-JP/+NZ,t+dt}$ on the current interest rate differential $r_{J,t} - r_{US,t}$. We assume the following structure for the covariance matrix of regression errors $\epsilon_{J,t}$. Within each country J we assume a GARCH(1,1) structure to model the heteroskedasticity in the times series of error $\epsilon_{J,t}$. Contemporaneous correlations $Corr\left(\epsilon_{J,t}, \epsilon_{I,t}\right), \forall I, J, t$ are constant through time and we assume $Corr\left(\epsilon_{J,t}, \epsilon_{I,\tau}\right) = 0, \forall I, J, t \neq \tau$.[43] The results are reported in Table 5. In different specifications, we control for country fixed effects as well as the carry trade volatility.[44]

[43] We estimate the covariance matrix using errors from a first stage OLS regression. We re-estimate the covariance matrix using errors from the FGLS regression and it appears to be stable.

[44] Because of the diffusion invariance associated with this net-zero strategy (Proposition 1), in theory, the carry trade return differential equals the expected carry trade return differential, which is $(\eta^T_J - \eta^T_{US})(\eta_{JP} - \eta_{NZ})$. The

Table 5: Carry trade returns across denomination currencies vs. interest rates

	Panel A: 1999-2014			
	(1)	(2)	(3)	(4)
$r_J - r_{US}$	-0.015^{***}	-0.072^{***}	-0.010^{*}	-0.082^{***}
	(-2.67)	(-9.83)	(-1.83)	(-11.23)
$\text{Vol}_t(CT)$	-0.818^{***}	-9.852^{***}		
	(-87.96)	(-54.78)		
Country FE	Yes	No	Yes	No
R-squared	0.524	0.429	0.126	0.035
	Panel B: 1984-2014			
	(1)	(2)	(3)	(4)
$r_J - r_{US}$	-0.008^{***}	-0.036^{***}	-0.009^{***}	-0.040^{***}
	(-3.15)	(-10.90)	(-3.77)	(-14.37)
$\text{Vol}_t(CT)$	-0.801^{***}	-9.171^{***}		
	(-69.11)	(-57.27)		
Country FE	Yes	No	Yes	No
R-squared	0.455	0.340	0.137	0.024

Notes: FGLS panel regression $CT^{J}_{-JP/+NZ,t+dt} - CT^{US}_{-JP/+NZ,t+dt} = \alpha + \beta_1 (r_{J,t} - r_{US,t}) + \beta_2 \text{Vol}_t(CT) + d_J + \epsilon_{J,t}$ to examine the predictability of next-period carry trade return differential $CT^{J}_{-JP/+NZ,t+dt} - CT^{US}_{-JP/+NZ,t+dt}$ (between currency J's and USD denomination) by the country J's current interest rate differential $(r_{J,t} - r_{US,t})$ (relative to current US interest rate). Panel regression on country index $J \in 1, \ldots, N$ and time index $t \in \{1, \ldots, s\}$. Control variables include the carry trade volatility computed as $\text{Vol}_t(CT) = \frac{1}{N} \sum_J^N \left| CT^{J}_{-JP/+NZ,t} \right|$, and country fixed effect dummy d_J. Panel A uses data for period 1999-2014, Panel B for period 1984-2014. Values in parentheses (below each slope coefficient) are t-statistics of the associated slope coefficients. Tables E.7 and E.8 reports results of robustness checks where we divide the sample into two subsamples: NBER expansions versus contractions. 10%, 5%, 1% significance levels of two sided t-statistic are indicated by *, ** and ***, respectively.

Table 5 shows that the coefficient associated with the interest rate differential is negative and statistically significant across the specifications. This regression reinforces our previous finding that investors in countries of lower interest rates tends to earn higher returns on same carry trade strategies.[45] Table E.7 in the Appendix reports results of robustness checks where we divide the sample into two subsamples: NBER expansions versus contractions; the qualitative results are unchanged.

5.3 Macroeconomic Evidence

We now provide evidence that our construction of the SDF (21) are related to important macroeconomic risk, i.e., the output gap volatility. Output gap, which measures the difference between a country's actual output and its potential output, is a key macroeconomic input for monetary policy and is employed to chart business cycles in economies.[46] Fluctuations in output gap constitute a important risk in the economy.

To characterize this risk, let $x_{J,t}$ be the demeaned output gap time series. We conjecture that there is a close relationship between the volatility of SDF growth and the volatility of output gap. On a related note, Riddiough (2014) finds that currencies sorted based on output gaps generate considerable carry trade returns, while Cooper and Priestley (2009) find that output gap can predict stock returns. Figure 8 shows the relationship between our estimated SDF growth volatility $||\widehat{\eta_J}||$ in (17) and the standard deviation of output gap for the respective country J, for all J. We estimate $||\widehat{\eta_J}||$ using data from 1999-2014, whereas the output gap volatility is calculated using annual IMF data from 1980-2014 (all available).[47] The figure shows a positive alignment of output gap volatility and our estimated volatility of SDF growth. The associated regression gives a slope coefficient of 0.0764 with a t-statistics of 4.38 and adjusted R-squared of 64.5%. For data from 1984-2014, the

factor $\eta_{JP} - \eta_{NZ}$, which presumably is fixed in this strategy (of borrowing JPY and lending NZD), most likely fluctuates in the data. Consequently, in one specification, we control for this extra source of variability by the carry trade volatility, which is $||\eta_{JP} - \eta_{NZ}||$ and independent of the nomination currency.

[45] For robustness, we also examine and obtain same result for the returns (to different denomination currencies) on other most profitable carry strategies, including the popular long-short portfolio strategies on currencies sorted on last-period interest rates.

[46] Formally, the output gap is an (annual) time series, whose value at year t is,

$$\text{OutputGap}_{J,t} \equiv \frac{\text{RealizedOutput}_{J,t} - \text{PotentialOutput}_{J,t}}{\text{PotentialOutput}_{J,t}},$$

where $\text{RealizedOutput}_{J,t}$ is a proxy for country J's realized GDP in year t, and $\text{PotentialOutput}_{J,t}$ is proxy for the production capacity of the economy J (or optimal GDP).

[47] Because the IMF output gap is at low (annual) frequency, we need a reasonably long time series to construct a reliable estimate of the output gap volatility.

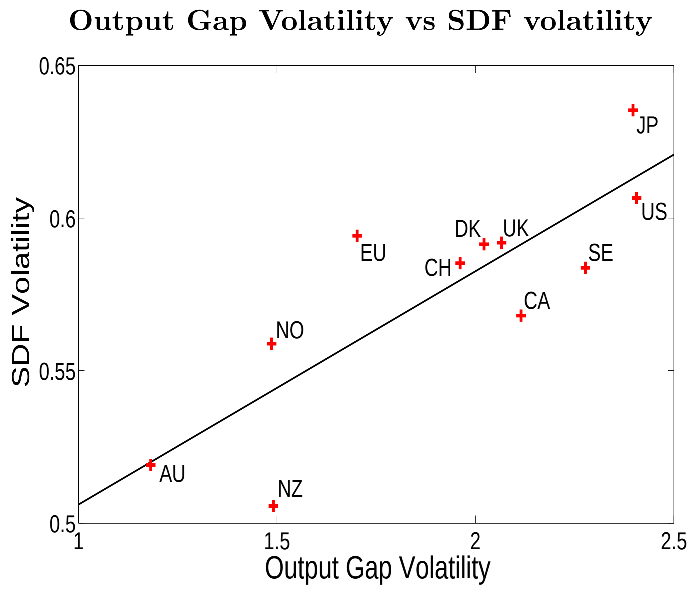

Figure 8: Positive relationship between unconditional volatility in output gap and volatility of estimated SDFs according to equation (18).

slope coefficient is 0.0486, with a t-statistics of 3.27 and adjusted R-squared of 49.28%.

The above regression only looks at the overall cross-sectional relationship between output gap volatility and the implied SDF growth for the entire period. Because our approach gives a construction of the time series (21), we now undertake a more stringent empirical evaluation of the implied SDF against the IMF output gap time series. We run a panel regression of SDF growth $\frac{dM_{J,t}}{M_{J,t}}$ on output gap series $x_{J,t}$ across countries J and time t, with both time fixed effects and country fixed effects (Table 6). We estimate a statistically significant negative slope coefficient, implying that an unexpected increase in our constructed SDF (a bad shock to the economy) comes with an unexpected decrease in the output gap (the economy produces less than its potential output). This striking negative correlation between our constructed SDFs and country-specific business cycles as measured by the respective output gaps is in strong support of our methodology.

6 Conclusion

In this paper, we argue that the variation in returns on the same carry trade strategy but denominated in different currencies is a natural and important test object to evaluate no-arbitrage pricing of risks in FX markets. We document an intriguing regularity that investors in countries of lower

Table 6: Model implied innovation to SDF growth and demeaned output gap

	Panel A: 1999-2014			Panel B: 1984-2014		
	(1)	(2)	(3)	(4)	(5)	(6)
Output gap	-0.015^{***}	-0.121^{***}	-0.010^{*}	-0.053^{**}	-0.058^{**}	-0.007^{**}
	(-6.18)	(-6.14)	(-1.89)	(-2.63)	(-2.62)	(-2.42)
Country FE	No	Yes	Yes	No	Yes	Yes
Time FE	No	No	Yes	No	No	Yes
R-squared	0.141	0.141	0.982	0.019	0.019	0.994

Notes: Panel regression $\frac{dM_{J,t}}{M_{J,t}} = \alpha + \beta x_{J,t} + \epsilon_{J,t}$ to examine the relationship between model implied innovation to SDF growth $\frac{dM_{J,t}}{M_{J,t}}$ of country J and the (demeaned) output gap of that country $x_{J,t}$. Control variables include country fixed effects and time fixed effects. Panel A uses data for period 1999-2014, Panel B for period 1984-2014. Values in parentheses (below the slope coefficient) are t-statistics of the slope coefficients All t-statistics are computed with country clustering. 10%, 5%, 1% significance levels of two sided t-statistic are indicated by *, ** and ***, respectively.

interest rates tend to earn higher returns on popular and profitable carry strategies. We propose a non-parametric procedure to estimate country-specific SDFs. Though the SDFs are constructed solely from FX market prices (and do not require any assumptions on preferences or wealth distributions or rely on macroeconomic data), they are linked to important macroeconomic quantities (output gap) and stock returns. We find that the first two PCs in exchange rate growths explain our cross-denomination pattern as well as a substantial part in both the time-series variation of international stock returns and the cross-sectional variation of average stock returns. We leave the investigation of non-diffusion risks and time variations in market prices of risk for future research.

References

Backus, D., S. Foresi, and C. Telmer. 2001. Affine Models of Currency Pricing: Accounting for the Forward Premium Anomaly. *Journal of Finance* 56:279–304.

Bakshi, G., and G. Panayotov. 2013. Predictability of Currency Carry Trades and Asset Pricing Implications. *Journal of Financial Economics* 110:139–163.

Brandt, M. W., J. H. Cochrane, and P. Santa-Clara. 2006. International Risk-Sharing is Better Than You Think (or Exchange Rates are Much Too Smooth). *Journal of Monetary Economics* 53:671–698.

Brunnermeier, M. K., S. Nagel, and L. H. Pedersen. 2008. Carry Trades and Currency Crashes. In K. Rogoff, M. Woodford, and D. Acemoglu (eds.), *NBER Macroeconomics Annual 2008*, vol. 23, pp. 313–347.

Brusa, F., T. Ramadorai, and A. Verdelhan. 2015. The International CAPM Redux. Working paper, University of Oxford.

Burnside, C. 2011. The Cross-Section of Foreign Currency Risk Premia and Consumption Growth Risk: Comment. *American Economic Review* 101:3456–3476.

Burnside, C. 2012. Carry Trades and Risk. In J. James, I. Marsh, and L. Sarno (eds.), *Handbook of Exchange Rates*. Hoboken: John Wiley & Sons.

Burnside, C., M. Eichenbaum, I. Kleshchelski, and S. Rebelo. 2011. Do Peso Problems Explain the Returns to the Carry Trade? *Review of Financial Studies* 24:853–891.

Cenedese, G. 2012. Safe Haven Currencies: A Portfolio Perspective. Working paper, Bank of England.

Chernov, M., J. Graveline, and I. Zviadadze. 2013. Crash Risk in Currency Returns. Working paper, London School of Economics.

Cooper, I., and R. Priestley. 2009. Time-Varying Risk Premiums and the Output Gap. *Review of Financial Studies* 22:2801–2833.

Daniel, K., R. J. Hodrick, and Z. Lu. 2014. The Carry Trade: Risks and Drawdowns. Working paper, Columbia University.

Della Corte, P., T. Ramadorai, and L. Sarno. forthcoming. Volatility Risk Premia and Exchange Rate Predictability. *Journal of Financial Economics* .

Dobrynskaya, V. 2014. Downside Market Risk of Carry Trades. *Review of Finance* 18:1885–1913.

Dobrynskaya, V. 2015. Currency Exposure to Downside Risk: Which Fundamentals Matter? Working paper, London School of Economics.

Dong, S. 2006. Monetary Policy Rules and Exchange Rates: A Structural VAR Identified by No Arbitrage. Working paper, Columbia University.

Engel, C., N. C. Mark, and K. D. West. 2007. Exchange Rate Models are not as Bad as You Think. In D. Acemoglu, K. Rogoff, and M. Woodford (eds.), *NBER Macroeconomics Annual 2007*, vol. 22, pp. 381–441.

Fama, E. F., and J. D. MacBeth. 1973. Risk, Return, and Equilibrium: Empirical Tests. *Journal of Political Economy* 81:607–636.

Farhi, E., S. Fraiberger, X. Gabaix, R. Ranciere, and A. Verdelhan. 2014. Crash Risk in Currency Markets. Working paper, Harvard University.

Galsband, V., and T. Nitschka. 2013. Currency Excess Returns and Global Downside Risk. Working paper, Deutsche Bundesbank.

Gavazzoni, F., B. Sambalaibat, and C. Telmer. 2013. Currency Risk and Pricing Kernel Volatility. Working paper, ssrn no. 2179424.

Greenaway-McGrevy, R., N. C. Mark, D. Sul, and J.-L. Wu. 2012. Exchange Rates as Exchange Rate Common Factors. Working paper, Bureau of Economic Analysis.

Habib, M. M., and L. Stracca. 2012. Getting beyond Carry Trade: What Makes a Safe Haven Currency? Working paper, European Central Bank.

Hassan, T. 2013. Country Size, Currency Unions, and International Asset Returns. *Journal of Finance* 68:2269–2308.

Hassan, T., and R. Mano. 2014. Forward and Spot Exchange Rates in a Multi-currency World. working paper no. 20294, NBER.

Hodrick, R. J., and E. C. Prescott. 1997. Postwar U.S. Business Cycles: An Empirical Investigation. *Journal of Money, Credit, and Banking* 29:1–16.

Jurek, J. W. 2014. Crash-Neutral Currency Carry Trades. *Journal of Financial Economics* 113:325–347.

Koedij, K., and P. Schotman. 1989. Dominant Real Exchange Rate Movements. *Journal of International Money and Finance* 8:517–531.

Lettau, M., M. Maggiori, and M. Weber. 2014. Conditional Risk Premia in Currency Markets and Other Asset Classes. *Journal of Financial Economics* 114:197–225.

Lustig, H., N. Roussanov, and A. Verdelhan. 2011. Common Risk Factors in Currency Returns. *Review of Financial Studies* 24:3731–3777.

Lustig, H., N. Roussanov, and A. Verdelhan. 2014. Countercyclical Currency Risk Premia. *Jounal of Financial Economics* 111:527–553.

Lustig, H., and A. Verdelhan. 2007. The Cross-section of Foreign Currency Risk Premia and Consumption Growth Risk. *American Economic Review* 97:89–117.

Lustig, H., and A. Verdelhan. 2011. The Cross-section of Foreign Currency Risk Premia and Consumption Growth Risk: A Reply. *American Economic Review* 101:3477–3500.

Maasoumi, E., and L. Bulut. 2012. Predictability and Specification in Models of Exchange Rate Determination. Working paper, Emory University.

Mancini, L., A. Ranaldo, and J. Wrampelmeyer. 2013. Liquidity in the Foreign Exchange Market: Measurement, Commonality, and Risk Premiums. *Journal of Finance* 68:1805–1841.

Maurer, T. A., T.-D. To, and N.-K. Tran. 2016. Optimal Factor Strategy in FX Markets. Working paper, Washington University.

Maurer, T. A., and N.-K. Tran. 2015. Entangled Risks in Incomplete FX Markets. Working paper, Washington University.

Meese, R., and K. Rogoff. 1983. Empirical Exchange Rate Models of the Seventies: Do They Fir Out of Sample? *Journal of International Economics* 14:3–24.

Menkhoff, L., L. Sarno, M. Schmeling, and A. Schrimpf. 2012a. Carry Trades and Global Foreign Exchange Volatility. *Journal of Finance* 67:681–718.

Menkhoff, L., L. Sarno, M. Schmeling, and A. Schrimpf. 2012b. Currency Momentum Strategies. *Journal of Financial Economics* 106:620–684.

Mueller, P., A. Stathopoulos, and A. Vedolin. 2013. International Correlation Risk. Working paper, London School of Economics and Political Science.

Rafferty, B. 2012. Currency Returns, Skewness and Crash Risk. Working paper, Duke University.

Rapach, D., and M. Wohar. 2006. The Out-of-Sample Forecasting Performance of Nonlinear Models of Real Exchange Rate Behavior. *International Journal of Forecasting* 22:341–361.

Riddiough, S. 2014. The Mystery of Currency Betas. Working paper, Warwick Business School.

Sarno, L., P. Schneider, and C. Wagner. 2012. Properties of Foreign Exchange Risk Premiums. *Journal of Financial Economics* 105:279–310.

Verdelhan, A. 2010. A Habit-Based Explanation of the Exchange Rate Risk Premium. *Journal of Finance* 65:123–145.

Verdelhan, A. 2015. The Share of Systematic Risk in Bilateral Exchange Rates. *Journal of Finance* .

Online Appendices

A Data

Our empirical work focuses on the currencies of developed countries, which have more liquid exchange rate markets, better data quality as well as better conform to covered interest rate parity.[48].

The creation of the Economic and Monetary Union (a.k.a Eurozone) in 1999 combines 15 developed currencies into one new currency, the Euro.[49] Recent application of carry trade strategies have to use this single Euro currency. To keep our work relevant, we focus on the era after Euro is introduced. However, as a robustness check, we also repeat empirical work using data from 1984 (when exchange rate data starts to be available for research) to 2014 where we use German data to backfill data for the Euro before 1999. More details on currencies used, different data types and sources are as follows.

Spot and forward exchange rates: Spot and forward exchange rates against the US dollar are downloaded from Datastream. There are two major sources, Barclays Capital and WM/Reuters (WMR), each have data on different currencies. In cases where data for one currency is available from both sources, the longer series is used. We check the discrepancies between the two sources and they are negligible. We use 1-month forward rate for our carry trade strategy. This is standard in the literature where carry trade strategy is determined each month based on past month interest rates. Tickers for spot and 1-month forward exchange rates are in Table A.1. The portfolio is rebalanced monthly at bid and offered prices. Forward bid and offered rates are also downloaded from the same source. For bid rates, replace (ER) in the code by (EB), whereas code (EO) will give the offered rates. Data for all of the series start from December 1984, except for the Eurozone (January 1999), Japan (October 1983), Switzerland (October 1983), the United Kingdom (spot data start from December 1957, forward data start from January 1997), and Germany (sport data start from December 1993, forward data start from January 1997). We use data up to April 2014 for all series.

[48]As in standard carry trade literature, we rely on covered interest rate parity to calculate carry trade returns based on forward and spot exchange rates.

[49]The following 7 developed countries joined the Eurozone (with respective currency adopting date in the parenthesis): Austria (01/01/1999), Belgium (01/01/1999), Finland (01/01/1999), France (01/01/1999), Germany (01/01/1999), Italy (01/01/1999), Netherlands (01/01/1999). Other countries in the Eurozone that are not considered developed countries, or data not available, in standard carry trade literature are Estonia, Ireland, Luxembourg, Portugal, Slovak Republic, Slovenia, and Spain.

Table A.1: Exchange rate data

Country	Abbr.	Source	Spot rate code	Forward rate code
Australia	AU	Barclays	BBAUDSP(ER)	BBAUD1F(ER)
Canada	CA	Barclays	BBCADSP(ER)	BBCAD1F(ER)
Denmark	DK	Barclays	BBDKKSP(ER)	BBDKK1F(ER)
Eurozone	EU	WMR	EUDOLLR(ER)	EUDOL1F(ER)
Japan	JP	Barclays	BBJPYSP(ER)	BBJPY1F(ER)
New Zealand	NZ	Barclays	BBNZDSP(ER)	BBNZD1F(ER)
Norway	NO	Barclays	BBNOKSP(ER)	BBNOK1F(ER)
Sweden	SE	Barclays	BBSEKSP(ER)	BBSEK1F(ER)
Switzerland	CH	Barclays	BBCHFSP(ER)	BBCHF1F(ER)
United Kingdom	UK	WMR	UKDOLLR(ER)	UKUSD1F(ER)
Germany	DE	WMR	DMARKE$(ER)	USDEM1F(ER)

Interest rates: US interest rates are from the Center for Research in Security Prices (CRSP) US Treasury Databases, series "CRSP Monthly Treasury - Fama Risk Free Rates". This series is designed by Professor Eugene F. Fama that contains 1-month risk free rates for use in pricing and macroeconomic models. We use the average between bid and ask rates.

Interest rates for other countries are downloaded from the website of Professor Adrien Verdelhan.[50] This source contains data used in Verdelhan (2015), and provides monthly interest rate differentials between various countries and the US from November 1983 to December 2010. From 2011-2014, we use the forward and spot exchange rates to construct interest rate differentials (covered interest rate parity).

Macroeconomic data: Output gap data is downloaded from IMF World Economic Outlook (WEO) Database, series "Output Gap in percent of potential GDP". Data is only available at annual frequency, and from 1980. There is no output gap data for Switzerland. We therefore construct the series by first estimating the trend in real output of Switzerland using an Hodrick and Prescott (1997) filter. The output gap is then calculated as the difference between the actual and trend level of real output. Real output for Switzerland is from "Real GDP, National Currency at 2010 prices, Seasonally Adjusted" series downloaded from IMF International Finance Statistics (IFS) database, whereas Switzerland population is from the "Total Population" series, downloaded from World Bank World Development Indicators (WDI).

[50] The file is under the heading "Monthly Changes in Exchange Rates and Global Risk Factors". http://web.mit.edu/adrienv/www/Data.html.

Stock market data: Stock return indexes are downloaded from Datastream, series "MSCI Total Return Index". The MSCI Total Return Indexes measure the price performance of markets with the income from constituent dividend payments. Gross total return indexes reinvest as much as possible of a company's dividend distributions. The reinvested amount is equal to the total dividend amount distributed to persons residing in the country of the dividend-paying company. Gross total return indexes do not, however, include any tax credits. All indexes are in local currency.

We use monthly index data to match with our carry trade return frequency. MSCI index monthly data is available for our sample period, i.e., from 1984-2014. The ticker for each series in Datastream is presented in Table A.2.

Table A.2: MSCI Total Return Index - Ticker in Datastream

Country	Ticker	Country	Ticker
Australia	MSAUSTL(MSRI)	Norway	MSNWAYL(MSRI)
Canada	MSCNDAL(MSRI)	Sweden	MSSWDNL(MSRI)
Denmark	MSDNMKL(MSRI)	Switzerland	MSSWITL(MSRI)
Eurozone	MSEMUIE(MSRI)	United Kingdom	MSUTDKL(MSRI)
Japan	MSJPANL(MSRI)	United States	MSUSAML(MSRI)
New Zealand	MSNZEAL(MSRI)	Germany	MSGERML(MSRI)

For the US market, we also use the daily value-weighted-index from CRSP. This index is built each period using all issues listed on the New York Stock Exchange (NYSE), the NYSE MKT exchange, the NYSE Arca Exchange (ARCA), and the National Association of Securities Dealers Automated Quotations (NASDAQ) Stock Exchange with available shares outstanding and valid prices in the current and previous periods, excluding American Depositary Receipts. Issues are weighted by their Market Capitalization at the end of the previous period.

Lustig-Roussanov-Verdelhan (2011) carry trade return: Lustig et al. (2011) study the currency markets and build carry trade strategies. We download the monthly returns of their strategy from Professor Verdelhan's website, excel file under the heading "Monthly Currency Excess Returns".

B Extensions

Generic Asset Returns across Currency Denominations

Our basic results on differential risk pricing across different currency denominations also hold for any generic investments (beyond carry trades). For an illustration of a mixed investment of bonds and equities, let us consider a net-zero strategy of selling (i.e., short) country B's bond and buying (i.e., long) country L's stock index, for the period $(t, t+dt)$. The proceeds are converted to the denomination currency I at closing date $t+dt$, thus from investor I's perspective.[51] Assume that a generic country L's the stock price (measured in its own currency L) follows the diffusion process,

$$\frac{dP_{L,t} + D_{L,t}dt}{P_{L,t}} = \mu_L dt + \sigma_L^T dZ_t,$$

the realized excess return of this strategy is,

$$\begin{aligned} S^I_{-B/+L,t+dt} &= \frac{M_{I,t}}{M_{L,t}} \frac{P_{L,t+dt} + D_{L,t}dt}{P_{L,t}} \frac{M_{L,t+dt}}{M_{I,t+dt}} - \frac{M_{I,t}}{M_{B,t}} \left(1 + r_B dt\right) \frac{M_{B,t+dt}}{M_{I,t+dt}} \\ &= \frac{M_{I,t}}{M_{I,t+dt}} \left[\frac{M_{L,t+dt}}{M_{L,t}} \left(1 + \mu_L dt + \sigma_L^T dZ_t\right) - \frac{M_{B,t+dt}}{M_{B,t}} \left(1 + r_B dt\right) \right] \\ &= \left[\mu_L - r_L - \eta_L^T \sigma_L + \eta_I^T \left(\eta_B - \eta_L + \sigma_L\right)\right] dt + \left(\eta_B^T - \eta_L^T + \sigma_L^T\right) dZ_t. \end{aligned}$$

where we have the differential form (1) of the SDFs and Ito's lemma. Note that because stock L is traded in country L,

$$1 = E_t \left[\frac{M_{L,t+dt}}{M_{L,t}} \frac{P_{L,t+dt} + D_L dt}{P_{L,t}} \right] \Longrightarrow \mu_L - r_L - \eta_L^T \sigma_L = 0. \tag{25}$$

This is (24). Thus, the realized excess return of the above strategy can be simplified to,

$$S^I_{-B/+L,t+dt} = \eta_I^T \left(\eta_B - \eta_L + \sigma_L\right) dt + \left(\eta_B^T - \eta_L^T + \sigma_L^T\right) dZ_t. \tag{26}$$

Clearly, the shocks are independent of denomination currency, thus this net-zero strategy indeed exhibits diffusion invariance. However, the same shocks are priced differently in different country I (depending on their correlation with I's SDF) as seen in the expected return of the strategy. Similarly, the realized excess return of a net-zero strategy of selling (i.e., short) country B's bond

[51] Long and short positions in stock indices can be achieved though exchange-traded funds (ETF).

and buying (i.e., long) a portfolio $\mathcal{L}$ of countries' stock indices, from investor I's perspective is,

$$S^I_{-B/+\mathcal{L},t+dt} = \eta_I^T \left[\eta_B + \sum_{L\in\mathcal{L}} \theta_L \left(\sigma_L - \eta_L\right)\right] dt + \left[\eta_B^T + \sum_{L\in\mathcal{L}} \theta_L \left(\sigma_L^T - \eta_L^T\right)\right] dZ_t, \tag{27}$$

where $\{\theta_L\}$ are portfolio weights, $\sum_{L\in\mathcal{L}} \theta_L = 1$. Any other more sophisticated net-zero investment's return follows straight from the above return. For instance, a strategy of selling a basket $\mathcal{B}$ of stocks, and buying another basket $\mathcal{L}$ yields the realized excess return,

$$S^I_{-\mathcal{B}/+\mathcal{L},t+dt} = S^I_{-B/+\mathcal{L},t+dt} - S^I_{-B/+\mathcal{B},t+dt}$$

$$= \eta_I^T \left[\sum_{L\in\mathcal{L}} \theta_L \left(\sigma_L - \eta_L\right) - \sum_{B\in\mathcal{B}} \theta_B \left(\sigma_B - \eta_B\right)\right] dt + \left[\sum_{L\in\mathcal{L}} \theta_L \left(\sigma_L^T - \eta_L^T\right) - \sum_{B\in\mathcal{B}} \theta_B \left(\sigma_B - \eta_B\right)\right] dZ_t,$$

C Technical Details and Proofs

C.1 Diffusion Risk Model of FX Markets

Exchange rates: Consider a random payoff Y_{t+dt} to be realized at $t + dy$ ($Y_{t+dt} \in \mathcal{F}_{t+dt}$) in units of currency I. Let $EX_{J/I,t}$ be the exchange rate of currency J per one unit of currency I at a generic time t. The time-t value Y_t of this payoff can either be computed directly in currency I (using I's SDF M_I), or in currency J (using J's SDF M_J) and exchange to currency I,

$$E_t\left[\frac{M_{I,t+dt}}{M_{I,t}} Y_{t+dt}\right] = Y_t = \frac{1}{EX_{J/I,t}} E_t\left[\frac{M_{J,t+dt}}{M_{J,t}} \left(EX_{J/I,t+dt} Y_{t+dt}\right)\right].$$

Assuming complete financial markets so that SDFs M's are unique, and the above pricing equations hold for any Y_t. These imply that the exchange rate unambiguously is the ratio of the two SDFs, $EX_{J/I,t} = \frac{M_{I,t}}{M_{J,t}}$, $\forall t$, which is (2).

Realized carry trade excess returns: Consider the following net-zero strategy denominated in currency I: (i) at time t, borrow $\frac{M_{I,t}}{M_{B,t}}$ units of currency B (worth one unit of currency I) paying interest rate r_B, and simultaneously lend $\frac{M_{I,t}}{M_{L,t}}$ units of currency L (also worth one unit of currency I) earning interest rate r_L, (ii) at time $t + dt$, close all positions and convert the proceeds to denomination currency I. The realized excess return of the strategy is, after using differential representation (1) and applying Ito's lemma,

$$CT^I_{-B/+L,t+dt} = \frac{M_{I,t}}{M_{L,t}}(1+r_L dt)\frac{M_{L,t+dt}}{M_{I,t+dt}} - \frac{M_{I,t}}{M_{B,t}}(1+r_B dt)\frac{M_{B,t+dt}}{M_{I,t+dt}}$$

$$= \frac{M_{I,t}}{M_{I,t+dt}} \times \left[\frac{M_{L,t+dt}}{M_{L,t}}(1+r_L dt) - \frac{M_{B,t+dt}}{M_{B,t}}(1+r_B dt)\right]$$

$$= \frac{1}{1-r_I dt - \eta_I^T dZ_t} \times \left[(1-r_L dt - \eta_L^T dZ_t)(1+r_L dt) - (1-r_B dt - \eta_B^T dZ_t)(1+r_B dt)\right]$$

$$= \left[1+(r_I + ||\eta_I||^2)dt + \eta_I^T dZ_t\right] \times \left[\eta_B^T dZ_t - \eta_L^T dZ_t\right] = \eta_I^T \left(\eta_B - \eta_L\right) dt + \left(\eta_B^T - \eta_L^T\right) dZ_t,$$

which yields (4). We can also relate $CT^I_{-B/+L,t+dt}$ to the log returns on the underlying strategies. To this end, we follow the literature to define respectively the gross return $RX^I_{-I/+J,t+dt}$ and log return $rx^I_{-I/+J,t+dt}$ on buying currency J to investors of currency denomination I as follows (see, e.g., Lustig et al. (2011)),

$$RX^I_{-I/+J,t+dt} \equiv e^{rx^I_{-I/+J,t+dt}} \equiv \frac{F_{J/I,t}}{EX_{J/I,t+dt}} \quad \Longrightarrow \quad rx^I_{-I/+J,t+dt} = r_J dt - r_I dt - d\log EX_{J/I,t+dt}, \tag{28}$$

where $F_{J/I,t}$ is time-t forward price,[52] and in the last equality we have employed the covered interest rate parity (CIP) $F_{J/I,t}e^{-r_J dt} = EX_{J/I,t}e^{-r_I dt}$. Because the above net-zero currency strategy underlying return $CT^I_{-B/+L,t+dt}$ can be decomposed into two net-zero strategies (the first strategy borrows I and lends L, the second borrows B and lends I), both denominated in currency I, we have,

$$CT^I_{-B/+L,t+dt} = RX^I_{-I/+L,t+dt} - RX^I_{-I/+B,t+dt} = e^{rx^I_{-I/+L,t+dt}} - e^{rx^I_{-I/+B,t+dt}}. \tag{29}$$

This identifies a relationship between simple and log returns on carry trades.[53] Substituting (28) into above relationship also reestablishes (4). It then follows an important observation that the mean of the simple return $CT^I_{-B/+L,t+dt}$ (but not the mean of log returns $rx^I_{-I/+L,t+dt}$, $rx^I_{-I/+B,t+dt}$) captures the risk-based compensation on the strategy to investors I (see also Footnote 19 in the main text),

$$E_t[CT^I_{-B/+L,t+dt}] = -Cov_t\left[\frac{M_{It+dt}}{M_{It}}, CT^I_{-B/+L,t+dt}\right] \neq E_t[rx^I_{-I/+L,t+dt}] - E_t[rx^I_{-I/+B,t+dt}].$$

[52]Similar to the convention of the spot exchange rate $EX_{J/I,t}$ (2), the currency forward contract initiated at time t allows the long party to buy one unit of currency I at the price of $F_{J/I,t}$ units of currency J at time $t+dt$.

[53]In particular, when the funding currency is also the denomination currency, $B \equiv I$, relationship (29) simplifies to $CT^I_{-I/+L,t+dt} = \exp\left(rx^I_{-I/+L,t+dt}\right) - 1$.

Consequently, the simple return $CT^I_{-B/+L,t+dt}$ and its mean are the appropriate study objects of the risk-based pricing of carry trade strategies in our paper.

We can also consider a net-zero strategy denominated in currency I, of notional initial value of unit of that currency, which borrows a set of currencies $B \in \mathcal{B}$ with corresponding weights satisfying $\sum_{B\in\mathcal{B}} \theta_B = 1$, and lends currencies $L \in \mathcal{L}$ with $\sum_{L\in\mathcal{L}} \theta_L = 1$. The realized excess return of this strategy on currency portfolio is, (again using (1) and Ito's lemma),

$$CT^I_{-\mathcal{B}/+\mathcal{L},t+dt} = \frac{M_{I,t}}{M_{I,t+dt}} \times \left[\sum_{L\in\mathcal{L}} \theta_L \frac{M_{L,t+dt}}{M_{L,t}}(1+r_L dt) - \sum_{B\in\mathcal{B}} \theta_B \frac{M_{B,t+dt}}{M_{B,t}}(1+r_B dt)\right]$$

$$= \eta_I^T \left(\sum_{B\in\mathcal{B}} \theta_B \eta_B - \sum_{L\in\mathcal{L}} \theta_L \eta_L\right) dt + \left(\sum_{L\in\mathcal{B}} \theta_B \eta_B^T - \sum_{L\in\mathcal{L}} \theta_L \eta_L^T\right) dZ_t,$$

which yields (6).

C.2 Proof of Proposition 1

First, if a strategy is net-zero investment when denominated in a currency, it must also be net-zero investment when denominated in any other currencies. Thus, without loss of generality we consider a net-zero strategy denominated in currency I, for the period $(t, t+dt)$, earning the realized excess return of $R_{I,t+dt} = \mu_I dt + \sigma^T dZ_t$. We conventionally fix the strategy's initial (at t) notional value to equal one unit of currency I. so that the realized payoff of this strategy to investor I at $t+dt$ is also $R_{I,t+dt}$. Using the exchange rate (2), assuming no friction and no arbitrage in FX markets, the realized payoff to investor J (i.e., denominated in currency J) is,

$$P^J_{t+dt} = \frac{M_{I,t+dt}}{M_{J,t+dt}} R_{I,t+dt} = \frac{M_{I,t}}{M_{J,t}} \frac{M_{I,t+dt}/M_{I,t}}{M_{J,t+dt}/M_{J,t}} R_{I,t+dt} = \frac{M_{I,t}}{M_{J,t}} \left\{\left[\mu_I + \left(\eta_J^T - \eta_I^T\right)\sigma\right] dt + \sigma^T dZ_t\right\}.$$

where we have employed (1) and Ito's lemma. Since the strategy has initial notional value of $\frac{M_{I,t}}{M_{J,t}}$ in units of currency J, its net realized return to investor J reads,

$$R_{J,t+dt} = \frac{P^J_{t+dt}}{M_{I,t}/M_{J,t}} = \mu_J dt + \sigma^T dZ_t, \quad \text{with} \quad \mu_J \equiv \mu_I + \left(\eta_J^T - \eta_I^T\right)\sigma.$$

Clearly, $R_{I,t+dt}$ and $R_{J,t+dt}$ have identical innovations, so any net-zero strategy is a diffusion-invariant investment.

Second, in the other way around, to show that any diffusion-invariant strategy is a net-zero investment, it suffices to show that any investment requiring initial capitals is not a diffusion-invariant strategy. So again let $\overline{R}_{I,t+dt} = \overline{\mu}_I dt + \overline{\sigma}^T dZ_t$ be the realized net return of an investment (denominated in a generic currency I) of one unit of currency I in net initial capital conventionally. The investment's realized payoff at $t + dt$ in unit of any other currency J is $\frac{M_{I,t+dt}}{M_{J,t+dt}}\left(1 + \overline{R}_{I,t+dt}\right)$. Thus, the strategy's realized net return to investor J is,[54]

$$\overline{R}_{J,t+dt} = \frac{\frac{M_{I,t+dt}}{M_{J,t+dt}}\left(1 + \overline{\mu}_I dt + \overline{\sigma}^T dZ_t\right) - \frac{M_{I,t}}{M_{J,t}}}{\frac{M_{I,t}}{M_{J,t}}} = \overline{\mu}_J dt + \left(\overline{\sigma}^T + \eta_J^T - \eta_I^T\right) dZ_t.$$

Clearly, $\overline{R}_{I,t+dt}$ and $\overline{R}_{J,t+dt}$ generally do not have identical innovations, except when $\eta_J = \eta_I$ (or exchange rate between currencies I and J is locally deterministic). Thus any diffusion-invariant investment is a net-zero strategy ■

C.3 Details on Principal Component Analysis

We begin with the innovations $X_{J/I,t} \equiv \left(\eta_{J,t}^T - \eta_{I,t}^T\right) dZ_t$ in exchange rate growths (3),

$$X_{J/I,t} = \sum_i^n dZ_{i,t}(\eta_{J,i} - \eta_{I,i}) = \sum_i^n dZ_{i,t}\eta_{J/I,i}; \quad t \in [0, s]; \quad \forall J \in \{1, N\}, \tag{30}$$

$$\text{where } \eta_{J/I} \text{ denotes vector:} \quad \eta_{J/I} \equiv (\eta_J - \eta_I) \in \mathbf{R}^n; \quad \forall J \in \{1, N\},$$

We arrange these N *demeaned* time series into N columns of a matrix $X = [X_{1/I}; \ldots; X_{N/I}]$ of dimension $s \times N$, where s is the number of observations in each exchange rate time series, $dZ_{i,t}$ is normally distributed random variable with mean zero and variance dt, and $\frac{1}{dt}$ is the number of observations for each time series per year and $s \times dt$ is the length of our time series in years.[55] Note

[54] Where $\overline{\mu}_J = \overline{\mu}_I + r_J - r_I + \left(\eta_J^T - \eta_I^T\right)(\eta_J + \overline{\sigma})$.

[55] $X_{J/I}$ is a $s \times 1$ column vector, η_J is a $n \times 1$ column vector for each $J \in \{1, \ldots, N\}$, dZ_t is a $n \times 1$ column vector for each $t \in \{1, \ldots, s\}$.

that each column of X has empirical zero mean, as required by PCA,

$$X \equiv \begin{bmatrix} X_{1/I,1} & & X_{J/I,1} & & X_{N/I,1} \\ \vdots & & \vdots & & \vdots \\ X_{1/I,t} & \ldots & X_{J/I,t} & \ldots & X_{N/I,t} \\ \vdots & & \vdots & & \vdots \\ X_{1/I,s} & & X_{J/I,s} & & X_{N/I,s} \end{bmatrix} =$$

$$\begin{bmatrix} dZ_{1,1} & & dZ_{j,1} & & dZ_{n,1} \\ \vdots & & \vdots & & \vdots \\ dZ_{1,t} & \ldots & dZ_{j,t} & \ldots & dZ_{n,t} \\ \vdots & & \vdots & & \vdots \\ dZ_{1,s} & & dZ_{j,s} & & dZ_{n,s} \end{bmatrix} \times \begin{bmatrix} \eta_{1/I,1} & & \eta_{J/I,1} & & \eta_{N/I,1} \\ \vdots & & \vdots & & \vdots \\ \eta_{1/I,j} & \ldots & \eta_{J/I,j} & \ldots & \eta_{N/I,j} \\ \vdots & & \vdots & & \vdots \\ \eta_{1/I,n} & & \eta_{J/I,n} & & \eta_{N/I,n} \end{bmatrix} \equiv dZ \times \boldsymbol{\eta}. \qquad (31)$$

where each element of $s \times n$ matrix dZ is normally distributed random variable with mean zero and variance dt, $\frac{1}{dt}$ is the number of observations for each time series per year, $s \times dt$ is the length of our time series in years, and the definitions for $n \times N$ matrix $\boldsymbol{\eta}$ is self-evident. Since the sum of each column of X is zero, the symmetric matrix $X^T X$ is proportional to the empirical covariance matrix of the exchange rate fluctuations represented by $n \times N$ matrix X,

$$[X^T X]_{JK} = \sum_t^n X_{J/I,t} X_{K/I,t} = s \operatorname{Cov}\left[X_{J/I}\,,\, X_{K/I}\right], \qquad \forall J, K \in \{1, \ldots, N\}.$$

In the PCA, we solve for the eigenvalues and eigenstates of this $N \times N$ empirical covariance matrix $X^T X$ to identify and sort out the most important risks in FX markets. Because $X^T X$ is symmetric, it can be diagonalized by an $N \times N$ orthogonal matrix W (that is $W^T W = W W^T = \mathbf{1}_{N \times N}$): $W^T \left[X^T X\right] W = \mathrm{Diag}[\lambda_1; \ldots; \lambda_N]$. Elements and columns of orthogonal matrix W are respectively referred to as *loadings* and loading vectors. We next define the $n \times N$ *score* matrix Π as follows,

$$\Pi \equiv XW \Longrightarrow \Pi^T \Pi = \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \lambda_N \end{bmatrix}.$$

The columns of the score matrix Π are commonly referred to as *principal component* (PC) vectors. Obviously, the PCs, or score vectors, are linear combinations of the original exchange rate fluctuation data in X in such a way that (i) score vectors are pair-wise orthogonal, and (ii) empirical variance of scores in J-th PC[56] is the corresponding eigenvalue λ_J, or $\sum_K \Pi^2_{KJ} = \lambda_J$. Hence when we arrange the eigenvalues in descending order, $\lambda_1 \geq \ldots \lambda_N$, the PCs exhibit the portions of exchange rate fluctuations also in the descending order. Specifically, J-th PC captures a fraction $\frac{\lambda_J}{\sum_K^N \lambda_K}$ of the total variations in the original exchange rate data X.

Since the shocks in matrix dZ are uncorrelated, we have $dZ^T dZ = s \times dt \times \mathbf{1}_{n\times n}$, we have $X^T X = (dZ\,\boldsymbol{\eta})^T (dZ\,\boldsymbol{\eta}) = s \times dt \times \boldsymbol{\eta}^T \boldsymbol{\eta}$. The orthogonal matrix W (that diagonalizes the matrix $X^T X$ must also diagonalize the symmetric matrix $\boldsymbol{\eta}^T \boldsymbol{\eta}$,

$$\underbrace{W^T \left[X^T X\right] W}_{\Pi^T \Pi} = s \times dt \times \underbrace{W^T \left[\boldsymbol{\eta}^T \boldsymbol{\eta}\right] W}_{\overline{\boldsymbol{\eta}}^T \overline{\boldsymbol{\eta}}} = \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \lambda_N \end{bmatrix}. \tag{32}$$

Let us now define the *principal price-of-risk vectors* (PPoRs) as columns of matrix $\overline{\boldsymbol{\eta}} \equiv \boldsymbol{\eta} W$, or,

$$\begin{bmatrix} \overline{\eta}_{1/I,1} & & \overline{\eta}_{J/I,1} & & \overline{\eta}_{N/I,1} \\ \vdots & & \vdots & & \vdots \\ \overline{\eta}_{1/I,j} & \ldots & \overline{\eta}_{J/I,j} & \ldots & \overline{\eta}_{N/I,j} \\ \vdots & & \vdots & & \vdots \\ \overline{\eta}_{1/I,n} & & \overline{\eta}_{J/I,n} & & \overline{\eta}_{N/I,n} \end{bmatrix} \equiv \begin{bmatrix} \eta_{1/I,1} & & \eta_{J/I,1} & & \eta_{N/I,1} \\ \vdots & & \vdots & & \vdots \\ \eta_{1/I,j} & \ldots & \eta_{J/I,j} & \ldots & \eta_{N/I,j} \\ \vdots & & \vdots & & \vdots \\ \eta_{1/I,n} & & \eta_{J/I,n} & & \eta_{N/I,n} \end{bmatrix} \times \begin{bmatrix} W_{1,1} & \ldots & W_{1,N} \\ \vdots & \ddots & \vdots \\ W_{N,1} & \ldots & W_{N,N} \end{bmatrix}$$

so that (32) implies the orthogonality among PPoRs (defined as the column vectors of the matrix $\overline{\eta}$),

$$\begin{bmatrix} \overline{\eta}_{1/I,1} & & \overline{\eta}_{J/I,1} & & \overline{\eta}_{N/I,1} \\ \vdots & & \vdots & & \vdots \\ \overline{\eta}_{1/I,j} & \ldots & \overline{\eta}_{J/I,j} & \ldots & \overline{\eta}_{N/I,j} \\ \vdots & & \vdots & & \vdots \\ \overline{\eta}_{1/I,n} & & \overline{\eta}_{J/I,n} & & \overline{\eta}_{N/I,n} \end{bmatrix}^T \times \begin{bmatrix} \overline{\eta}_{1/I,1} & & \overline{\eta}_{J/I,1} & & \overline{\eta}_{N/I,1} \\ \vdots & & \vdots & & \vdots \\ \overline{\eta}_{1/I,j} & \ldots & \overline{\eta}_{J/I,j} & \ldots & \overline{\eta}_{N/I,j} \\ \vdots & & \vdots & & \vdots \\ \overline{\eta}_{1/I,n} & & \overline{\eta}_{J/I,n} & & \overline{\eta}_{N/I,n} \end{bmatrix} = \frac{1}{s \times dt} \times \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \lambda_N \end{bmatrix}.$$

This matrix equation is equivalent to the system in (11).

Because carry trades' realized excess returns exhibit the diffusion-invariant feature (Proposition

[56] Because each column of matrix X is demeaned, the sum of scores in each column of matrix Π (which is also a PC vector) is also zero, $\sum_t \Pi_{t,J} = \sum_t \sum_K X_{t,K} W_{K,J} = \sum_K W_{K,J} \left(\sum_t X_{t,K}\right) = 0$.

1), the return differentials across denomination currencies always is the additive combination of term of the form $\eta^T_{J/I}\eta_{H/I}$, for which PCA is handy. Indeed, for any $J, H \in \{1, \ldots, N\}$, using the relationship $\eta = \overline{\eta} W^T$ and the orthogonality (32) of the PPoRs $\{\overline{\eta}_{J/I}\}$ obtains,

$$\eta^T_{J/I}\eta_{H/I} = \sum_i^n \eta_{J/I,i}\eta_{H/I,i} = \sum_i^n \left(\sum_K^N \overline{\eta}_{K/I,i}[W^T]_{K,J}\right)\left(\sum_L^N \overline{\eta}_{L/I,i}[W^T]_{L,H}\right)$$

$$= \sum_{K,L}^N W_{J,K}W_{H,L}\sum_i^n \overline{\eta}_{K/I,i}\overline{\eta}_{L/I,i} = \frac{1}{s \times dt}\sum_K \lambda_K W_{J,K}W_{H,K}$$

Next using (4), the above result immediately yields the carry trade's realized (as well as expected) return differential (recall the notation (30), $\eta_{J/I} \equiv \eta_J - \eta_I$),

$$CT^J_{-B/+L,t+dt} - CT^H_{-B/+L,t+dt} = ECT^J_{-B/+L,t} - ECT^H_{-B/+L,t} = \left(\eta^T_J - \eta^T_H\right)(\eta_B - \eta_L)$$

$$= \left(\eta^T_{J/I} - \eta^T_{H/I}\right)\left(\eta_{B/I} - \eta_{L/I}\right) = \frac{1}{s \times dt}\sum_K \lambda_K \left(W_{J,K}W_{B,K} - W_{J,K}W_{L,K} - W_{H,K}W_{B,K} + W_{H,K}W_{L,K}\right)$$

$$= \frac{1}{s \times dt}\sum_K \lambda_K \left(W_{J,K} - W_{H,K}\right)\left(W_{B,K} - W_{L,K}\right). \quad (33)$$

This proves (13).

The $N \times F$ matrix V in (20) is defined as follows,

$$V \equiv -\frac{1}{s \times dt}\widehat{W}\,\mathrm{Diag}\left[\lambda_1, \ldots, \lambda_F\right] = -\frac{1}{s}\begin{bmatrix} W_{1,1} & \ldots & W_{1,F} \\ \vdots & \ddots & \vdots \\ W_{N,1} & \ldots & W_{N,F} \end{bmatrix} \times \begin{bmatrix} \lambda_1 & \ldots & 0 \\ 0 & \ddots & 0 \\ 0 & \ldots & \lambda_F \end{bmatrix}. \quad (34)$$

Since the columns of W (or $\widehat{W}$) are orthonormal, $V^T V$ is a diagonal matrix, $V^T V = \frac{1}{s^2 dt^2}\mathrm{Diag}\left[\lambda_1^2, \ldots, \lambda_F^2\right]$ and coefficients of vector $\widehat{\beta}^I$ in (20) have the following explicit expressions,

$$\begin{bmatrix} \widehat{\beta}^I_1 \\ \vdots \\ \widehat{\beta}^I_N \end{bmatrix} = -s \times dt \begin{bmatrix} \frac{1}{\lambda_1} & \ldots & 0 \\ 0 & \ddots & 0 \\ 0 & \ldots & \frac{1}{\lambda_F} \end{bmatrix} \begin{bmatrix} W_{1,1} & \ldots & W_{1,F} \\ \vdots & \ddots & \vdots \\ W_{N,1} & \ldots & W_{N,F} \end{bmatrix}^T \begin{bmatrix} ECT^I_{-I/+1,t} \\ \vdots \\ ECT^I_{-I/+N,t} \end{bmatrix} \quad (35)$$

Similar to (18), we can also have an expression for the covariance between countries' SDF growths

$$\widehat{\eta}_J^T \widehat{\eta}_H = \frac{1}{s \times dt} \sum_K^F \lambda_K \widehat{\beta}_K^J \widehat{\beta}_K^H = ||\widehat{\eta}_I||^2 + \frac{1}{s \times dt} \sum_K^F \left(\widehat{\beta}_K^I W_{H,K} + \widehat{\beta}_K^I W_{J,K} + W_{J,K} W_{H,K} \right), \quad \forall J, H.$$

C.4 Details on Constructing Prices of Principal Risks in FX Markets

While we do not observe N PPoRs $\overline{\eta}_{J/I}$,[57] we can construct a portfolio to mimic any J-th PC, $\Pi_{J/I} = \overline{\eta}_{J/I}^T dZ_t$ (12), which loads on the respective J-th PPoR $\overline{\eta}_{J/I}$. The mimicking is in the *conditional* sense that the innovations in the portfolio's realized return are identical (up to a non-material multiplicative factor) to the innovations in the corresponding PC. In light of the realized return on carry trade portfolio (6), the combination of equations (4) and (9) implies that,

$$\begin{aligned} F_t^J \equiv \sum_K \theta_K^J CT_{-K/+I,t+dt}^H &= \sum_K \theta_K^J \eta_H^T (\eta_K - \eta_I)\, dt + \sum_K \theta_K^J dZ_t^T (\eta_K - \eta_I), \\ &\equiv \eta_H^T \left(\breve{\eta}^J - \eta_I \right) dt + dZ_t^T \left(\breve{\eta}^J - \eta_I \right), \quad \forall H, \end{aligned} \tag{36}$$

$$\text{where:} \quad \theta_K^J \equiv \frac{W_{KJ}}{\sum_{K'} W_{K'J}}, \qquad \sum_K \theta_K^J = 1, \qquad \breve{\eta}^J \equiv \sum_K \theta_K^J \eta_K.$$

The net-zero portfolio F^J of lending currency I, and borrowing a fraction θ_K^J in each of the other currencies, $K \neq I$, replicates J-th PC conditionally. Indeed, the innovations in F^J matches those of $\Pi_{J/I}$ in (10) up to a non-material multiplicative coefficient. Several observations are in order here.

First, because J-th replicating portfolio F_J is a net-zero investment, the matching between F^J and J-th principal risk is observed under any denomination currency H, by virtue of Proposition 1. Second, the principal prices of risks $\overline{\eta}_{J/I}$ are common to all currencies. Therefore, they are not characteristic of any specific currency or investor, but of global FX markets. The country-specific prices of risks $\overline{\eta}_J$ however can be constructed on the basis of $\{\overline{\eta}_{J/I}\}$ (see (16) below). Third, the mimicking portfolio F^J is pre-deterministic in the sense that its composition is known at time t, while its return's innovations, engineered to match the innovations of J-th principal risk, are realized at $t + dt$. Fourth, because the PCs are orthogonal to one another by construction, conditionally, J-th mimicking portfolio bears exclusively the exchange-rate risk associated with J-th PC (J-th

[57] We do not observe n components of each price-of-risk vectors $\eta_{J/I}$ or $\overline{\eta}_{J/I}$. In the PCA, instead we observe the loadings $\{W_{J,H}\}$ (as well as the principal eigenvalues $\{\lambda_J\}$). The relationships in (9) show that observing W is not sufficient to observe neither the original price-of-risk vectors $\eta_{J/I}$, nor the rotated $\overline{\eta}_{J/I}$. Instead, we observe the scores $dZ_t^T \overline{\eta}_{J/I}$ (12).

principal risk of FX markets),[58] and its conditional expected return fully reflects the respective compensation. This compensation, or the drift term in (36), however, depends on the currency of denomination H, in form of a correlation. Fifth, risk factors in FX markets, in the descending order of importance, can be faithfully represented by the returns of these mimicking portfolios. Finally, weights $\{\theta_K^J\}$ for some currencies K can be negative, in which case we actually borrow these currencies K in the portfolio mimicking J-th principal risk.

C.5 Currency Returns across Denominations: A Numerical Illustration

This appendix presents a simple numerical illustration showing that the well-known sorting pattern of carry trade returns to US investors on short-term interest rates (or the forward premium puzzle) is not sufficient to imply that a given carry trade strategy offers higher returns when denominated in currencies of lower interest rates.

For simplicity of the illustration, we consider 4 countries: Japan, US, Norway and New Zealand. We calibrate our model so that interest rates are consistent with the data for 1999-2014 (Table E.5), $r_{JP} - r_{US} = -2.4\%$, $r_{NO} - r_{US} = 1.3\%$, $r_{NZ} - r_{US} = 2.6\%$. Ranking currencies according to interest rates, we get $r_{JP} < r_{US} < r_{NO} < r_{NZ}$. On average, US investors earn higher returns on carry strategies of lending currencies of higher interest rates. Again we choose to match average carry trade returns denominated in USD (borrowing USD and lending JPY, NOK, NZD) to be consistent with the data for 1999-2014 (Table E.5),

$$\begin{aligned} ECT^{US}_{-US/+JP} &= \eta^T_{US}(\eta_{US} - \eta_{JP}) &= -1.2\%, \\ ECT^{US}_{-US/+NO} &= \eta^T_{US}(\eta_{US} - \eta_{NO}) &= 3.5\%, \\ ECT^{US}_{-US/+NZ} &= \eta^T_{US}(\eta_{US} - \eta_{NZ}) &= 6.5\%. \end{aligned} \quad (37)$$

By construction, therefore, the interest rates and carry trade returns sort well in our example, $ECT^{US}_{-US/+JP} < ECT^{US}_{-US/+NO} < ECT^{US}_{-US/+NZ}$. We observe that this sorting of carry trade returns to US investors is only informative about (relative) country-specific prices of risks priced by US investors.[59] In contrast, country-specific prices of risks, which are not spanned by components of vector η_{US}, are not constrained by carry trade returns to US investors. The prices of these "US-orthogonal" risks thus have the potential to void any pattern between returns (on a given carry

[58] We recall that J-th PC depicts and explains the J-th most important variation in the exchange rate data.

[59] That is, carry trade average returns to US investors have implication only on the projections of $(\eta_I - \eta_J)$ onto η_{US}.

trade strategy) denominated across different currencies and the respective interest rates associated with those currencies. Exploiting this flexibility, in a two-dimensional diffusion setting, the following two (hypothetical) numerical configurations (i) precisely produce the carry trade returns (37) to US investors, but (ii) one configuration generates a larger return (on the same strategy of borrowing JPY and lending NZD) to US than to Norway investors while the second configuration generates the opposite cross-denomination pattern.[60] Thus, our example illustrates that in a simple 2-factor model which is able to generate realistic carry trade returns to US investors, our cross-denomination pattern may or may not hold. Accordingly, our empirical regularity imposes additional restrictions on the factor structure.

First configuration: The following country-specific prices of risks,

$$\eta_{JP} = \begin{bmatrix} 34\% \\ 13\% \end{bmatrix}, \quad \eta_{US} = \begin{bmatrix} 30\% \\ 0 \end{bmatrix}, \quad \eta_{NO} = \begin{bmatrix} 18\% \\ 11\% \end{bmatrix}, \quad \eta_{NZ} = \begin{bmatrix} 8\% \\ 6\% \end{bmatrix},$$

generate both (i) carry trade returns (37), and (ii) *lower* returns to Norway investors than to US investors (on the strategy of borrowing JPY and lending NZD),[61]

$$ECT^{NO}_{-JP/+NZ} = \eta^T_{NO} (\eta_{JP} - \eta_{NZ}) = 5.5\% < ECT^{US}_{-JP/+NZ} = \eta^T_{US} (\eta_{JP} - \eta_{NZ}) = 7.8\%. \quad (38)$$

Accordingly, carry trade returns differences across denominations sort well with interest rate differentials and the 2-factor model specification is able to generate our empirical cross-denomination pattern (Figure 1). Moreover, we compute the bilateral exchange rate volatilities $Vol\left(\frac{dEX_{J/I,t}}{EX_{J/I,t}}\right) = (\eta_{J,t} - \eta_{I,t})^T (\eta_{J,t} - \eta_{I,t})\, dt, \forall J, I.$[62] Consistent with our empirical finding (Figure 2), the bilateral exchange rate volatilities sort well with absolute interest rate differentials; the correlation between these two characteristics is 74%.

Second configuration: The following country-specific prices of risks,

$$\eta_{JP} = \begin{bmatrix} 34\% \\ 0 \end{bmatrix}, \quad \eta_{US} = \begin{bmatrix} 30\% \\ 0 \end{bmatrix}, \quad \eta_{NO} = \begin{bmatrix} 18\% \\ 20\% \end{bmatrix}, \quad \eta_{NZ} = \begin{bmatrix} 8\% \\ -30\% \end{bmatrix},$$

generate both (i) carry trade returns (37), and (ii) *higher* returns to Norway investors than to US

[60] Note that in any possible configuration Japanese investors always earn the highest and New Zealand investors the lowest expected returns on the carry trade -JPY/+NZD.

[61] Moreover, $ECT^{JP}_{-JP/+NZ} = \eta^T_{JP} (\eta_{JP} - \eta_{NZ}) = 9.6\%$ and $ECT^{NZ}_{-JP/+NZ} = \eta^T_{NZ} (\eta_{JP} - \eta_{NZ}) = 2.5\%$.

[62] There are six bilateral exchange rates for the four currencies.

investors (on the strategy of borrowing JPY and lending NZD),[63]

$$ECT^{NO}_{-JP/+NZ} = \eta^{T}_{NO}\left(\eta_{JP} - \eta_{NZ}\right) = 10.7\% > ECT^{US}_{-JP/+NZ} = \eta^{T}_{US}\left(\eta_{JP} - \eta_{NZ}\right) = 7.8\%. \quad (39)$$

Clearly, the return pattern in (39) is opposite to that in (38). Thus, this second model specification is inconsistent with our empirical cross-denomination pattern (Figure 1). Again, we compute the bilateral exchange rate volatilities $Vol\left(\frac{dEX_{J/I,t}}{EX_{J/I,t}}\right) = \left(\eta_{J,t} - \eta_{I,t}\right)^{T}\left(\eta_{J,t} - \eta_{I,t}\right)dt,\ \forall J, I$. This time, bilateral exchange rate volatilities appear independent of absolute interest rate differentials as their correlation is only 4%, which is inconsistent with our empirical finding (Figure 2).

Our numerical illustration demonstrates that, the pattern of the forward premium puzzle to US investors alone is not sufficient to infer unambiguously whether the returns (on a carry trade strategy) denominated across currencies systematically sort with respective interest rates. Moreover, the relationship between bilateral exchange rate volatilities and absolute differences in interest rates is also ambiguous and crucially depends on the factor structure. Therefore, empirically documenting carry trade returns across currency denominations provides us with novel and non-redundant restrictions to discipline international asset pricing models. The exception is the case of a single factor model, where our cross-denomination pattern does not imply any additional restrictions. However, a single factor model is restrictive and it is unlikely that FX markets are explained by a single factor. Indeed, a large literature of empirical research argues that we need multiple factors to explain both the time-series and the cross-sectional variation in carry trade returns (see our paragraph on the related literature for a discussion of many important factors in FX markets, as well as our own empirical results suggesting that we need (at least) two factors to explain the empirical facts studied in this paper).

D Additional Figures

This Appendix provides additional figures on the differences in expected returns and Sharpe ratios across currency denominations. Figure 9 shows a striking negative relationship with a correlation of 91%: investors in countries of lower interest rates tend to earn substantially higher returns on the same carry trade strategy. Average annual returns range from 6% for a New Zealand investor to almost 9% for a Japanese investor for the period 19992014.

[63] Moreover, $ECT^{JP}_{-JP/+NZ} = \eta^{T}_{JP}\left(\eta_{JP} - \eta_{NZ}\right) = 8.8\%$ and $ECT^{NZ}_{-JP/+NZ} = \eta^{T}_{NZ}\left(\eta_{JP} - \eta_{NZ}\right) = -6.9\%$.

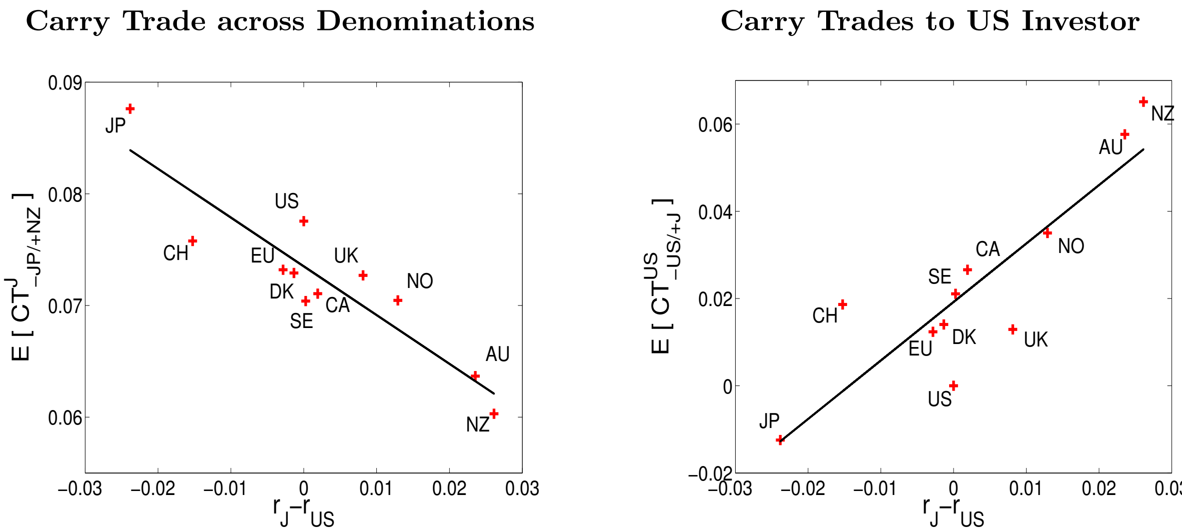

Figure 9: Left Panel: Negative relationship between the average of the carry trade return of borrowing JPY and investing in NZD from the perspective of an investor in country J and the time series average of the interest rate differential between country J and the USA. Right Panel: Positive relationship between the average of the carry trade return of borrowing USD and investing in currency J from the perspective of a US investor and the time series average of the interest rate differential between country J and the USA. Red crosses indicate data points and the black line is the best linear fit of the data according to an OLS estimation. Sample: 1999-2014. For statistical test of the illustrated relations see Tables E.2 and 5.

In Figure 10, instead of focusing on the profitable carry trade -JPY/+NZD, we explore how the High minus Low interest rate carry trade strategy (HML) studied by Lustig and Verdelhan (2007, 2011) differs across currency denominations. The results are similar to the results in section 2.1: investors in low interest rates countries tend to earn substantially higher returns and Sharpe ratios than investors in high interest rate countries. The correlation in the left hand side picture is -91% and in the right hand side picture -92%.

Carry Trade across Denominations

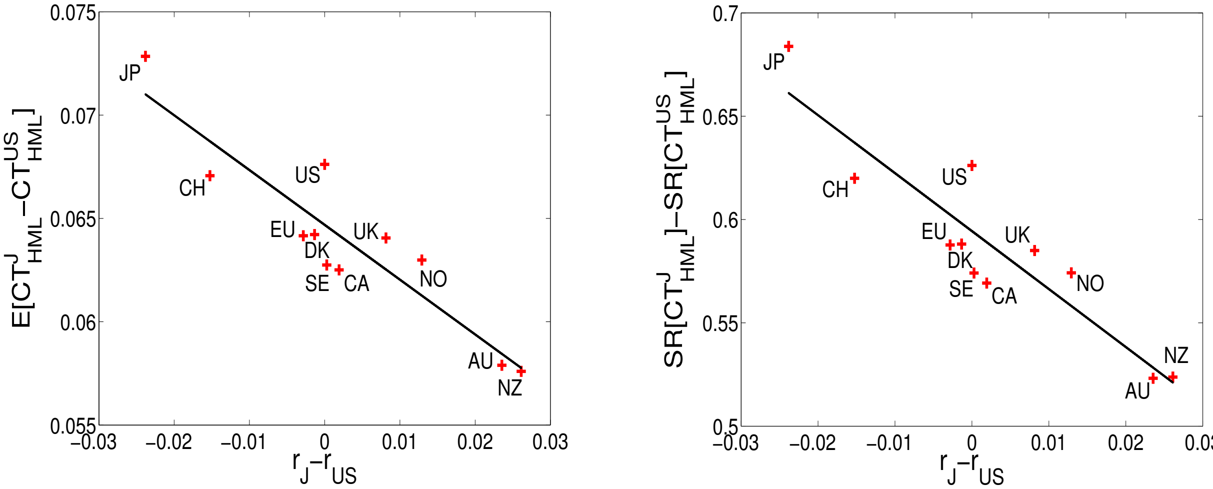

Figure 10: Negative relationship between the time series average (Left Panel) and Sharpe ratio (Right Panel) of the HML carry trade return studied by Lustig and Verdelhan (2007, 2011) from the perspective of an investor in country J and the time series average of the interest rate differential between country J and the USA. Red crosses indicate data points and the black line is the best linear fit of the data according to an OLS estimation. Sample: 1999-2014.

E Additional Tables

This Appendix presents Tables of further empirical results referenced in the main text.

Table E.1: Cross-country Regressions on SDF Volatility $||\eta_J||$

	Panel A: 1999-2014				
(1)	(2)	(3)	(4)	(5)	(6)
	Ave Interest Rate Diff $r_J - r_{US}$	Average CT -US/+J to US investor	Sharpe Ratio CT -US/+J to US investor	Average CT -JP/+NZ to investor J	Sharpe Ratio CT -JP/+NZ to investor J
$\|\|\eta_I\|\|$	-0.35^{***}	-0.61^{***}	-4.67^{***}	0.18^{***}	1.10^{***}
	(6.06)	(39.22)	(15.93)	(8.86)	(8.87)
constant	Yes	Yes	Yes	Yes	Yes
R^2	79%	99%	96%	89%	89%

	Panel B: 1984-2014				
(1)	(2)	(3)	(4)	(5)	(6)
	Ave Interest Rate Diff $r_J - r_{US}$	Average CT -US/+J to US investor	Sharpe Ratio CT -US/+J to US investor	Average CT -JP/+NZ to investor J	Sharpe Ratio CT -JP/+NZ to investor J
$\|\|\eta_I\|\|$	-0.59^{***}	-0.67^{***}	-5.04^{***}	0.17^{***}	1.12^{***}
	(4.45)	(31.88)	(13.98)	(3.82)	(3.82)
constant	Yes	Yes	Yes	Yes	Yes
R^2	66%	99%	95%	59%	59%

Notes: OLS regression $Y_J = \alpha + \beta||\eta_J|| + \epsilon_J$ to examine the relationship between quantity Y and the SDF volatility in country J denoted by $||\eta_J||$ (equation (18)). Variable Y is: Y = average interest rate differential $r_J - r_{US}$ (Column 2), Y = average carry trade return $CT^{US}_{-US/+J}$ (Column 3), Y = Sharpe ratio of $CT^{US}_{-US/+J}$ (Column 4), Y = average carry trade return $CT^{J}_{-JP/+NZ}$ (Column 5), and Y = Sharpe ratio of $CT^{J}_{-JP/+NZ}$ (Column 6). Panel A uses data for period 1999-2014, Panel B for period 1984-2014. Values in parentheses below each regression coefficient are t-statistics. We have 11 observations. 10%, 5%, 1% significance levels of two sided t-statistic are indicated by *, ** and ***, respectively.

Table E.2: Cross-country Regressions on Interest Rate Differential $r_J - r_{US}$

	Panel A: 1999-2014			
(1)	(2)	(3)	(4)	(5)
	Average CT -US/+J to US investor	Sharpe Ratio CT -US/+J to US investor	Average CT -JP/+NZ to investor J	Sharpe Ratio CT -JP/+NZ to investor J
$r_J - r_{US}$	1.34***	10.26***	−0.91***	−2.70***
	(5.57)	(5.24)	(7.14)	(7.20)
constant	Yes	Yes	Yes	Yes
R^2	76%	73%	84%	84%

	Panel B: 1999-2014 NBER expansions			
(1)	(2)	(3)	(4)	(5)
	Average CT -US/+J to US investor	Sharpe Ratio CT -US/+J to US investor	Average CT -JP/+NZ to investor J	Sharpe Ratio CT -JP/+NZ to investor J
$r_J - r_{US}$	1.36***	11.15***	−0.44***	−3.29***
	(4.93)	(4.56)	(6.41)	(5.97)
constant	Yes	Yes	Yes	Yes
R^2	71%	67%	80%	78%

	Panel C: 1999-2014 NBER contractions			
(1)	(2)	(3)	(4)	(5)
	Average CT -US/+J to US investor	Sharpe Ratio CT -US/+J to US investor	Average CT -JP/+NZ to investor J	Sharpe Ratio CT -JP/+NZ to investor J
$r_J - r_{US}$	3.45**	2.24	−3.03***	−6.18***
	(2.74)	(0.70)	(3.95)	(5.09)
constant	Yes	Yes	Yes	Yes
R^2	43%	5%	61%	72%

Notes: OLS regression $Y_J = \alpha + \beta\,(r_J - r_{USA}) + \epsilon_J$ to examine the relationship between quantity Y and the interest rate differential between country J and the USA ($r_J - r_{US}$). Variable Y is: Y = average carry trade return $CT^{US}_{-US/+J}$ (Column 2), Y = Sharpe ratio of $CT^{US}_{-US/+J}$ (Column 3), Y = average carry trade return $CT^{J}_{-JP/+NZ}$ (Column 4), and Y = Sharpe ratio of $CT^{J}_{-JP/+NZ}$ (Column 5). Panel A uses all data for period 1999-2014, Panel B only data from periods of NBER expansions for 1999-2014, Panel C only data from periods of NBER contractions for 1999-2014. Values in parentheses below each regression coefficient are t-statistics. We have 11 observations. 10%, 5%, 1% significance levels of two sided t-statistic are indicated by *, ** and ***, respectively.

Table E.3: Cross-country Regressions on Interest Rate Differential $r_J - r_{US}$

	Panel A: 1984-2014			
(1)	(2)	(3)	(4)	(5)
	Average CT -US/+J to US investor	Sharpe Ratio CT -US/+J to US investor	Average CT -JP/+NZ to investor J	Sharpe Ratio CT -JP/+NZ to investor J
$r_J - r_{US}$	0.72^{***}	5.95^{***}	-0.27^{***}	-1.83^{***}
	(3.91)	(3.93)	(6.85)	(6.87)
constant	Yes	Yes	Yes	Yes
R^2	61%	61%	82%	83%

	Panel B: 1984-2014 NBER expansions			
(1)	(2)	(3)	(4)	(5)
	Average CT -US/+J to US investor	Sharpe Ratio CT -US/+J to US investor	Average CT -JP/+NZ to investor J	Sharpe Ratio CT -JP/+NZ to investor J
$r_J - r_{US}$	0.68^{***}	5.81^{***}	-0.28^{***}	-2.06^{***}
	(3.40)	(3.52)	(7.86)	(8.55)
constant	Yes	Yes	Yes	Yes
R^2	54%	55%	86%	88%

	Panel C: 1984-2014 NBER contractions			
(1)	(2)	(3)	(4)	(5)
	Average CT -US/+J to US investor	Sharpe Ratio CT -US/+J to US investor	Average CT -JP/+NZ to investor J	Sharpe Ratio CT -JP/+NZ to investor J
$r_J - r_{US}$	0.16	-5.08	-0.34^{**}	-1.83^{**}
	(0.19)	(1.67)	(2.82)	(2.97)
constant	Yes	Yes	Yes	Yes
R^2	0%	22%	44%	47%

Notes: OLS regression $Y_J = \alpha + \beta\,(r_J - r_{USA}) + \epsilon_J$ to examine the relationship between quantity Y and the interest rate differential between country J and the USA $(r_J - r_{US})$. Variable Y is: Y = average carry trade return $CT^{US}_{-US/+J}$ (Column 2), Y = Sharpe ratio of $CT^{US}_{-US/+J}$ (Column 3), Y = average carry trade return $CT^{J}_{-JP/+NZ}$ (Column 4), and Y = Sharpe ratio of $CT^{J}_{-JP/+NZ}$ (Column 5). Panel A uses all data for period 1984-2014, Panel B only data from periods of NBER expansions for 1984-2014, Panel C only data from periods of NBER contractions for 1984-2014. Values in parentheses below each regression coefficient are t-statistics. We have 11 observations. 10%, 5%, 1% significance levels of two sided t-statistic are indicated by *, ** and ***, respectively.

Table E.4: Exchange Rate Volatility against Interest Rate Differential

	Panel A: 1999-2014		
	All data	Good times	Bad times
(1)	(2)	(3)	(4)
	Volatility of EX I/J growth	Volatility of EX I/J growth	Volatility of EX I/J growth
$\|r_I - r_J\|$	1.39***	1.43***	3.12***
	(7.14)	(7.06)	(3.84)
constant	Yes	Yes	Yes
R^2	49%	48%	21%
	Panel A: 1984-2014		
	All data	Good times	Bad times
(1)	(2)	(3)	(4)
	Volatility of EX I/J growth	Volatility of EX I/J growth	Volatility of EX I/J growth
$\|r_I - r_J\|$	0.93***	0.97***	4.33***
	(5.20)	(5.76)	(4.90)
constant	Yes	Yes	Yes
R^2	33%	38%	31%

Notes: OLS regression $Vol\left(\frac{dEX_{I/J}}{EX_{I/J}}\right) = \alpha + \beta\|r_I - r_J\| + \epsilon_{I,J}$ to examine the relationship between exchange rate volatilities and interest rate differentials for all currency pairs I and J. Panel A uses data for period 1999-2014, Panel B for period 1984-2014. Values in parentheses below each regression coefficient are t-statistics. We have 11 observations. 10%, 5%, 1% significance levels of two sided t-statistic are indicated by *, ** and ***, respectively.

Table E.5: Summary I: SDF Volatility, Interest Rate and $CT^{US}_{-US/+J}$

	Panel A: 1999-2014				
(1)	(2)	(3)	(4)	(5)	(6)
Country J	Volatility of SDF in country J	Ave Interest Rate Diff $r_J - r_{US}$	Average CT -US/+J to US investor	Standard Dev CT -US/+J to US investor	Sharpe Ratio CT -US/+J to US investor
New Zealand	0.50562	0.026116	0.065117	0.13392	0.48624
Australia	0.51904	0.023546	0.05764	0.12936	0.4456
Norway	0.55882	0.012914	0.035022	0.11988	0.29214
Canada	0.56805	0.0019203	0.026598	0.08901	0.29882
Sweden	0.5837	0.00027313	0.021081	0.12214	0.17259
Switzerland	0.58519	-0.015253	0.018632	0.10843	0.17184
Denmark	0.59139	-0.001341	0.014059	0.099524	0.14126
UK	0.59194	0.0081466	0.012919	0.090964	0.14202
Euro	0.59418	-0.0028366	0.012377	0.099238	0.12472
USA	0.60659	N/A	N/A	N/A	N/A
Japan	0.63533	-0.023818	-0.012439	0.10398	-0.11963

	Panel B: 1984-2014				
(1)	(2)	(3)	(4)	(5)	(6)
Country J	Volatility of SDF in country J	Ave Interest Rate Diff $r_J - r_{US}$	Average CT -US/+J to US investor	Standard Dev CT -US/+J to US investor	Sharpe Ratio CT -US/+J to US investor
New Zealand	0.56393	0.041041	0.067604	0.12265	0.55121
Norway	0.60646	0.021695	0.04136	0.11105	0.37245
Australia	0.60832	0.030421	0.04086	0.11654	0.35061
Denmark	0.61094	0.008453	0.037726	0.1025	0.36805
UK	0.61357	0.018859	0.035567	0.09701	0.36663
Sweden	0.62168	0.015584	0.031973	0.11063	0.289
Euro	0.62849	-0.0044802	0.026957	0.10359	0.26024
Switzerland	0.63202	-0.015924	0.025877	0.11409	0.22682
Canada	0.64203	0.0071077	0.01556	0.071642	0.21719
Japan	0.65484	-0.024491	0.010449	0.10737	0.097325
USA	0.66196	N/A	N/A	N/A	N/A

Notes: Columns are sorted according to Column 2. Column 2 shows our estimates of SDF volatilities across countries. Column 2 is sorted in ascending order. Column 3 lists the time series average of the interest rate in country J vs the USA. Column 3, 4, and 5 show the average, standard deviation and Sharpe ratio of the carry trade return of borrowing USD and lending in currency J from the perspective of a US investor.

Table E.6: Summary II: SDF Volatility, Interest Rate and $CT^{J}_{-JP/+NZ}$

	Panel A: 1999-2014				
(1)	(2)	(3)	(4)	(5)	(6)
Country J	Volatility of SDF in country J	Ave Interest Rate Diff $r_J - r_{US}$	Average CT -JP/+NZ to investor J	Standard Dev CT -JP/+NZ to investor J	Sharpe Ratio CT -JP/+NZ to investor J
New Zealand	0.50562	0.026116	0.060308	0.16594	0.36342
Australia	0.51904	0.023546	0.063675	0.16605	0.38346
Norway	0.55882	0.012914	0.07046	0.16543	0.42592
Canada	0.56805	0.0019203	0.071059	0.16536	0.42973
Sweden	0.5837	0.00027313	0.0704	0.16533	0.42582
Switzerland	0.58519	-0.015253	0.075778	0.16512	0.45891
Denmark	0.59139	-0.001341	0.07291	0.16528	0.44114
UK	0.59194	0.0081466	0.072695	0.16542	0.43945
Euro	0.59418	-0.0028366	0.073202	0.1653	0.44284
USA	0.60659	N/A	0.077556	0.16517	0.46954
Japan	0.63533	-0.023818	0.087616	0.16467	0.53207

	Panel B: 1984-2014				
(1)	(2)	(3)	(4)	(5)	(6)
Country J	Volatility of SDF in country J	Ave Interest Rate Diff $r_J - r_{US}$	Average CT -JP/+NZ to investor J	Standard Dev CT -JP/+NZ to investor J	Sharpe Ratio CT -JP/+NZ to investor J
New Zealand	0.56393	0.041041	0.04369	0.15345	0.28472
Norway	0.60646	0.021695	0.05521	0.15288	0.36114
Australia	0.60832	0.030421	0.048438	0.15333	0.31592
Denmark	0.61094	0.008453	0.056945	0.15277	0.37276
UK	0.61357	0.018859	0.05594	0.15287	0.36592
Sweden	0.62168	0.015584	0.054993	0.15283	0.35984
Euro	0.62849	-0.0044802	0.057359	0.15278	0.37544
Switzerland	0.63202	-0.015924	0.059001	0.15267	0.38645
Canada	0.64203	0.0071077	0.053469	0.15288	0.34974
Japan	0.65484	-0.024491	0.067036	0.15236	0.43997
USA	0.66196	N/A	0.057154	0.15275	0.37416

Notes: Columns are sorted according to Column 2. Column 2 shows our estimates of SDF volatilities across countries. Column 2 is sorted in ascending order. Column 3 lists the time series average of the interest rate in country J vs the USA. Column 3, 4, and 5 show the average, standard deviation and Sharpe ratio of the carry trade return of borrowing JPY and lending NZD from the perspective of an investor in country J.

Table E.7: Carry trade returns across denomination currencies vs. interest rates

	Panel A: 1999-2014 NBER expansions			
	(1)	(2)	(3)	(4)
$r_J - r_{US}$	-0.023^{***}	-0.110^{***}	-0.023^{***}	-0.140^{***}
	(4.25)	(20.77)	(3.77)	(26.78)
$\text{Vol}_t(CT)$	-0.617^{***}	-8.202^{***}		
	(17.82)	(15.15)		
Country FE	Yes	No	Yes	No
R-squared	38%	38%	3%	1%
	Panel A: 1999-2014 NBER contractions			
	(1)	(2)	(3)	(4)
$r_J - r_{US}$	0.316^{***}	-0.620^{***}	-3.305^{***}	-1.200^{***}
	(10.39)	(12.42)	(14.66)	(18.26)
$\text{Vol}_t(CT)$	-0.834^{***}	-5.898^{***}		
	(59.05)	(9.94)		
Country FE	Yes	No	Yes	No
R-squared	89%	68%	1%	1%

Notes: FGLS panel regression $CT^{J}_{-JP/+NZ,t+dt} - CT^{US}_{-JP/+NZ,t+dt} = \alpha + \beta_1 (r_{J,t} - r_{US,t}) + \beta_2 \text{Vol}_t(CT) + d_J + \epsilon_{J,t}$ to examine the predictability of next-period carry trade return differential $CT^{J}_{-JP/+NZ,t+dt} - CT^{US}_{-JP/+NZ,t+dt}$ (between currency J's and USD denomination) by the country J's current interest rate differential $(r_{J,t} - r_{US,t})$ (relative to current US interest rate). Panel regression on country index $J \in 1, \ldots, N$ and time index $t \in \{1, \ldots, s\}$. Control variables include the carry trade volatility computed as $\text{Vol}_t(CT) = \frac{1}{N}\sum_J^N \left|CT^{J}_{-JP/+NZ,t}\right|$, and country fixed effect dummy d_J. Panel A uses data for period 1999-2014, Panel B for period 1984-2014. Values in parentheses (below each slope coefficient) are t-statistics of the associated slope coefficients. All reported returns and volatilities are annualized. Values in parentheses below each regression coefficient are t-statistics. 10%, 5%, 1% significance levels of two sided t-statistic are indicated by *, ** and ***, respectively.

Table E.8: Carry trade returns across denomination currencies vs. interest rates

	Panel A: 1984-2014 NBER expansions			
	(1)	(2)	(3)	(4)
$r_J - r_{US}$	-0.019^{***}	-0.059^{***}	-0.017^{***}	-0.066^{***}
	(10.23)	(28.44)	(7.86)	(36.67)
$\text{Vol}_t(CT)$	-0.546^{***}	-6.517^{***}		
	(12.20)	(8.93)		
Country FE	Yes	No	Yes	No
R-squared	36%	33%	4%	1%
	Panel A: 1984-2014 NBER contractions			
	(1)	(2)	(3)	(4)
$r_J - r_{US}$	0.095	-0.433^{***}	-1.319^{***}	-0.943^{***}
	(1.54)	(7.31)	(6.22)	(6.46)
$\text{Vol}_t(CT)$	-0.166^{***}	-3.779^{***}		
	(5.00)	(6.02)		
Country FE	Yes	No	Yes	No
R-squared	27%	47%	1%	1%

Notes: FGLS panel regression $CT^{J}_{-JP/+NZ,t+dt} - CT^{US}_{-JP/+NZ,t+dt} = \alpha + \beta_1 (r_{J,t} - r_{US,t}) + \beta_2 \text{Vol}_t(CT) + d_J + \epsilon_{J,t}$ to examine the predictability of next-period carry trade return differential $CT^{J}_{-JP/+NZ,t+dt} - CT^{US}_{-JP/+NZ,t+dt}$ (between currency J's and USD denomination) by the country J's current interest rate differential $(r_{J,t} - r_{US,t})$ (relative to current US interest rate). Panel regression on country index $J \in 1, \ldots, N$ and time index $t \in \{1, \ldots, s\}$. Control variables include the carry trade volatility computed as $\text{Vol}_t(CT) = \frac{1}{N} \sum_J^N \left| CT^{J}_{-JP/+NZ,t} \right|$, and country fixed effect dummy d_J. Panel A uses data for period 1999-2014, Panel B for period 1984-2014. Values in parentheses (below each slope coefficient) are t-statistics of the associated slope coefficients. All reported returns and volatilities are annualized. Values in parentheses below each regression coefficient are t-statistics. 10%, 5%, 1% significance levels of two sided t-statistic are indicated by *, ** and ***, respectively.